CHAPTER 6
SPIN OF ELECTRON

A. Introduction

In Stern-Gerlach experiment that conducted in 1922, silver atom beams is passed through in inhomogeneus magnetic field $\vec{B}$ which have perpendicular direction from the incident atom beams. The results of experiment show that there is space quantification of magnetic dipole moment of silver atom.

Figure 6.1. Stern-Gerlach Experiment

Further more in 1927, Phipps and Taylor use hydrogen atom with low tempwrature (most hydrogen atom have orbital quantum number l = 0 and magnetic quantum number m = 0) and the beams are passed through in magnetic field as Stern-Gerlach experiment. The experiment result show that space quantification for hydrogen atom with orbital quantum number l = 0 and magnetic quantum number m = 0; especially in 2 different orientation. Because of the magnitude of angular momentum L = 0 and L_z = 0, so it is not possible that phenomenon come from orbital electron. They said that electron rotate its axis, as earth rotate its axis. This phenomenon is called spin motion.

B. Magnetic Dipole Moment

Magnitude of magnetic dipole moment of electron caused by spin motion fulfilled equation:

$$\vec{\mu}_S = -\frac{g_s \mu_B}{\hbar}\vec{S} \tag{6.1}$$

where: μ_B = Bohr magneton, g_s = gyration factor, and $\vec{S}$ spin angular momentum of electron that rotate its axis. It is probable that magnetic dipole that observed is from nucleus. If this is occurred, it means that the magnitude of magnetic dipole moment was in area $\frac{e\hbar}{2M}$, where M is the mass of nucleus. The experiment result show that magnitude of magnetic dipole moment near from $\frac{e\hbar}{2m_e}$. It is predicted that the magnetic dipole moment is from electron.

Goudsmit and Uhlenbeck studied spectral lines of hydrogen and alkaline metal. They found that certain lines in optical spectrum of hydrogen and alkaline metal show that it is found line pairs. This phenomenon is called *fine structure*. Sommerfeld can explaine the fine structure using Bohr atomic model. Fine structure in hydrogen atom occurred caused by the change of electron mass which move in high speed. This assumption can not explaine fine structure in optical spectrum of alkaline atom. The truth of Sommerfeld assumption is questioned.

To explain fine structure Goudsmit and Uhlenbeck in 1925 suggested an assumtion:" *Electron has angular momentum and magnetic dipole moment intrinsically, which has z component which stated by spin magnetic quantum number m_s which only has 2 value of +1/2 and -1/2*".

If the assumption accepted, then electron in hydrogen atom need four quantum number as follow:

n : principal quantum number
l : orbital quantum number
m: magnetic quantum number (usually has symbol of m_l)
m_s: spin magnetic quantum number (+1/2 dan -1/2)

C. Spin Elektron dan Nilai Eigen Atom H

Analog with angular momentum, magnitude of spin angular momentum of electron $\vec{S}$ can be written:

$$S = \sqrt{s(s+1)}\hbar \qquad (6.2)$$

Component of $\vec{S}$ in z direction is:

$$S_z = m_s\hbar \qquad (6.3)$$

In this case *s* related with m_s, for m_s has 2 values which has difference of 1 ($\Delta m_s = 1$), mean while its value is laid between –s and +s.

m_s = -1/2, +1/2
s = 1/2

The result of experiment show that $g_s m_s = \pm 1$, then it is obtained gyration factor or spin factor has value of :

$g_s = 2$

Further more if spin motion is applied in hydrogen atom system, the motion must be represented in a function of χ_{ms}, and operator S_{op} and S_{zop} operate with rule as follow:

$$S^2{}_{op}\chi_{ms} = s(s+1)\hbar^2\chi_{ms} = \frac{3}{4}\hbar^2\chi_{ms} \qquad (6.4)$$

$$S_{zop}\chi_{ms} = m_s\hbar\chi_{ms} \qquad (6.5)$$

There are 2 spin functions:

χ_+ for m_s = +1/2, and
χ_- for m_s = -1/2

Operator S_{op} and S_{zop} only work in function of χ_+ and χ_- and does mot work in coordinat function (r,θ,φ). Also differential operator does not work in a function of χ_{ms}. Space quantification of spin angular momentum is presented in Figure 6.2.

Wave function of hydrogen atom can be notated:

$$\psi_{n,l,m_l,m_s} = \left| n\ l\ m_l\ m_s \right\rangle \qquad (6.6)$$

By this model the relation of its eigen value fulfilled:

$$H_{op}\left| n\ l\ m_l\ m_s \right\rangle = E_n\left| n\ l\ m_l\ m_s \right\rangle \qquad (6.7)$$

$$L^2{}_{op}\left| n\ l\ m_l\ m_s \right\rangle = l(l+1)\hbar^2\left| n\ l\ m_l\ m_s \right\rangle \qquad (6.8)$$

$$L_{Zop}\left| n\ l\ m_l\ m_s \right\rangle = m_l\hbar\left| n\ l\ m_l\ m_s \right\rangle \qquad (6.9)$$

$$S^2{}_{op}\left| n\ l\ m_l\ m_s \right\rangle = s(s+1)\hbar^2\left| n\ l\ m_l\ m_s \right\rangle \qquad (6.10)$$

$$S_{Zop}\left| n\ l\ m_l\ m_s \right\rangle = m_s\hbar\left| n\ l\ m_l\ m_s \right\rangle \qquad (6.11)$$

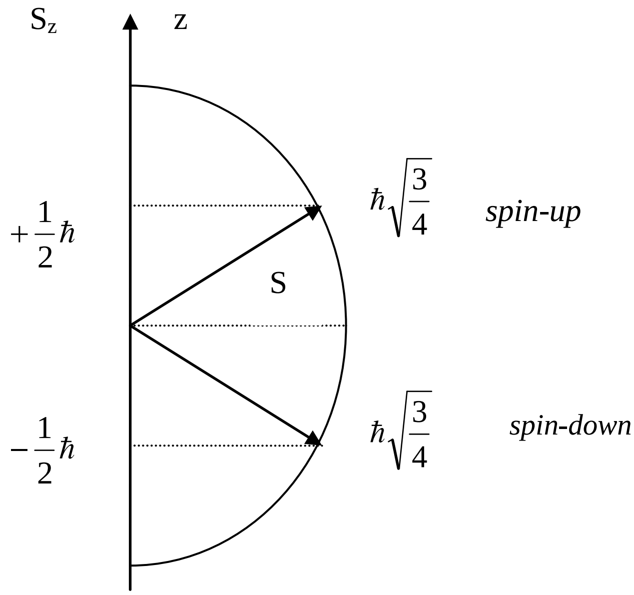

Figure 6.2. Space Quantification of Spin Angular Momentum

For normalized wave function:

$$\langle n\ l\ m_l\ m_s | n\ l\ m_l\ m_s \rangle = 1 \tag{6.12}$$

$$\langle \chi_+ | \chi_+ \rangle = 1 \tag{6.14}$$

$$\langle \chi_- | \chi_- \rangle = 1 \tag{6.15}$$

$$\langle \chi_+ | \chi_- \rangle = \langle \chi_- | \chi_+ \rangle = 0 \tag{6.16}$$

D. Hydrogen Atom with Spin in External Magnetic Field

Magnetic dipole moment of hydrogen atom is:

$$\vec{\mu}_H = \vec{\mu}_L + \vec{\mu}_S \quad \text{where H (hydrogen)} \tag{6.17}$$

Then it can be determined:

$$\begin{aligned} \vec{\mu}_H &= -\frac{eg_l}{2m_e}\vec{L} - \frac{eg_s}{2m_e}\vec{S} \\ \vec{\mu}_H &= -\frac{e}{2m_e}\left(g_l\vec{L} + g_s\vec{S}\right) \end{aligned} \tag{6.18}$$

It have been known that $g_l = 1$ and $g_s = 2$; then magnetic dipole moment of hydrogen atom fullfilled:

$$\vec{\mu}_H = -\frac{e}{2m_e}\left(\vec{L} + 2\vec{S}\right) \tag{6.19}$$

Potential energy of hydrogen atom in extdernal magnetic field:

$$V_B = -\vec{\mu}_H.\vec{B} = \frac{e}{2m_e}\left(\vec{L}.\vec{B} + 2\vec{S}.\vec{B}\right)$$

$$V_B = \frac{eB}{2m_e}\left(L_z + 2S_z\right) \text{ direction of } \vec{B} \text{ is parallel with z axis} \tag{6.20}$$

The attendance of external magnetic field $\vec{B}$ cause the shift of total energy in hydrogen atom:

$$\Delta E = \frac{eB}{2m_e}\left(m_l + 2m_s\right) = \frac{B\mu_B}{\hbar}\left(m_l + 2m_s\right) \tag{6.21}$$

E. Diagram of Energy States in Hydrogen Atom

Further more in Figure 6.3 is presented diagram of energy states in hydrogen atom in external magnetic field $\vec{B}$.

- Quantum state with different principal quantum number are placed in vertical arrange.
- Each column in the diagram according to the same orbital quantum number l.
- In the left side is energy states without external magnetic field.
- In the right side, energy states are splited caused by the effect of extderal magnetic field $\vec{B}$.
- The number beside the splitting energy state are number of different energy state in external magnetic field $\vec{B}$.
- For example 3p; it has 5 different energy states for 6 sets of quantum number.
- $\Delta E = \frac{B\mu_B}{\hbar}\left(m_l + 2m_s\right) = A\left(m_l + 2m_s\right)$

$$
\begin{array}{lclcllll}
 & & & & n & l & m_l & m_s \\
\Delta E & = & +2A & \Leftrightarrow & 3 & 1 & +1 & +\frac{1}{2} \\
\Delta E & = & +A & \Leftrightarrow & 3 & 1 & 0 & +\frac{1}{2} \\
\Delta E & = & 0 & \Leftrightarrow & 3 & 1 & +1 & -\frac{1}{2} \\
\Delta E & = & 0 & \Leftrightarrow & 3 & 1 & -1 & +\frac{1}{2} \\
\Delta E & = & -A & \Leftrightarrow & 3 & 1 & 0 & -\frac{1}{2} \\
\Delta E & = & -2A & \Leftrightarrow & 3 & 1 & -1 & -\frac{1}{2}
\end{array}
\qquad (6.22)
$$

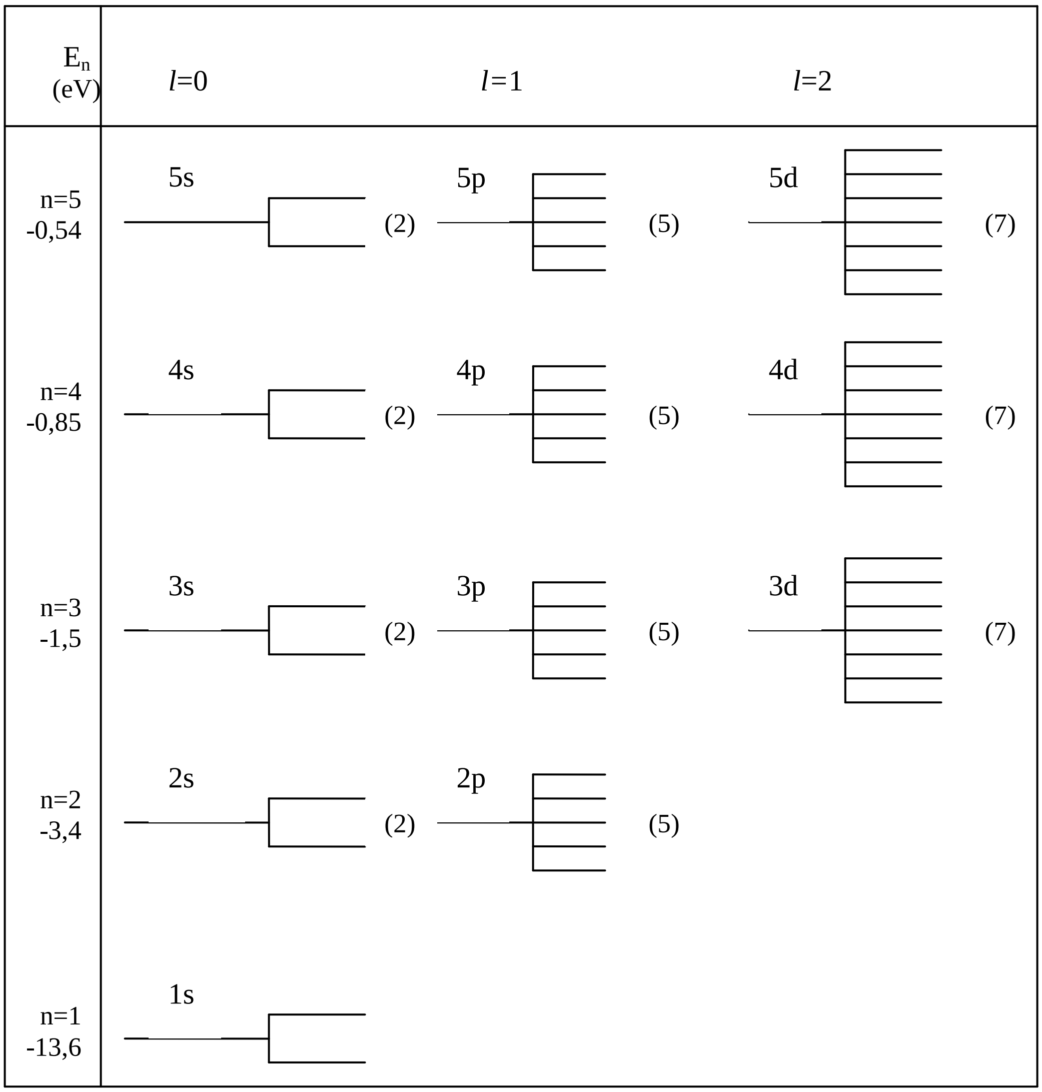

Figure 6.3. Diagram of Energy States in Hydrogen Atom

Kaedah seleksi yang berlaku adalah:

$$\Delta l = \pm 1$$
$$\Delta m_l = 0,\ \pm 1$$
$$\Delta m_s = 0$$

F. Precisition Motion (Larmor)

In this model, there are no realtion between orbital angular momentum $\vec{L}$ and spin angular momentum $\vec{S}$. Each angular momentum is quantificatied independenly each other. Magnetic induction field $\vec{B}$, each angular momentum is quantificatied in space by itself, and both angular momentum have precision motion from the same z axis, as shown in Figure 6.4.

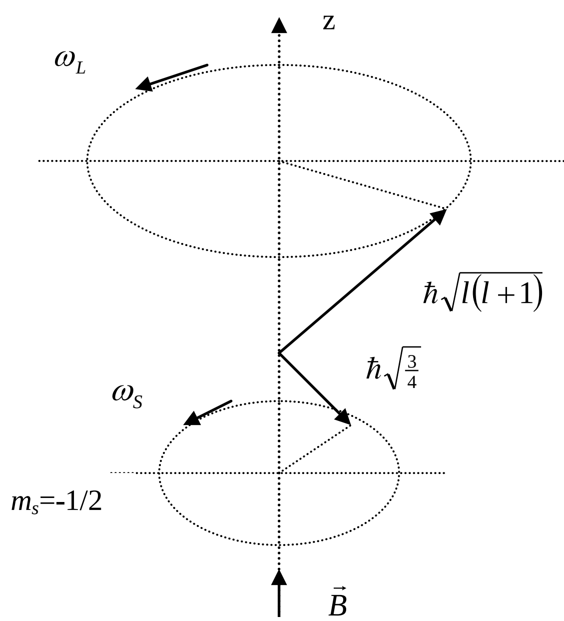

Figure 6.4. Precision Motion of $\vec{L}$ and $\vec{S}$

In orbital motion $\vec{L}$ it is produced force moment:

$$\vec{\tau} = \vec{\mu}_L x\vec{B} = -\frac{g_l \mu_B}{\hbar}\vec{L}x\vec{B} \tag{6.23}$$

Larmor frequency for precision motion of $\vec{L}$ around z axis:

$$\vec{\omega}_L = \frac{g_l \mu_B}{\hbar} \vec{B} \qquad (6.24)$$

For precision motion $\vec{S}$, it is produced force moment:

$$\vec{\tau} = \vec{\mu}_S x\vec{B} = -\frac{g_s \mu_B}{\hbar} \vec{S} x\vec{B} \qquad (6.25)$$

Larmor frequency for precision motion of $\vec{S}$ around z axis:

$$\vec{\omega}_S = \frac{g_s \mu_B}{\hbar} \vec{B} \qquad (6.26)$$

The value of Larmor frequency caused by spin angular momentum is twice of Larmor frequency caused by orbital angular momentum, because g_s =2 while $g_l = 1$.

G. Problem

1. Determine the energy shift in hydrogen atom with orbital quantum number of l = 3 for each possible value of m_s.

2. Hydrogen atom is placed in external magnetic field of 1 Tesla in z axis direction. Detemine the Larmor frequency for precision motion of orbital angular momentum and spin angular momentum!

3. In external magnetic field, how many splitting energy states is occurred for subshell 6f?

H. Reference

Yusman Wiyatmo. (2008). Fisika Atom dalam Perspektif Klasik, Semiklasik, dan Kuantum. Yogyakarta: Pustaka Pelajar.