

**DEVELOPING THE ATTITUDE AND CREATIVITY
IN MATHEMATICS EDUCATION**

**Paper to be Presented at
The International and The Fourth National Conference on Mathematics
Education 21-22 July 2011
FMIPA UNY**

**By
Marsigit
Department of Mathematics Education,
Faculty of Mathematics and Science,
Yogyakarta State University**

DEVELOPING THE ATTITUDE AND CREATIVITY IN MATHEMATICS EDUCATION

By Marsigit

Faculty of Mathematics and Science, Yogyakarta State University

ABSTRACT

The structures in a traditionally-organized classroom of mathematics teaching can usually be linked readily with the routine classroom activities of teacher-exposition and teacher-supervised desk work, teacher's initiation, teacher's direction and strongly teacher's expectations of the outcome of student learning. If the teacher wants to develop appropriate attitude and creativities in mathematics teaching learning it needs for him to develop innovation in mathematics teaching. The teacher may face challenge to develop various style of teaching i.e. various and flexible method of teaching, discussion method, problem-based method, various style of classroom interaction, contextual and or realistic mathematics approach.

To develop mathematical attitude and creativity in mathematics teaching learning processes, the teacher may understand the nature and have the highly skill of implementing the aspects of the following: mathematics teaching materials, teacher's preparation, student's motivation and apperception, various interactions, small-group discussions, student's works sheet development, students' presentations, teacher's facilitations, students' conclusions, and the scheme of cognitive development.

In the broader sense of developing attitude and creativity of mathematics learning, the teacher may needs to in-depth understanding of the nature of school mathematics, the nature of students learn mathematics and the nature of constructivism in learning mathematics.

Key Word: mathematical attitude, creativity in mathematics, innovation of mathematics teaching, school mathematics,

A. Introduction

New trend of mathematics teaching describes that the role of teacher should change from teaching of mathematics to facilitate their students to learn mathematics. And thus the innovative of mathematics teaching needs to develop its curriculum and syllabus, good combination of various and different of students competencies, flexibility of teaching style and strategy, and a concern for the mathematical attitude and creativity as well as the cognitive need of pupils. It is greatly recommended the teacher requires the use of a range of teaching styles and develop students' knowledge of mathematics and ways of working as well as on mathematical attitude, method and content.

This is actually in accordance with what the Cockroft Report suggested that it is not possible to indicate a definitive style for teaching of mathematics (ibid, p.158). The report also

suggested that approaches to the teaching of a particular piece of mathematics need to be related to the topic itself and the abilities and experience of both teachers and children; that methods which may be extremely successful with one teacher and one group of children will not necessarily be suitable for use by another teacher or with another group of children (ibid, p.158).

In-depth study of mathematics teaching practice at schools may lead to the conclusion that different context of teaching has significantly different approach implications. Concepts for the teaching and learning of mathematics – more specifically: goals and objectives, syllabi, textbooks, curricula, teaching methodologies, didactical principles, learning theories, mathematics educational research, teachers' conceptions of mathematics and mathematics teaching as well as students' perception of mathematics – carry with them or even rest upon particular philosophical and epistemological views of mathematics (Steiner, 1987 in Ernest, 1994).

The National Middle School Association in Marsigit (2010) identified the nature of students in term of their intellectual, social, physical, emotional and psychological, and moral. Young adolescent learners¹ are curious, motivated to achieve when challenged and capable of problem-solving and complex thinking. There is an intense need to belong and be accepted by their peers while finding their own place in the world. They are engaged in forming and questioning their own identities on many levels. The students may mature at different rates and experience rapid and irregular growth, with bodily changes causing awkward and uncoordinated movements. In term of emotional and psychological aspect, they are vulnerable and self-conscious, and often experience unpredictable mood swings. While in the case of moral, they are idealistic and want to have an impact on making the world a better place.

Most of the teachers always pay much attention to the nature of student's ability. We also need to have an answer how to facilitate poor and low-ability children in understanding, learning and schooling. Intellectual is really important to realize mental ability; while, their work depend on motivation. It seems that motivation is the crucial factor for the students to perform their ability. In general, some teachers are also aware that

the character of teaching learning process is a strong factor influencing student's ability. We need to regard the pupils as central to our concerns if our provision for all the pupils is to be appropriate and effective; some aspects of teaching for appropriateness for students might be: matching their state of knowledge, identifying and responding to their particular difficulties, extending them to develop their potential in mathematics, providing some continuity of teaching with a demonstrated interest in progress, developing an awareness of themselves as learners using the teacher as a resource, and providing regular feedback on progress. Those who teach mathematics must take into account the great variations which exist between pupils both in their rate of learning and also in their level of attainment at any given age (Cockroft Report in Marsigit 2010).

John Munchak (2004) indicated that in order to provide lessons that are both engaging and challenging to each individual, it is necessary to know the students as people. Each individual will come to my class with their own set of abilities, motivations, attitudes, goals, and cultural background. Further he stated that getting to know these various facets of my students will allow the teachers to excel because they can tap into their talents, resources, and knowledge to make the classroom more interesting, dynamic, and personal. Further he stated that establishing a familiar bond and some trust between the teacher and the students, as well as among the students themselves, contributes to a safe and caring learning environment. Accordingly, learning their interests, the activities they enjoy, their academic strengths and weaknesses, their future plans and motivations informs how the teacher will teach each individual. He assumed that this personal information is important in order to differentiate learning in a classroom with students of various levels of motivation, career goals, and academic abilities. He concluded that caring for the students means "honor their humanity, hold them in high esteem, expect high performance from them, and use strategies to fulfill their expectations".

Mathematics must be close to children and be relevant to every day life situations. However, the word 'realistic', refers not just to the connection with the real-world, but also refers to problem situations which real in students' mind (Zulkardi, 2006). For the problems to be presented to the students this means that the context can be a real-world but this is not always necessary. De Lange (1996) stated that problem situations can also be seen as applications or modeling. The idea of mathematics as a human activity is stressed. Mathematics education organized as a process of *guided reinvention*, where students can experience a similar process compared to the process by which mathematics was invented. The meaning of invention is steps in learning processes while the meaning of guided is the instructional

environment of the learning process. Moreover, the reinvention principle can also be inspired by informal solution procedures.

As the Programme for International Student Assessment (PISA) concerned with the capacities of students to analyse, reason, and communicate ideas effectively as they pose, formulate and interpret mathematics in a variety of situations. In line with the effort to prepare students for society and for future schooling and work; and thus, it is reasonable for Indonesian government to implement Realistic Mathematics Education (RME) in the schema of Lesson Study, in which the extents of above issues are able to be handled. Realistic Mathematics Education (RME) in the schema of Lesson Study let the teachers to improve instructional approach from traditional to progressive one.

Isoda, M. (2006), outlined that mathematical thinking is open ideas; thus, it is very difficult to discuss its development without having a window to discuss. When we focus on each lesson, we easily focus on specific knowledge and skills (Understanding), and easily forget to develop Attitude, Mathematical Thinking and Representation. The present form of RME is mostly determined by Freudenthal's view on mathematics (Freudenthal, 1991). Two of his important points of views are mathematics must be connected to reality and mathematics as human activity. First, mathematics must be close to children and be relevant to every day life situations. However, the word 'realistic', refers not just to the connection with the real-world, but also refers to problem situations which real in students' mind (Zulkardi, 2006). For the problems to be presented to the students this means that the context can be a real-world but this is not always necessary. De Lange (1996) stated that problem situations can also be seen as applications or modeling. Second, the idea of mathematics as a human activity is stressed. Mathematics education organized as a process of *guided reinvention*, where students can experience a similar process compared to the process by which mathematics was invented. The meaning of invention is steps in learning processes while the meaning of guided is the instructional environment of the learning process. Moreover, the reinvention principle can also be inspired by informal solution procedures. Informal strategies of students can often be interpreted as anticipating more formal procedures (Zulkardi, 2006).

Two types of mathematization which were formulated explicitly in an educational context by Treffers, 1987, in Zulkardi, 2006, are horizontal and vertical mathematization. In horizontal mathematization, the students come up with mathematical tools which can help to organize and solve a problem located in a real-life situation. The following activities are examples of horizontal mathematization: identifying or describing the specific mathematics in a general context, schematizing, formulating and visualizing a problem in different ways, discovering relations, discovering regularities, recognizing isomorphic aspect in different problems, transferring a real world problem to a mathematical problem, and transferring a real world problem to a known

mathematical problem (Zulkardi, 2006). On the other hand, vertical mathematization is the process of reorganization within the mathematical system itself. The following activities are example of vertical mathematization: representing a relation in a formula, proving regularities, refining and adjusting models, using different models, combining and integrating models, formulating a mathematical model, and generalizing. The implementations of primary mathematics curriculum in class-rooms thus need to develop problem solving skills covering both closed and open problems. In solving the problems, students need to creatively develop many ways and alternatives, to develop mathematical models, and to estimate the results. It was suggested that in teaching learning of primary mathematics, students have the chances to identify mathematical problems contextually and realistically. Contextual and realistic approaches are recommended to be developed by the teachers to encourage mathematical thinking in primary schools. With these approaches, there is a hope that the students step-by-step learn and master mathematics enthusiastically. To make their teaching learning of primary mathematics more effective, teachers also need to develop resources such as information technology, teaching aids and other media.

Specifically, our primary mathematics curriculum outlines the aims of teaching learning of mathematics are as follows:

- a. to understand the concepts of mathematics, to explain the relationships among them and to apply them to solve the problems accurately and efficiently.
- b. to develop thinking skills to learn patterns and characteristics of mathematics, to manipulate them in order to generalize, to proof and to explain ideas and mathematics propositions.
- c. to develop problems solving skills which covers understanding the problems, outlining mathematical models, solving them and estimating the outcomes.
- d. to communicate mathematics ideas using symbols, tables, diagrams and other media.
- e. to develop appreciations of the uses of mathematics in daily lives, curiosity, consideration, and willingness to learn mathematics as well as tough and self-confidence.

B. Attitude Toward Mathematics, Mathematical Attitude and Creativity Across Asia Pacific Countries

In Singaporean Context of mathematics teaching (Banhar Y , 2007) indicated that pupils are expected to possess mathematical skills and concepts. Skills include computation including mental computation and visualization. Key concepts in elementary school include numerical, geometrical and algebraic concepts. Pupils are also expected to possess the ability to engage in processes such as reasoning, communicating, making connections, modeling, and

using thinking skills and heuristics. Pupils are expected to possess good problem-solving attitudes and habits as well as the ability to engage in meta-cognition. Good habits of mind such as flexibility, which is a component of creative thinking, facilitate the ability to see the relationship. Flexibility is another example of a supporting generic competency. Some of the supporting generic competencies are heuristic skills while others are habits of mind.

In Malaysian context of mathematics (Lim Chap Sam & Hwa Tee Yong, 2007) revealed that the right attitudes or dispositions toward attainment of content knowledge are very important and serve as the ground force to execute cognitive skills and strategies in mathematics problem-solving. In sum, to become a successful and effective mathematical thinker, one needs to possess and internalize all these three components: content knowledge, cognitive skills cum strategies and thinking dispositions. The mathematical thinking was inferred from the students' attitudes towards mathematics and problem solving. Mathematical thinking was taught as related to higher order thinking, critical and analytical thinking as well as problem solving. Hence, there is generally a lack of clear understanding about what is mathematical thinking among Malaysian mathematics teachers. Many teachers perceived mathematical thinking to problem solving or higher level of questioning. Others referred mathematical thinking to critical and creative thinking skills.

In the Philippines context of mathematics teaching (Soledad A.Ulep, 2007) reported that Creative thinking results to an output that is new, different, and the like. All the competencies stated at the beginning were all addressed in the activity in the lesson. Moreover, different thinking skills were used in the process. She recommended that to promote mathematical attitude and creativity the students need to : Focussing skills: defining problems, setting goals; Information-gathering skills: observing, formulating questions; Remembering skills: encoding, recalling; Organizing skills: comparing, classifying, ordering, representing; Analysing skills: identifying attributes and components, identifying, relationships and patterns, identifying errors; Generating skills: inferring, predicting, elaborating; Integrating skills: summarizing, restructuring; Evaluating skills: establishing criteria, verifying. Further, she also stressed that the teachers should appreciated how much thinking the students used while doing mathematics. They also liked how, by asking related questions, a simple problem was made more complex. They were amazed that the methods that worked for the simple cases were also the basis of the methods for the more complex ones.

In the Brunei context of mathematics teaching (Madiah Khalid, 2007) learned that the children need to be provided with opportunities to develop positive attitude and confidence as

well as understanding in a number of topics, develop the ability to calculate both mentally and on paper; estimate; communicate mathematical ideas clearly and hence be able to tackle and solve real problems systematically as well as conduct mathematical investigations. From these objectives it can be assumed that mathematical thinking is embedded through out the teaching of mathematical content to give children the opportunity to appreciate the aesthetic nature of mathematics. She found that assessment should be conducted on a continuous basis through out the school year and assessment tasks should include “problem solving, mathematical thinking and creative work”

In the South Korean of mathematics teaching (Jung Inchul, 2007) noted that in mathematics learning the students should be able to experience the joy of discovery and maintain their interest in mathematics by pursuing the following instructional methods in their classrooms: to emphasize concrete operational activities in order to help students to discover principles and rules and solve problems embedded in such a discovery; to have students practice basic skills to help students be familiar with them and problem solving abilities in order to use mathematics in their everyday life; to present concepts and principles in the direction from the concrete to the abstract in order to activate self-discovery and creative thinking; to induce students to recognize and formulate problems from situations both within and outside mathematics; to select appropriate questions and subsequently provide feedback in a constructive way in order to consider the stages of students' cognitive development and experiences; to use open-ended questions in order to stimulate students' creativity and divergent thinking; to value the application of mathematics in order to foster a positive attitude toward mathematics; to help students understand the problem-solving process and use basic problem-solving strategies in order to enhance students' problem-solving abilities.

In Thailand context of mathematics teaching (Imprasitha and Loipha, 2007) found that most of the school students have positive attitudes towards learning through the open-approach method. They have more opportunities to act, think, play an active role, do something original, and conclude things by oneself. They show some interesting responses as follows; more reasonable, more skillful in observation, more cool-hearted, know how to work cooperatively and more confidence in asking “*why?* “ and “*how come?*“ type questions. The students had realized that they were able to apply their daily-life encounters to their learning activities in classroom. They became more eager to learn and were able to work cooperatively. They had become more expressive. They spoke louder and were able to do class presentation very nicely. They liked to participate in class activities and to present ideas and methods of their own group

(especially the grade-1 students). They dared to think differently and in various ways, enjoyed talking and expressing their own opinions. More significantly, they stuck fast with the activities or problems they encountered for a longer period of time and wouldn't give up easily. They had developed a more positive attitude towards group work or working with other people.

In the China context of mathematics teaching (Wang, S, et al., 2007) indicated that the students should possess some degree of creative spirits and practical abilities, and develop sufficiently in areas of general abilities, affection and attitudes. They should be able to develop four dimensions of mathematics learning : Knowledge and Skill, Mathematical Thinking, Problem Solving, Affection and Attitudes. During teaching, teachers should promote democracy in teaching, so as to become organizers, guides, and collaborators of student mathematical activities. Teachers should be good at inspiring students' learning potentials, encourage students to innovate and practice boldly. They should deploy teaching materials creatively, exploit actively, and use all sorts of teaching materials, so as to provide students rich and multifarious learning materials. They should pay attention to students' individual differences and practice individualized teaching effectively, so that students receive sufficient developments. Application of modern educational technologies to teaching should be emphasized. For those areas where conditions permit, teachers should try hard to use computers and related software reasonably and effectively to raise teaching efficiency.

In the Taiwanese context of mathematics teaching (Shou Lin and Chun Lin, 2007) claimed that recently, there is a trend in mathematical education to emphasize the mathematical thinking, more than to ask students mastering computational skills. As a mathematician, he certainly understand how important the mathematical thinking is. However, in many instances, the mathematical thinking is often misunderstood as "creative pedagogy". He said that if we carefully exam the cases of creative pedagogy, we find most of them are kind of improvisations for creative pedagogy, full of creative improvisation but no mathematics at all! This is one of the key window we are considering: Improvisation for creative pedagogy vs. Mathematical thinking. Along the reasoning with the above paragraph, he said he would like to make some remarks to stress the importance of assessing quality of mathematical thinking. He suggested that we should keep in mind that developing mathematical thinking is not only for fun, but with the definite purpose for helping students to learn mathematics. Thus, the question whether a program of mathematical thinking fits or not becomes essentially important when we come to assess the mathematical thinking. The assessment should be a key window of mathematical thinking. While designing the curriculum in mathematics, we should know mathematics is not

only a local stream of logic sequence, but should be designed according to student's cognitive ability at different stages. In his opinion, it is better that mathematical thinking should be developed along with curriculum. Neither any mathematical thinking in the program should be picked up randomly, nor a mathematical-thinking-developing curriculum is a collection of isolated ideas. Instead, it should be interwoven together as a whole, so that mathematical ideas could be developed further at the next stage.

In the Hong Kong context of mathematics teaching (Litwin Chor, 2007) stressed that mathematics teaching should aim at developing pupils' creativity, and their ability to think, communicate and solve problems; developing pupils' number sense and spatial sense, and their ability to appreciate patterns and structures of number and shapes; and enhancing pupils' lifelong learning abilities through basic mathematical knowledge. The ability to think is included in the aims of the curriculum and the attitude that students are capable of "independent thinking and persistence in solving problems" is one of the objectives of the curriculum. It is advocated in the curriculum guide that pupils should learn pleurably through various learning activities and develop their imagination, creativity and thinking skills. These thinking skills include: Inquiring, Communicating, Reasoning, Conceptualizing, and Problem Solving. Accordingly, the Hong Kong Mathematics Curriculum defines five high order-thinking skills in the mathematics secondary curriculum. These are the same as defined in the primary curriculum: problem-solving skills, inquiring skills, communicating skills, reasoning skills, and conceptualizing skills. Further he stated that the document remind the teachers that there are no simple and clear cut definitions for high order thinking and the five high order thinking can be arranged under several categories such as meta-cognitive skills, critical thinking. These can be overlapping in a sense. Having said that, the common technique of posing open-ended problems in mathematics classes is usually advocated and practiced in the classroom. He believed that these five skills are not easily isolated. And though these skills can be taught in isolation, the popular way is to incorporating them into content areas.

In the Mexican context of mathematics teaching, Cedillo and Santillan (2007) studied that a positive attitude towards mathematics consists in motivate and develop students' curiosity and interest for investigate and solve problem situations. Besides, it means creativity to put forward conjectures, flexibility to modify their own view, and intellectual autonomy to confront unknown situations and gain self confidence on their capability to learn. Collaborative and critical thinking will be supported by organizing group activities in which students are required to formulate, communicate and provide arguments to validate mathematical statements

by properly using mathematical rules for debating that led them to appropriate decisions in each situation. According to them, mathematical attitude and creativities need to be promoted through posing and solving problems, argumentation, communication, and procedural skills. Their research (2006) concluded that the teacher needs to *frame the classroom events according to the students' ways of reasoning instead of students following their teacher's ways of reasoning*. By adopting such a principle they assumed that learning is an active construction process that is socially shared by the learners and the teacher. Accordingly, the teacher requires to play a different role. In order to briefly describe how it is conceived that teacher's role, they used the metaphor of conceiving the activity in the classroom as a chess game; in such game the teacher is the expert player who simultaneously plays against 30 other players who can communicate and discuss amongst themselves before making a move. The expert makes the first move and has to be prepared to receive up to 30 different challenging responses; upon the second move on the chess board the expert player (teacher) has to give specific and challenging responses to each player, and so on. A major difference between the conventional chess game and the version we use in the metaphor is that in our chess game the teacher must manage the game in such ways that, eventually, the students legally win.

In the Chilean context of mathematics teaching, Bonomo Cerda (2007) insisted that school mathematics education should aims to help students achieve four following objectives: knowledge, skills, thinking and attitudes. According to him, the mathematics curriculum should provides opportunities for students to develop: the ability of observing, predicting, rational reasoning and logical reasoning; the ability of expressing precisely and clearly their own ideas and understanding the ideas of others; spatial imagination; the characteristics of thinking, especially the flexible, independent and creative thinking; thinking operations: comparison, analogy, generalization, and specialization. He suggested that in developing mathematical attitude, the teacher should encourage or provide the students for: self-study, active and confident in learning mathematics; having characteristics of being: honest, hard working, overcoming difficulty, careful, accurate, disciplinal, and creative; having a sense of cooperation, appreciate their own works and others; and recognizing and valuing the beauty of mathematics and like mathematics. Accordingly, the teacher need to develop teaching strategies: promote active, initiative and self-conscious learning of the learners; form and develop the ability of self-study; cultivate the characteristics of flexible, independent, and creative thinking; develop and practice the logical thinking; apply problem solving approaches; and apply mathematics to real life situations.

In the Japanese context of mathematics teaching, Katagiri (2007) outlined that mathematical attitude is one of the type of mathematical thinking. According to him, mathematical thinking includes mathematical attitudes, mathematical methods and mathematical contents. He further indicated that student's mathematical attitude consists of :

1. Attempting to grasp one's own problems or objectives or substance clearly, by oneself
 - a. Attempting to have questions
 - b. Attempting to maintain a problem consciousness
 - c. Attempting to discover mathematical problems in phenomena
2. Attempting to take logical actions
 - a. Attempting to take actions that match the objectives
 - b. Attempting to establish a perspective
 - c. Attempting to think based on the data that can be used, previously learned items, and assumptions
3. Attempting to express matters clearly and succinctly
 - a. Attempting to record and communicate problems and results clearly and succinctly
 - b. Attempting to sort and organize objects when expressing them
4. Attempting to seek better things
 - a. Attempting to raise thinking from the concrete level to the abstract level
 - b. Attempting to evaluate thinking both objectively and subjectively, and to refine thinking
 - c. Attempting to economize thought and effort

Further, Katagiri (2007) stated that although he have discussed two types of mathematical thinking (the meanings of mathematical thinking regarding mathematical methods, and of mathematical thinking regarding the substance of mathematics), there is another type of mathematical thinking referred to as "mathematical attitudes" that drives the a fore mentioned types. Therefore, these "mathematical attitudes" are also a type of "mathematical thinking." According to him, the correctness of formulas one has created is not something that one must have another person recognize, but rather something one must determine and recognize through one's own ability. In general, when it comes to arithmetic and mathematics, the correctness of a solution is based on the meaning of the original problem and the four arithmetic operations, which can be judged independently. Further, he suggested that the correctness of a solution is not something that is determined by the authority of a vote held among the children, or based on the authority of the teacher's assertion that it is correct. He outlined that arithmetic and mathematics are characterized by the independence of judgment, and independent judgment is easy in this field. This is why Katagiri concluded that arithmetic/mathematics is an appropriate subject for the cultivation of a desire and attitude aimed at learning things independently; the attitude of seeking precision in one's own problems, objectives, and matters, as well as the attitude of trying to solve problems through one's own ability, are both crucial.

C. Promoting Mathematical Attitude and Creativity Through The Series of School-Based Lesson Study Activities in Indonesia

Various studies were held by the writer in the form of *School-Based Lesson Study* in which some teachers of Primary and Secondary Schools were participated to establish lesson study goal and develop lesson study cycles by developing common vision of systematic and consistent pedagogical approach to facilitate students need in performing their mathematical thinking.

1. The Developed Schemes and Implementation

The steps of developing School-Based Lesson Study covered recruiting teachers, developing the theme, planning the Lesson Study, preparing observation and reflecting the results. Lesson study teams composed of 4th-grade teachers and 5th-grade teachers of Primary Schools and 8th-grade teachers of Junior High Schools who work in three different schools: SD MIN I Yogyakarta, SD Percobaan Bulaksumur Yogyakarta and SMP N II Depok Yogyakarta. The researcher facilitated the teachers to provide perspective and a broader view of the issues as well as to serve as outside commentator, evaluator, or outside advisor. The researcher emphasized that the selected teachers should come in with the mindset of being a learner and ready to share and to communicate findings. The researcher and the selected teachers built open communication and set time-table for the related activities.

Most of the lesson study theme captures the school goals as well as the academic content goals for students to develop approaches and to perform *mathematical thinking*. The researcher exposed the important of mathematical thinking in the sense of students' thinking, educational theories, and international trends. The selected teachers choose a subject area in which to focus on *mathematical thinking*. They needed to identify a unit or lesson on which to focus on *mathematical thinking* and thoroughly discuss the unit and agree about what they are trying to achieve with the lesson. They also needed to expect what did they want students to know and be able to perform mathematical thinking. The researcher strived that the selected teachers must understand how their lesson would significantly supporting and facilitating mathematical thinking. To achieve this goal the researcher carried out firstly the socialization of Katagiri's notions of mathematical thinking. Prior the study implementation, the researcher shared and discussed with the teachers to prepare lessons related to the topic. The researcher and the selected teachers developed the lessons and set the stage for the observation in which the lesson and the learning processes would be reflected. A piece of planning the lesson included the schema of student responses to various aspects of the lesson and preparing appropriate teacher responses as well as the logical implication of mathematical thinking.

There was conformity among the goal of the overall Lesson Study, the aim of teaching and the aim for student learning. The researcher and the selected teachers developed lesson design and lesson plan to bring these goals. The developed Lesson Plan referred to the School-Based Curriculum (KTSP). The selected teachers implemented teaching learning processes while the researchers collect data on students' mathematical thinking and their aspects. Some other teachers and the researcher were set to observe the study lesson in a scheduled time and place. The researcher and the selected teachers shared the data collected covering of the evidence that goals for promoting students' mathematical thinking; and then found out the solutions of how to improve the lessons. Lesson debriefing was proposed to give the chance for the selected teacher to reflect his/her teaching; while getting inputs from other teachers or researcher. In while teaching, the researcher collected the data that need for debriefing. The data covered the comments of students and the work students produce during the lesson. At some occasions the researcher needed to observe closely the work and comments of particular students. The researcher and the selected teachers prepared copies of the lesson plan, teaching aids, and any students' worksheets that students would be using. The study prepared the classroom so that the observers can circulate freely among students during whole-class teaching. The researcher developed instrument to investigate the structure of lesson, the schema of interaction, and the schema of mathematical thinking in the frame of the effort of achieving the mathematical competences.

In order to observe intensively, the researcher develop Observation Instrument and Observation Check-List consisting of the elaboration of Katagiri's notions of mathematical thinking related to the series of events in which the students strived to perform their *mathematical thinking*. Having observed mathematical classroom, the researcher strived to enable the data to be learned so much about the lesson being taught. The researcher motivated the teachers that the lesson study experience was well worth the time; however, the value of collaborative between the researcher and the teacher allowed for considerations certain aspect that may not think about when they are planned alone. The teachers may reflect their teaching activities within their schema of perception of *mathematical thinking*. However, the teacher might find wonderful inputs from the researcher of the different angle of sight.

2. The First Sample of Evidences Observing Mathematical Attitude and Creativity at Primary Mathematics Teaching

The aim of the study is to uncover the aspect of mathematical attitude and creativity based on the theory of Katagiri. Following was the research conducted by Marsigit et.al (2006) at the Teaching Learning of Least Common Multiple at Fourth Grade of Primary School. The

standard competency is to understand and to apply factors and multiple of numbers to solve problems. And the basic competency are: to understand the Least Common Multiple (LCM), to determine the Least Common Multiple (LCM), and to solve problems which is related to LCM.

a. Lesson I:

- 1) Introduction
- 2) Describing *prepared contextual problems* (problems situated in reality as follow):

Since the early of the year 2006, Shinta has two activities i.e. swimming and gardening. She is periodically going to swim once a week and gardening every 8 days, as shown in the following calendar:

Question:

When Shinta is going for swimming and gardening on the same days?

- 3) Developing (group discussion)
- 4) Reason and explanation (presentation)
- 5) Conclusion (homework)
- 6) Closing

b. Lesson II :

- 1) Introduction
- 2) Continue to developing (group discussion)
- 3) Reason and explanation (presentation)
- 4) Conclusion (homework)
- 5) Closing

c. Uncovered Data of Lesson I:

- 1) Apperception (prepare for emotional and awareness):
Teacher started the lesson by delivering the question of whose the students have routine activities.

Analyses:

For a moment teacher could not identify the satisfied problems from students, and therefore she posed the prepared problem

- 2) Preparing for group discussion:

The teacher distributed prepared problem written in Worksheet to every student. The teacher let the student first worked individually i.e. read and learn the contextual problem in calendar format:

Analyses:

The calendar format lead the students to develop their solution also in calendar format.

- 3) Developing in Group Discussion:

The teacher let the students to have group discussion to solve the problem.

Analyses:

Most of the group employed calendar to find the multiple of 7 days and the multiple of 8 days in one year. Most of the group constructed the complete one year calendar.

- 4) Monitoring developing in group discussion

The teacher let the students to develop their own methods and resources

Analyses:

The students by themselves employed various aids to solve the problems such as calendar, hand-phone, and blank table prepared by the teacher.

The teacher let the students to continue to solve the problem as home-work.

5) Translation of one of the the result of group discussion:

- *Shinta goes to swim once every 7 days and to gardening once every 8 days*
- *Shinta goes to swim 5 times a month and to gardening 4 times a month*
- *For 10 month, Shinta goes to swim $10 \times 5 = 50$ times*
- *For 10 month, Shinta goes to gardening $10 \times 4 = 40$ times*
- *Total number of Shinta's activities = 50 times + 40 times = 90 times*
- *90 times : (7 + 8) days = 6*
- *90 times : 15 days = 6*

Analyses:

The series of sentences produces by the group indicated first horizontal mathematization then followed by vertical mathematization.

d. Uncovered Data of Lesson II:

1) Reflection (reason and explanation):

Representatives of selective group presented their results of yesterday discussion.

Analyses:

Most of the students employed subtraction, addition, multiplication and division to list multiple of 7 and 8. Most of the students indicated Common Multiple, as the mathematical concepts to answer the common Shintas activities in one year.

2) Reflection (reason and explanation):

Student explaining their finding/solution of the problems

Analyses:

There was a transition from daily language of mathematical language i.e. from common activities to common multiple. There was a student who jumped their concept to LCM due to he got it from "informal private lesson".

The teacher encouraged the students to list more the multiple of 7 and the multiple of 8. The teacher encouraged the students to indicate the common multiple of 7 and 8. The teacher let the students to find the Least Common Multiple of 7 and 8, i.e 56. The teacher delivered class assignment to find the LCM of 2 and 3, and pointed out some students to write their answer in front of the class

e. Results of the First Sample

From the analyses of based on observation of the lesson, it was indicated that the students strived to develop mathematical attitude and creativity through some following activities:

1) Identifying or describing the specific mathematics :

- a) in the routine activities there are the concept of addition and subtraction i.e. $7 + 7 + 7 + 7 \dots$ or subtracting by 7 (for swimming)
 $8 + 8 + 8 + 8 \dots$ or subtracting by 8 (for gardening)
- b) in the question of " how many times common activities" there is the concept of "frequency" or "repeating addition or subtraction" i.e. th concept of multiple of number:
 - *For 10 month, Shinta goes to swim $10 \times 5 = 50$ times*
 - *For 10 month, Shinta goes to gardening $10 \times 4 = 40$ times*

2) Schematizing, formulating and visualizing a problem in different ways :

- a) There are various ways in determining the multiple number of 7 and 8 e.g. using calendar, using series of numbers, using calculator and manipulating different symbols for 7 and 8.
- b) There are different schemas on determining the common multiple of 7 and 8 i.e. some students calculate the multiple of 7 for the whole year first then

multiple for 8; and followed by counting the number of common activities in one year. Some students indicated first the common multiple of 7 and 8 (i.e. 56) and then counting the number of common activities in one year.

- 3) Discovering relations
The students discovered the relationship between “common activities” and ”common multiple” i.e. 7 days and 8 days compare with “multiple of 7 and 8 = 56”
- 4) Discovering regularities
The concepts of regularities arouse from the concepts of “routine activities”
- 5) Recognizing isomorphic aspect in different problems
The students identified that the activities to be manipulated were not only about “swimming” and “gardening”, but also for others their daily activities such as “study club”, “laboratory activities” or “going to library”
- 6) Transferring a real world problem to a mathematical problem
There are the key concepts reflecting by the key word of how the students can transfer the real world problems to mathematical problem e.g. the concepts of “common”, “regular”, “routine”, “number of” e.g.:
a) to add regularly: $7 + 7 + 7 + 7 + 7 \dots$ common activities, b) common multiple (56) number of common activities, and c) number of common multiple.
- 7) The students strived to represent daily problems in a related mathematical formula
- 8) The students strived to prove regularities of consisting concepts
- 9) Some students refined and adjusting models by not continuing to complete the whole year calendar; and found effective ways to count the common multiple.
- 10) Some students performed vertical mathematization by employing different models
- 11) Some students formulating a mathematical model, and generalizing in such away that they found $90 \text{ times} : 15 \text{ days} = 6$.
- Shinta goes to swim once every 7 days and to gardening once every 8 days
- Shinta goes to swim 5 times a month and to gardening 4 times a month
- For 10 month, Shinta goes to swim $10 \times 5 = 50$ times
- For 10 month, Shinta goes to gardening $10 \times 4 = 40$ times
- Total number of Shinta’s activities = 50 times + 40 times = 90 times
- $90 \text{ times} : (7 + 8) \text{ days} = 6$
- $90 \text{ times} : 15 \text{ days} = 6$

3. The Second Sample of Evidences Developing Mathematical Attitude and Creativity by Junior Secondary Mathematics Teachers

In a PMRI workshop, there are 6 (six) group of teachers each consisting of 6 (six) teachers developed the mathematical attitude and creativity for teaching fraction in Junior High School. Each group consists of five teachers. Followings are their works:

a. Scheme and Implementation

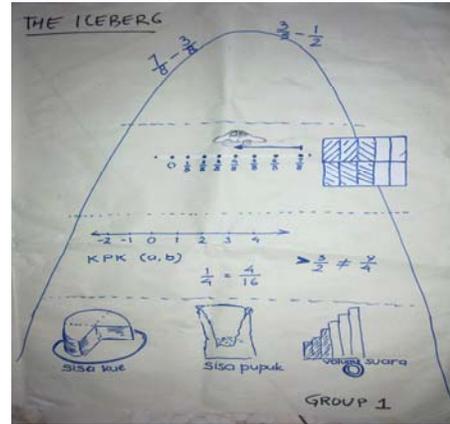
Group 1: Subtraction of Fraction Number

Posed Problem :

Agus has bought Tart Cake. He cut it into eight similar parts. He and his three younger brothers will each take one part of it. And he will save the rest. How many parts of the Cake Agus will save?

Teachers' activities:

By illustrating with Iceberg approach of Realistic Mathematics, the teachers then discuss the developed mathematical attitude and creativity may emerges.



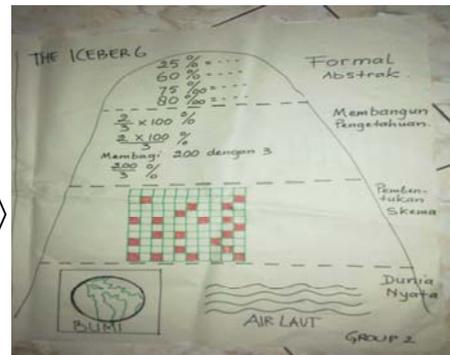
Group 2: Percentage and Permil

Posed Problem:

Two third of the Hemisphere consist of water. State into the percentage the amount of water?

Teachers' activities:

By illustrating with Iceberg approach of Realistic Mathematics, the teachers then discuss the developed mathematical attitude and creativity may emerges.



Group 3: Compare the Fractions

Posed Problem:

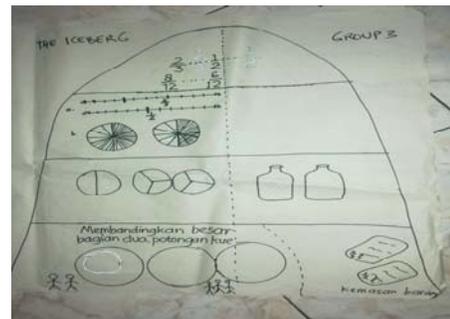
Compare the following fraction :

$2/3$ and $1/2$

Which is the bigger?

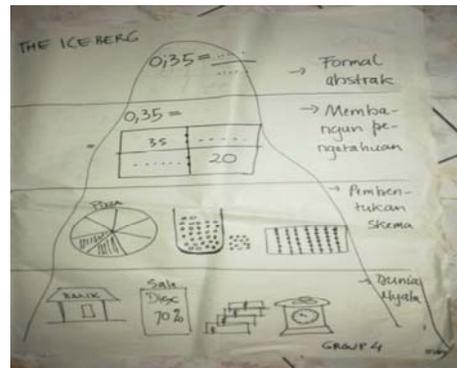
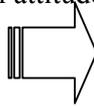
Teachers' activities:

By illustrating with Iceberg approach of Realistic Mathematics, the teachers then discuss the developed mathematical attitude and creativity may emerges.



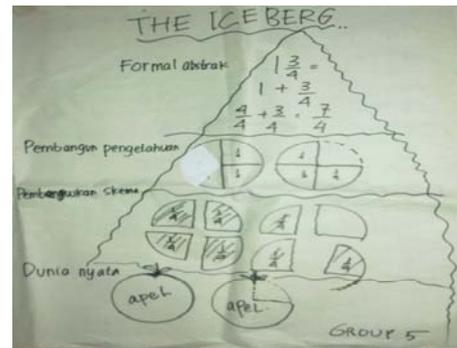
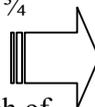
Group 4: Desimal Fraction

Posed Problem:
Write 0.35 into the simple form of fraction?
Teachers' activities:
By illustrating with Iceberg approach of Realistic Mathematics, the teachers then discuss the developed mathematical attitude and creativity may emerges.



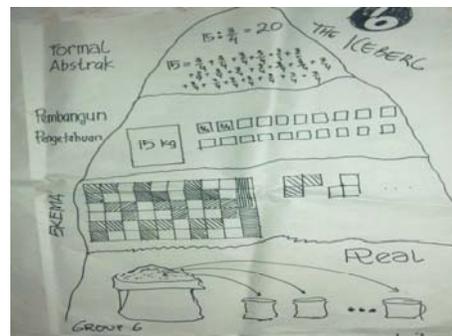
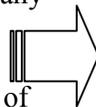
Group 5: Mix Number

Posed Problem:
Write the following mix number $1 \frac{3}{4}$ as simple fraction?
Teachers' activities:
By illustrating with Iceberg approach of Realistic Mathematics, the teachers then discuss the developed mathematical attitude and creativity may emerges.



Group 6: Division of Fraction

Posed Problem:
Fifteen kilogram of rice will be put into some smaller containers size $\frac{3}{4}$ kg. How many containers does it need?
Teachers' activities:
By illustrating with Iceberg approach of Realistic Mathematics, the teachers then discuss the developed mathematical attitude and creativity may emerges.



b. Result of the Second Sample

The results of teachers' activities and their reflections resulting the description of developed mathematical attitude and creativities covering 4 (four) aspects of the Iceberg approach of realistic mathematics: mathematical world orientation, material model, building mathematical relationship, and formal notation.

- 1) **Mathematical Attitude and Creativity through World Orientation**
 In the first start, it is not so easy for the teacher to develop and manipulate the concrete material as a mathematical world orientation. It seemed that there are some gaps between teachers' habit in doing formal mathematics and informal mathematics. Some teachers seemed uncertain whether initiating to introduce concrete model to their students or waiting until their students find for themselves. However most of the teachers believed that mathematical world orientation is important step to develop students' mathematical attitude and creativity.
- 2) **Mathematical Attitude and Creativity Through Material Model**
 For the material models, the teacher tried to identify the role of visual representation in setting up the relationship among fraction concepts, its relations and operations. To some extent the teachers need to manipulate the concrete model in such a way that they represent their and the students' knowledge of fractions. However it seemed that it was not automatically that the teachers find the supporting aspects of transition toward students' mathematical attitude. Most of the teacher understood that there were problems of intertwining between informal attitude and formal mathematical attitude.
- 3) **Mathematical Attitude and Creativity in forming Mathematical Relationship**
 In building mathematical relationship, the teachers perceived that the students need to develop their mathematical attitude as well as mathematical method. It needed for the teacher to facilitate students' questions, students' interactions and students' activities. Uncovering the pattern from material model and trying to connect with mathematical concepts are important aspects. There were possibilities for the students to find out various mathematical methods. To compare some mathematical method lead the students have a clear picture of the problems they faced.
- 4) **Mathematical Attitude at the level of Formal Mathematics**
 The teachers perceived that the formal notions of fractions, its relations and operations come up in line with inclination of sharing ideas of fraction concepts through small group discussion. The attitude of students reflect their interest when they get a clear understanding of formal notions of fractions. The teachers believed that the ultimate attitude of students is that they feel to have the mathematical concepts they found. More than this, the students will have an important capacity to a more kreatif and sophisticated solution for mathematical problems.

D. Results and Discussion

The subjects of the study were the 4th grade students of Primary School. The aim of the lesson was to encourage the students to understand and to apply factors and multiple of numbers to solve problems. The specific aim of the lesson was to encourage the students to understand the Least Common Multiple (LCM), to determine the Least Common Multiple (LCM), and solve problems which is related to LCM. The search in this lesson indicated that the students developed their *mathematical attitude* through *identifying or describing the specific mathematics, schematizing, formulating and visualizing a problem in different ways, discovering relations, discovering regularities, recognizing isomorphic aspect in different problems, transferring a real world problem to a mathematical problem.*

It was not always easy for the teacher to start initiating the students to think mathematically on the problems of understanding the Least Common Multiple (LCM) and determining the Least Common Multiple (LCM). At a certain occasion, the students seemed to not be able to grasp teacher's expectation of *mathematical thinking*. When the students were not able to take examples, the teacher posed the prepared problems. In group discussion, *students' mathematical attitude and creativity* was always started when the teacher posed the prepared problems written in the Work Sheet. There was a high spirit when the students found out the relevant references and resources to solve the problems. The students employed their pre-requisite knowledge of Calendar to find the pattern and relation i.e. from the problem of finding *the multiple of 7 days and the multiple of 8 days in one year*. Additional skill might have been a supporting factor that the students by themselves employed various aids to solve the problems such as calendar, hand-phone, and blank table prepared by the teacher.

In the effort of identifying or describing the specific mathematics, the students found the routine activities there are the concept of addition and subtraction i.e. $7 + 7 + 7 + 7 \dots$ or subtracting by 7 (for swimming); $8+8+8+8+8\dots$ or subtracting by 8 (for gardening). The concept of "frequency" emerged when there was a question of "how many times common activities". The concept of "frequency" was interpreted as the concept of "repeating addition or subtraction" i.e. the concept of multiple of number such as - *For 10 month, Shinta goes to swim $10 \times 5 = 50$ times*; - *For 10 month, Shinta goes to gardening $10 \times 4 = 40$ times*. In the effort of schematizing, formulating and visualizing, the students employed different ways to indicate that there are various ways in determining the multiple number of 7 and 8 e.g. using calendar, using series of numbers, using calculator and manipulating different symbols for 7 and 8. The students employed different schemas on determining the common multiple of 7 and 8 i.e. some students calculate the multiple of 7 for the whole year first then multiple for 8; and followed by counting the number of common activities in one year. Some students indicated first the common multiple of 7 and 8 (i.e. 56) and then counting the number of common activities in one year.

In the effort of to think mathematically on the problems of understanding the Least Common Multiple (LCM) and determining the Least Common Multiple (LCM), the series of sentences produces by the group indicated first *horizontal mathematization* then followed by *vertical mathematizaion*. Most of the students developed their *mathematical attitude and creativity* by employing subtraction, addition, multiplication and division to list multiple of 7 and 8. They indicated Common Multiple , as the mathematical concepts to answer the common

Shintas activities in one year. Students' *mathematical attitude and creativity* employed transition from daily language of mathematical language i.e. from common activities to common multiple. There was a student who jumped their concept to LCM due to he got it from "*informal private lesson*". In performing the *Vertical Mathematization* the students need the assistances from the teacher. The teacher encouraged the students to list more the multiple of 7 and the multiple of 8 and encouraged them to indicate the common multiple of 7 and 8.

In the certain *aspect of mathematical attitude and creativity* i.e. effort of discovering relations, the students discovered the relationship between "common activities" and "common multiple" i.e. 7 days and 8 days compare with "multiple of 7 and 8 = 56". In the aspect of discovering regularities, the students found that the concepts of regularities arouse from the concepts of "routine activities". The students recognized the isomorphic aspect in different mathematics problems i.e. the regularities emerged from isomorphic activities such as "swimming" and "gardening", "study club", "laboratory activities" or "going to library". There are the key concepts reflecting by the key word of how the students can transfer the real world problems to mathematical problem e.g. the concepts of "common", "regular", "routine", "number of", etc. Students' thinking of the concept of LCM were much contributed by teacher's employing real-life contexts as a starting point for their learning and simultaneously affected by the use of their own productions of formulas and strategies. In thinking the concept of LCM, interactions between teacher and students, students and students are the essential of their *mathematical attitude and creativity*. In other aspects of their mathematical attitude i.e. students' thinking of the concepts of LCM were influenced by the connection among the strands of mathematical concepts developed previously e.g. the concept of factor of numbers and by the connection with meaningful problems in the real world.

In the subjects of the study of the 5th grade students of Primary School, another aspect of students' attitude and creativity were to be emerged. The aim of the research was to promote mathematical attitude through teaching learning the Volume of Cube and Rectangular Parallelepiped. The aim of the lesson was to encourage the students to find the volume of Cube and Rectangular Parallelepiped and applying them to solve related problems. The specific aim of the lesson were to find the volume of Cube and Rectangular Parallelepiped and to solve the problems related to the volume of Cube and Rectangular Parallelepiped. Most of the students strived very hard to understand the concepts of Pyramid, Prism, Cone and Cube (Lesson Object). There are some ways in which the students strive to understand the concept: manipulation of the Model of Three Dimensional Geometrical Object, questioning to the

teacher, questioning to other students, manipulation of mathematics net. Some students got the concept of geometrical shapes from informal learning i.e. from their parents or from additional lesson outside the school. The students were able to indicate the similar of geometrical shapes in daily lives; they also tried to identify the function of identical geometrical shape in daily live. The students perceived that their teacher has important role in helping them to perform *mathematical attitude and creativity*.

Different geometrical shapes have different level to be understood. The concept of a cone was the most difficult for the students to understand. The students have difficulties how to calculate the number of the side of the cone. To solve the problems some students delivered the questions to the teacher and the other asked to their classmates. The students tried to employ their pre-requisite knowledge in clarifying the difficult concepts. Some students developed the step in order to understand the difficult concepts i.e. by asking first about the nature of the concept of a cone and then to ask to the teacher about its characteristics. However, some students inevitably jumped without any pattern due to have no systematic knowledge of geometrical shapes. There were the students who tended to be silent and passive if they still do not understand the difficult concepts.

Students' attitude to understand the difficult concept of geometrical shapes depended on the context and the schema of teaching. If the teacher communicate with the students in less formal, the students' attitude reflected that they have no constraint to ask to their teachers. Some students performed they attitude that their teacher should provide the complete and good quality of teaching aids. However, they also reflected their attitude that they enjoy getting assignments from the teacher. Most of the students employ inductive thinking i.e. by trial and error to answer teacher's questions; some of them tried to sketch the geometrical shapes and compare with different size of the models. The students tended to re-state the explanations and get attention from their teacher and their classmates to confirm whether their ideas were true.

The subjects of the study of the 8th grade students of Junior High School also revealed students mathematical attitude and creativity. The aim of the lesson was to understand the characteristics of cylinder, cone, sphere and to determine their measures. The specific aims of the lesson were to identify the formula of the total area of right circular cylinder and to identify the formula of the area of sphere. The students revealed their *mathematical attitude and creativity* by manipulating *Concrete Model* of the *Right Circular Cylinder, Sphere and Right Circular Cone* in order to identify its components. They performed *mathematical abstractions* when the teacher gave them some questions or when the teacher let them to work in group.

Some students defined the *concept* of *Right Circular Cylinder* as its *functions* in daily life e.g. “A *Right Circular Cylinder* is the storage to keep something like pen, pencil, etc.” There were students who defined a *Sphere* by giving the example in daily life e.g. ball, tennis-ball, etc. Students’ *abstractions* of *Sphere* resulted the investigation of its components i.e. the *radius* and *diameter*.

There were many ways in which the students *idealized* the geometrical concept. They mostly confirmed the concept to the teacher and asked to their mates. Sometimes they performed their *idealization* by commenting others work. Some students asked to the teacher why the *lateral area of cylinder is equal to the area of its rectangle* and why the *volume of cylinder is equal to three times the volume of its cone*? *Analogical thinking* happened when the students perceived that finding the lateral area of *Right Circular Cylinder* is similar to finding the area of its rectangle; and, finding the area of *Sphere* is similar to finding the area of its surface i.e. covering its surface by twisting around with the rope. In sum, the concepts of geometrical shapes are mostly perceived to be analogical with examples in daily life e.g. the *right circular cone* was perceived as a *traditional hat*. In performing their *analogical thinking* the students frequently used strategic terminologies such as “*similar to*”, “*compare with*”, “*the example of*”, and “*the function of*”.

Students’ *inductive thinking* involved *Concretization and method of abstraction* in the area of *problem formation* and *comprehension*. When the students, they who had known the certain concepts, were paced to perform *inductive thinking* they tend to reconfirm their concepts. *Inductive thinking* was spread from the beginning activities to the ultimate accomplishment when the students were paced to do so. The students developed *method of abstraction* to observe the given model of right circular cylinder and strived to identify the components of the right circular cylinder in order to define the concept of right circular cylinder. Students’ *inductive thinking* were also related to *establishing perspective* in which the students employed concrete model to search the total area of right circular cylinder and brook-down the model of right circular cylinder into its components: two congruent circles and one oblong.

Logical organization of mathematical concept happened in all context of mathematical method: *idealization, abstraction, deduction, induction and simplification*. *Logical organizations* of mathematical concept can be indicated from the following example of students’ questions: Why the lateral area of cylinder is equal to the area of its rectangle?, Why the volume of cylinder is equal to three times the volume of its cone?, What happened if

we do not carefully cover the surface of the sphere in which we use the rope for twisting around?, and Is it true that that the area of the surface of sphere is equal to 4 times the area of its circle? Problem *formation* and *comprehension* emerged when the students: observe given model of right circular cylinder, observe given model of Sphere, and observe given model of right circular cone; identify the components of the right circular cylinder, sphere and right circular cone; define the concept of right circular cylinder, sphere and right circular cone; and get questions and notices from teacher to search the concepts.

E. Conclusion

Many teachers bring a great deal of informal understanding of mathematical concepts to their effort in developing the model for teaching. In developing the model of teaching, the teachers expected that there is a tendency that their students will perform their attitude and creativity not in the isolated context of learning. Although a certain model e.g. the iceberg model reinforces *the students' mathematical attitude and creativity* to construct their own concepts of mathematics, there are still difficulties for the students to solve problems expressed symbolically. However, they performed *theirs attitude* to solve similar problems expressed in the context of real-world situations. Most of the teachers admitted that representation of mathematical concepts lead students' mathematical attitude in the situation that they perceived mathematics be a very abstract and difficult task to learn. Meanwhile, the teachers also found that the iceberg model is very important and useful approach to develop mathematical attitude and creativity in Junior High School.

REFERENCES

- Cheng Chun Chor Litwin, 2007, Development of mathematical thinking in Hong Kong schools in *Progress report of the APEC project: "Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II) : Lesson Study focusing on Mathematical Thinking"* , Center for Research on International Cooperation in Educational Development (CRICED) , University of Tsukuba : Maeda Printing Co., Ltd
- Emilie A. Naiser ,et al., 2004, Teaching Fractions: Strategies Used for Teaching Fractions to Middle Grades Students. *Journal of Research in Childhood Education*, Vol. 18
- Ernest, P. (1991). *The Philosophy of Mathematics Education*. Hampshire: The Falmer Press.
- Fou Lai Lin, 2007, Designing mathematics conjecturing activities to foster thinking and constructing actively in *Progress report of the APEC project: "Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II) : Lesson Study focusing on Mathematical Thinking"* , Center for Research on International Cooperation in Educational Development (CRICED) , University of Tsukuba : Maeda Printing Co., Ltd

- Freudenthal, H. (1991). *Revisiting Mathematics Education. China Lectures*. Dordrecht: Kluwer Academic Publishers.
- Gravemeijer, K.P.E. (1994). *Developing Realistic Mathematics Education*. Utrecht: CD-β
- Imprasitha and Loipha, 2007, Developing student's mathematical thinking through lesson study in Thailand in *Progress report of the APEC project: "Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II) : Lesson Study focusing on Mathematical Thinking"* , Center for Research on International Cooperation in Educational Development (CRICED) , University of Tsukuba : Maeda Printing Co., Ltd
- Jung Inchul, 2007, Perspectives on mathematical thinking in Korea in *Progress report of the APEC project: "Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II) : Lesson Study focusing on Mathematical Thinking"* , Center for Research on International Cooperation in Educational Development (CRICED) , University of Tsukuba : Maeda Printing Co., Ltd
- Lim Chap Sam & Hwa Tee Yong , 2007, Promoting mathematical thinking in the Malaysian classroom: issues and challenges in *Progress report of the APEC project: "Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II) : Lesson Study focusing on Mathematical Thinking"* , Center for Research on International Cooperation in Educational Development (CRICED) , University of Tsukuba : Maeda Printing Co., Ltd
- Madidah Khalid, 2007, Lesson study: promoting student thinking on the concept of least common multiple (LCM) through realistic approach in the 4th grade of primary mathematics teaching in *Progress report of the APEC project: "Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II) : Lesson Study focusing on Mathematical Thinking"* , Center for Research on International Cooperation in Educational Development (CRICED) , University of Tsukuba : Maeda Printing Co., Ltd
- Mag. Francisco Cerda Bonomo, 2007, Mathematical thinking like angular stone in the understanding of real world phenomena in *Progress report of the APEC project: "Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II) : Lesson Study focusing on Mathematical Thinking"* , Center for Research on International Cooperation in Educational Development (CRICED) , University of Tsukuba : Maeda Printing Co., Ltd
- Marsigit, Atmini Dhoruri, Sugiman and Ali Mahmudi , 2007, Lesson study: promoting student thinking on the concept of least common multiple (LCM) through realistic approach in the 4th grade of primary mathematics teaching in *Progress report of the APEC project: "Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II) : Lesson Study focusing on Mathematical Thinking"* , Center for Research on International Cooperation in Educational Development (CRICED) , University of Tsukuba : Maeda Printing Co., Ltd
- Marsigit, 2008, Promoting Primary And Secondary Mathematical Thinking Through The Series Of School-Based Lesson Study Activities, Paper to be presented at 1st International Conference on Lesson Study FMIPA UPI 31 July to August 2, 2008
- Masami Isoda, 2007, Developing mathematical thinking in classroom in *Progress report of the APEC project: "Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II) : Lesson Study focusing on Mathematical Thinking"* , Center for Research on International Cooperation in Educational Development (CRICED) , University of Tsukuba : Maeda Printing Co., Ltd
- Shangzhi Wang, Dan Zhang and Siming Zhang, 2007, Mathematical thinking in primary school and junior high school in *Progress report of the APEC project: "Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II) : Lesson*

- Study focusing on Mathematical Thinking*” , Center for Research on International Cooperation in Educational Development (CRICED) , University of Tsukuba : Maeda Printing Co., Ltd
- Shigeo Katagiri, 2007, Mathematical thinking and how to teach it in *Progress report of the APEC project: “Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II) : Lesson Study focusing on Mathematical Thinking”* , Center for Research on International Cooperation in Educational Development (CRICED) , University of Tsukuba : Maeda Printing Co., Ltd
- Soledad A. Ulep, 2007, Enhancing elementary school mathematics teachers’ mathematical thinking through in-service training in *Progress report of the APEC project: “Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II) : Lesson Study focusing on Mathematical Thinking”* , Center for Research on International Cooperation in Educational Development (CRICED) , University of Tsukuba : Maeda Printing Co., Ltd
- Sutarto Hadi, 2008, RME in Primary School, Power Point Presentation
- Tenoch Cedillo and Marcela Santillan, 2007, Mexican perspective on mathematical thinking in *Progress report of the APEC project: “Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II) : Lesson Study focusing on Mathematical Thinking”* , Center for Research on International Cooperation in Educational Development (CRICED) , University of Tsukuba : Maeda Printing Co., Ltd
- Yeap Ban Har, 2007, Developing mathematical thinking in Singapore elementary schools in *Progress report of the APEC project: “Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II) : Lesson Study focusing on Mathematical Thinking”* , Center for Research on International Cooperation in Educational Development (CRICED) , University of Tsukuba : Maeda Printing Co., Ltd
- Zulkardi (2006). *How to Design Mathematics Lessons based on the Realistic Approach?*
Retreived 2006 <<http://www.google.com>>