LEARNING TO SOLVE MATHEMATICS PROBLEMS IN GROUP WORK SETTINGS

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ABSTRACT

The use of group-work settings at schools has recently become more popular compared to the individual settings. It might due to the assumption that students need to practice working in groups as various workplaces apparently require collaborative skills. Mathematics is studied by most students worldwide. The study reported in this article aimed at testing if students could learn solving mathematics better in group-work compared to in individual settings. Worked-example instructions to learn novel Arithmetic problems for Seven Graders, part-to-part and part-to-whole comparisons, were developed based on Cognitive Load Theory. The investigation included whether the worked-example instruction provided a powerful tool for learning mathematics in group-work settings compare to problem solving instruction. The results showed that students were benefited from learning in group-work as much as those in individual setting. Moreover, students who were provided worked-example instructions performed significantly better that those who learned solving problems without the worked-example.

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INTRODUCTION

Cognitive Load Theory (CLT) is an instructional design theory developed by John Sweller since 1980s (see Sweller, van Merrienboer & Paas, 1998, Sweller, 2004).This theory is based on human cognitive architecture which can be used to explain how we receive, construct and organise biologically secondary knowledge. Knowledge that is categorised into biologically secondary knowledge is material we intend to learn because it is seen as culturally relatively important knowledge, for examples table manner, handwriting or mathematics formula. Different to biologically primary knowledge is material that we acquire as we evolve over generations, for examples eating, face expressions or collaboration. The distinction of knowledge into these two categories proposed by Geary (2002) is significant because human cognitive architecture processes the knowledge in different manner according while we have prior knowledge relevant to it.

Cognitive load theory follows five principles to describe the construction process of knowledge (Sweller and Sweller, 2006). (1) the information store principle, meaning that human has long term memory to store unlimited amount of information; (2) the borrowing and organising principle, describing that almost all information stored in our long term memory is obtained by borrowing or imitating other’s long term memory; (3) the randomness as genesis principles, explaining that when information is not available to borrow from others, than new information must be generated randomly and followed by tests of effectiveness during problem solving; (4) the narrow limits of change principle, showing the consequence of the limitation of working memory when dealing with new information causes the process in the third principle is often clumsy and failed; (5) the environmental organising and linking principle, when long term memory has sufficient amount of prior knowledge to recognise presented information, the function of working memory can be focused on generating new links between information to gain deeper understanding.

This study used cognitive load theory to examine the effectiveness of learning in group work settings. Group work settings have been recommended to apply in classrooms since arguably students will learn better when they are solving problems by interacting with other students (Johnson & Johnson, 1994, NCTM, 2000). Not only learn the subject matter, students will also learn communication skill which is seen to be important. However, particularly when students are dealing with new information, complex problem solving can cause high cognitive load. Without sufficient prior knowledge, students have to generate random ideas followed by tests of the effectiveness (principle three). As suggested, this causes high cognitive load and it is not effective for learning. Cognitive load theory suggests the use of worked example method to provide students with information to borrow and organise new information (Sweller, 2006). Furthermore, it can be assumed that students in their group may share ideas to assist the organising of to-be learned material, although this does not always happen (Retnowati, Ayres & Sweller, 2010). This learning setting may be more efficient than individualised. To examine these, an experiment was designed to investigate whether: (1) Students will perform better and experience lower cognitive load by studying worked example rather than problem solving, (2) Students will perform better and experience lower cognitive load by studying in group work settings rather than in an individual setting. The result presented here is a preliminary study.

METHODS AND RESULTS

The study used a factorial repeated measures design varying in learning approaches (problem solving or worked example) and student settings (individual or group work learning) within two types of ratio problems (part-to-part or part-to-whole). In the worked-example approach, students were provided pairs of worked-example and problem solving during practice while in the problem solving approach, students were provided problem solving only during practice. In the individual setting, students were assigned to learn the instruction by themselves however in the group work setting, students were allocated into three or four students to learn the instruction together.

The participant was 56 students with an average age of 12.55 years (SD = 0.25) unequally distributed in the four experiment groups. Learning performance (score), cognitive load measurement (rating scale) were measured.

An arithmetic topic that is compulsory for the Year 7 students was used in the study. The topic covered a ratio problem consisting two types of ratio problem. The first type called part-to-part ratio consisted tasks to determine the quantity of subgroups when the ratio between the subgroups and the quantity of a subgroup are given. Here is the example of the problem:

We need 2 cups of flour to make 3 muffins. How many cups of flour needed to make 10 muffins?

The second type called part-to-whole ratio consisted tasks to determining the quantity of subgroups when the ratio between the subgroups and the total quantity of the group are given. Here is the example of the problem:

The ratio between the number of boys and girls in the 7th grade is 2 : 3. If the total number of the students is 30, how many boys are in the 7th grade? How many girls are in the 7th grade?

In the first session of the experiment, students learned 8 problems of type one and then were given a test consisting of 4 problems similar to those previously learned, and a transfer test consisting of 3 transfer problems were given consecutively. The following session (a day apart) students were remained at the same experimental groups and procedure. As well the group works consisted of the same students. They learned 6 problems of type two, finished a similar test consisting of 3 similar problems (to those used during acquisition phase) and a transfer test consisting of 2 transfer problems consisting of different problem contexts and structure.

The essential prerequisite knowledge for ratio problems that is fractions and solving linier equations are also part of the compulsory curriculum. These topics were already taught to students prior the experiments by the teacher. To ensure that students had sufficient prior knowledge, students received a general introductory session to rehearse the prior-knowledge before commencing the experimental stages. They were also told a general overview of the following lessons, that the material was part of the topic in the school's end of term examination and that their performance in all phases would be marked as well as that the teacher had decided to random the students in two rooms to study the same learning material in different learning conditions. Feedbacks during the following lesson would not be provided and in case questions were raised related to the content, students were asked to think deeper using the material provided or the worked example provided in the introduction phase at the beginning of each lessons.

LEARNING PERFORMANCE RESULTS

The results of learning task, similar test and transfer test were measured. Each question was given a minimum score of 0 and a maximum score of 1. Therefore the minimum score was 0 for all tasks but the maximum score was 4, 4 and 3 in session one and 3, 3, and 2 in session two for learning task, similar test and transfer test respectively. For statistical analysis, the performance scores were transformed into proportion by dividing a participant's total score with the maximum score.

Using ANOVA, significant main effects of the learning approach showing that the worked-example instruction (WE) were more useful to learn rather than the problem solving instruction (PS) were indicated on learning task, $F(1, 52) = 27.34$, $p < 0.05$, $MSE = 0.08$, partial $\eta^2 = 0.35$ (WE: $M = 0.91$, $SD = 0.16$; PS: $M = 0.63$, $SD = 0.28$) and similar test, $F(1, 52) = 6.61$, $p < 0.05$, $MSE = 0.10$, partial $\eta^2 = 0.11$ (WE: $M = 0.73$, $SD = 0.27$; PS: $M = 0.57$, $SD : 0.26$). However, the approach effect was not found on the transfer test performance, $F < 0$.

Significant differences of the performance between the type of the learning material were found on learning task $F(1, 52) = 27.21$, $p < 0.05$, $MSE = 0.02$, partial $\eta^2 = 0.34$ (Type one: $M = 0.71$, $SD = 0.31$; type two: $M = 0.83$, $SD = 0.22$), similar test, $F(1, 52) = 62.65$, $p < 0.05$, $MSE = 0.35$, partial $\eta^2 = 0.55$ (Type one: $M = 0.51$, $SD = 0.32$; type two: $M = 0.79$, $SD = 0.24$) as well as transfer test performance, $F(1, 52) = 19.93$, $p < 0.05$, $MSE = 0.03$, partial $\eta^2 = 0.27$ (Type one: $M = 0.36$, $SD = 0.21$; type two: $M = 0.51$, $SD = 0.26$). All measures showed that students gained higher performance on the problem type two.

There were significant interaction effects between learning approach and the problem type: on the learning task, $F(1, 52) = 21.43$, $p < 0.05$, $MSE = 0.02$, partial $\eta^2 = 0.29$ (Type one: WE: $M = 0.90$, $SD = 0.18$; PS: $M = 0.51$, $SD = 0.28$; Type two: WE: $M = 0.92$, $SD = 0.14$; PS: $M = 0.75$, $SD = 0.25$) and similar test performance $F(1, 52) = 7.95$, $p < 0.05$, $MSE = 0.35$, partial $\eta^2 = = 0.13$ (Type one: WE: $M = 0.64$, $SD = 0.30$; PS: $M = 0.38$, $SD = 0.29$; Type two: WE: $M = 0.82$, $SD = 0.24$; PS: $M = 0.76$, $SD = 0.24$) but no significant interaction effect on the transfer test performance.

Simple effect tests revealed that both on learning and test stages Type one, students who learned worked-examples significantly outperformed than students who learned problem solving, $t(54) = 6.17$, $p < 0.05$, $r = 0.64$ and $t(54) = 3.34$, $p < 0.05$, $r = 0.41$ respectively.

A significant main effect of learning setting in performance was not found on both test phases, $F < 0$, however was closely indicated on the learning phase, $F(1, 52) = 2.95$, $p = 0.09$, partial $\eta^2 = 0.05$. In addition, a significant interaction effect between learning setting and complexity was shown on the similar test phase, $F(1, 52) = 12.49$, $p < 0.05$, $MSE = 0.35$, partial $\eta^2 = 0.19$ (Type one: GW: M = 0.42, SD = 0.31; Ind: M = 0.61, SD = 0.30; Type two: GW: M = 0.82, SD = 0.22; Ind: M = 0.76, SD = 0.26).

A simple effect test showed that in problem type one, students who learn individually significantly performed better than students who learned in group-work, $t(54) = –2.33$, $p < 0.05$, $r = 0.30$. Despite the insignificant different, nonetheless, the average score showed that students who learned in group work in problem type two were higher than those learned individually.

COGNITIVE LOAD MEASUREMENTS

After each tasks in the learning and test phase, the students were asked to indicate the cognitive load they had invested to study or to answer the problem by rating a 9-point cognitive load rating scale ranging from 1 = ‘extremely easy’ to 9 = ‘extremely difficult’. The question was “How easy or difficult was it to study and solve these tasks? Circle your answer on the scale from “Extremely easy” to “Extremely difficult”.

Significant main effects of learning approach were again indicated in the cognitive load measurement on the learning phase $F(1, 52) = 26.41$, $p < 0.05$, $MSE = 2.82$, partial $\eta^2 = 0.34$ (WE: M = 3.44, SD = 1.55; PS: M = 5.07, SD = 1.27), similar test phase, $F(1, 52) = 18.18$, $p < 0.05$, $MSE = 5.0$, partial $\eta^2 = 0.06$ (WE: M = 3.77, SD = 1.64; PS: M = 4.59, SD = 3.46) and transfer test, $F(1, 52) = 11.19$, $p < 0.05$, $MSE = 4.36$, partial $\eta^2 = 0.18$ (WE: M = 4.91, SD = 1.67; PS: M = 6.23, SD = 1.52).

However, a significant main effect of material complexity was not found on cognitive load measurement in the transfer test, $F < 0$. The significant main effects of complexity were indicated on the learning phase, $F(1, 52) = 14.60$, $p < 0.05$, $MSE = 1.38$, partial $\eta^2 = 0.22$ (Type one: M = 4.66, SD = 1.60; Type two: M = 3.80, SD = 1.69) and similar phase, $F(1, 52) = 17.59$, $p < 0.05$, $MSE = 1.65$, partial $\eta^2 = 0.25$ (Type one: M = 4.68, SD = 1.86; Type two: M = 3.64, SD = 1.87).

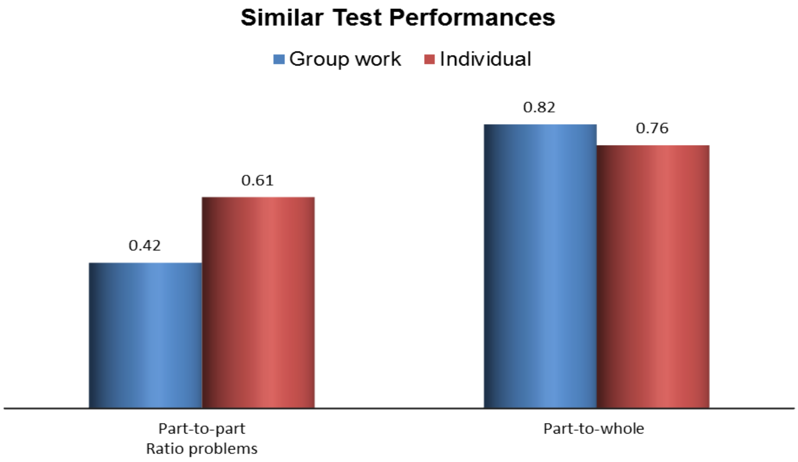
An interaction effect was shown for cognitive load measurement on the similar test phase, between learning approach and complexity, $F(1, 52) = 5.78$, $p < 0.05$, $MSE = 1.65$, partial $\eta^2 = 0.10$ (Type one: WE: M = 4.56, SD = 1.88; PS: M = 4.81, SD = 1.86; Type two: WE: M = 2.98, SD = 1.40, PS: M = 4.36, SD = 2.06). A simple effect analysis indicated that in Type two, students who studied problem solving significantly invested higher cognitive load than students who studied worked example, $t(54) = –2.95$, $p < 0.05$, $r = 0.37$.

Despite no significant main effect of learning setting was indicated for cognitive load measurement at all phases, a close significant interaction effect between learning setting and complexity on the similar test phase was found, $F(1,52) = 3.71$, $p = 0.06$, $MSE = 1.65$, partial $\eta^2 = 0.07$ (Type one: GW: M = 5.00, SD = 1.91, Ind: M = 4.36, SD = 1.79; Type two: GW: M = 3.51, SD = 2.14, Ind: M = 3.77, SD = 1.57). The other expected interaction effects were not found.

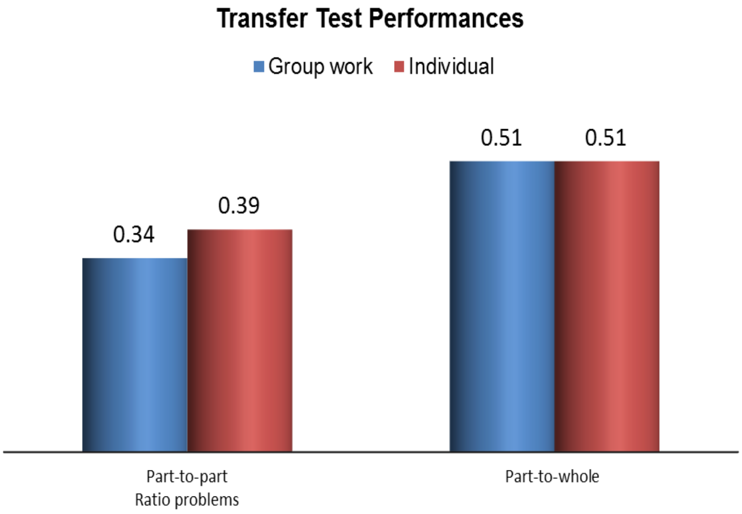
DISCUSSIONS

Two hypotheses were examined: (1) Students will perform better and experience lower cognitive load by studying worked example rather than problem solving, (2) Students will perform better and experience lower cognitive load by studying in group work settings rather than in an individual setting. Scores based on similar and transfer tests were recorded as well as cognitive load measures.

Two ratio topics named part-to-part and part-to-whole were used as the material. The results indicated that students in different settings did not show significant different in performances. This can be described in graph 1a for similar test and graph 1b for transfer test results.

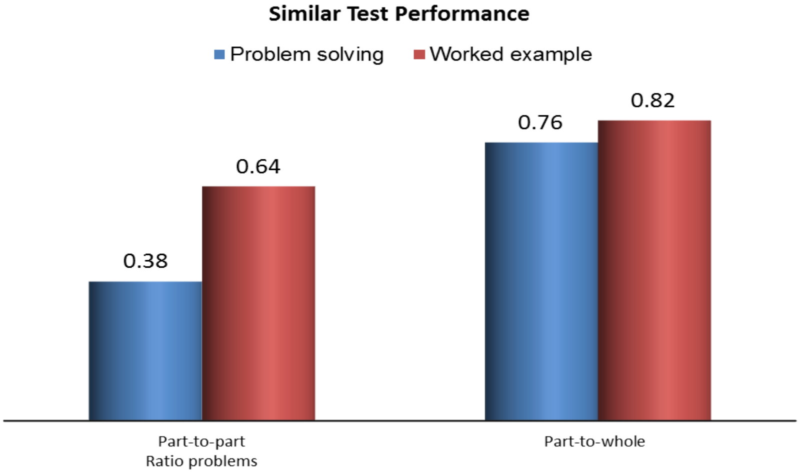


Graph 1a. Similar test results Group Work vs Individual

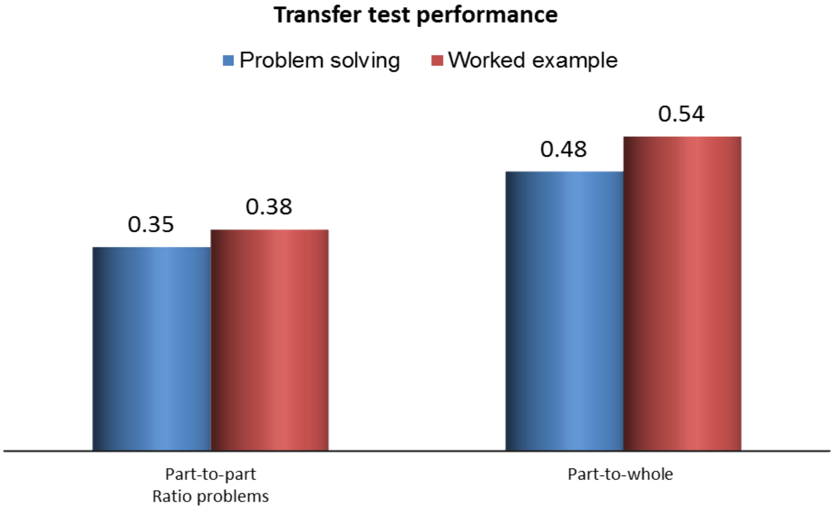


Graph 1b. Transfer test results Group Work vs Individual

Significant difference was found between the instructions where students who were provided the worked example instruction performed better that those with the problem solving instruction, regardless the learning material, as can be summarized in graph 2a and 2b below.



Graph 2a. Similar test results Worked example vs Problem solving



Graph 2b. Transfer test results Worked example vs Problem solving

It can be concluded that the worked example instruction were more advantageous for students either learning individually or in group work, compared to the problem solving instruction. Cognitive load theory, principle three supports these results that students will learn effectively when relevant prior knowledge is available to borrow. In this study, worked examples provided the prior knowledge. It can be explained that all group members had similar level of prior knowledge; therefore sharing relevant knowledge was very limited. Students seem gaining more benefit by studying the worked examples rather than problem solving with other students. Further studies to investigate the requirements or learning settings that support the effectiveness of group work should be followed up.

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