CHAPTER 5
ANGULAR MOMENTUM

A. Introduction

Physical observables as angular momentum and energy of hydrogen atom are quantificated (discret). The effect of magnetic field in hydrogen atom can change the symmetry characteristic in atom by degeneration. The effect of external magnetic field in hydrogen atom can split the energy of atom that has a consequence of energy spectrum splitting that emitted when transition from higher energy state to lower energy state is occurred. The effect of magnetic field in hydrogen atom also can quantificate the direction of angular momentum in space.

Further more, to discuss some thing above, in this chapter will be discussed obout: angular momentum, Quantification of angular momentum and energy of hydrogen atom, effect of homogeneus magnetic fied in hydrogen atom, Energy states of hydrogen atom, and transition of energy states in hydrogen atom.

B. Definition of Angular Momentum

$$\vec{L} = \vec{r} x \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix} = \hat{i}(yp_z - zp_z) - \hat{j}(xp_z - zp_x) + \hat{k}(xp_y - yp_x) \tag{5.1}$$

Related with hydrogen atom:

$\vec{r}$ is position vector of elektron from nucleus
$\vec{p}$ is linear momentum of electron motion from nucleus

C. Quantification of Angular Momentum and Energy in Hydrogen Atom

Angular momentum $\qquad\qquad : \langle L^2 \rangle = l(l+1)\hbar$ (5.2)

Z Componet of angular momentum : $\langle L_z \rangle = m\hbar$ (5.3)

Energy system of hydrogen atom $\qquad : E_n = -\dfrac{13,6}{n^2} eV$ (5.4)

Quantum number of l and m determine the orbit of electron.

D. Effect of External magnetic Field in Hydrogen Atom

Potential function V(r) of hydrogen atom has a spherical symmetry. The high symmetry gives a very high degeneration. Energy E_n of hydrogen atom system is characterized by principal quantum number n, the fact it relevan with n^2 eigen function which all of them have principal quantum number n.

For example for principal quantum number n = 2, hydrogen atom has total energy of $E_2 = -3,4eV$. The quantum state has 4 different eigen function ($\psi_{n,l,m}$), i.e: $\psi_{2,0,0}; \psi_{2,1,0}; \psi_{2,1,1}$; and $\psi_{2,1,-1}$. If the atom is placed in homogeneus magnetic field so the potential of electron is not spherical simetry. The attendance of magnetic field with magnetic induction $\vec{B}$ will decrease degree of its physical problem. Eigen function that have the same energy, then they have different energy.

The relation between magnetic dipol and angular momentum is written t:

$$\vec{\mu}_L = -\frac{e}{2m_e}\vec{L} \tag{5.5}$$

The positive z axis direction is the direction of magnetic field $\vec{B}$. Component of magnetic dipol in z axis direction is:

$$\vec{\mu}_{L_z} = -\frac{e}{2m_e}L_z = -\frac{\mu_B}{\hbar}L_z \tag{5.6}$$

where $\mu_B = Bohr\ magneton = \dfrac{e\hbar}{2m_e} = 9,2732.10^{-24}\ Joule\ Tesla^{-1} = 5,6564\ eV.Tesla^{-1}$

A magnetic dipole of $\vec{\mu}_L$ which placed in magnetic fied with intensity of $\vec{B}$ will have potential energy:

$$V = -\vec{\mu}_L.\vec{B} = \frac{e}{2m_e}\vec{L}.\vec{B} = \frac{\mu_B}{\hbar}\vec{L}.\vec{B} \tag{5.7}$$

Because z direction is taken parralely to $\vec{B}$ direction so:

$$V = \frac{\mu_B}{\hbar}BL_z \tag{5.8}$$

Kadang-kadang energi potensial V dapat dituliskan dalam bentuk:

$$V = g_l\frac{\mu_B}{\hbar}BL_z \tag{5.9}$$

where $g_l = 1$, orbital factor.

Total energy of hydrogen atom E' in external magnetic field $\vec{B}$ is:

$$\langle E' \rangle = E_n + m\hbar g_l \frac{\mu_B}{\hbar} B \tag{5.10}$$

The change of enegy of hydrogen atom which caused by the attendance of external magnetic field $\vec{B}$ is:

$$\Delta E = \langle E' \rangle - E_n$$
$$\Delta E = (g_l \mu_B B)m \tag{5.11}$$

where m is magnetic quantum number. Based on equation (5.11) it can be described that:
1. All quantum states with magneticquantum number m = 0, if the atom placed in magnetic field, its energy does not change.
2. If the quantum states have magnetic quantum number m>0 so ΔE>0. The energy of system will increase if atom is placed in magnetic $\vec{B}$.
3. On the other hand, it is occurred ΔE<0 if m<0. Energy of siytem will be decrease if atom is placer in external magnetic field $\vec{B}$.

E. Energy States of Hydrogen Atom

n	E_n(eV)	s(l=0)	p(l=1)	d(l=2)	f(l=3)
∞	0				
5	-0,54	—— 1	—— 3	—— 5	—— 7
4	-0,85	—— 1	—— 3	—— 5	—— 7
3	-1,50	—— 1	—— 3	—— 5	
2	-3,4	—— 1	—— 3		
1	-13,6	—— 1			

Figure 5.1. Energy States of Hydrogen Atom

In vertical direction are placed energy in varies of principal quantum number n, starting with n=1 which has energy of -13,6 eV. The highest state is placed by principal quantum number

n=∞ which has energy of 0. Energy states with the same n but different in l is placed in horizontal direction.   All energy states with the same   n although different in l are overlapping.

For example energy states with principal quantum number n=4, have:

- o   1 eigen function with l=0.
- o   3 eigen function with l =1 (m=-1,0, +1)
- o   5 eigen function with l =2 (m=-2, -1, 0, +1, +2)
- o   7 eigen function with l =3 (m=-3, -2, -1, 0, +1, +2, +3)

Numbers 3, 5, and 7 related with the splitting of energy states.
The splitting of energy states for principal quantum number n = 4 is:

1+3+5+7=16
or = n^2=16

F. Transition of Energy States in Hydrogen Atom

Transition from state   to another state can be drawn in vertical scheme.   For example transition from subshell 3p to 2s is shown in Figure 5.2 as follow:
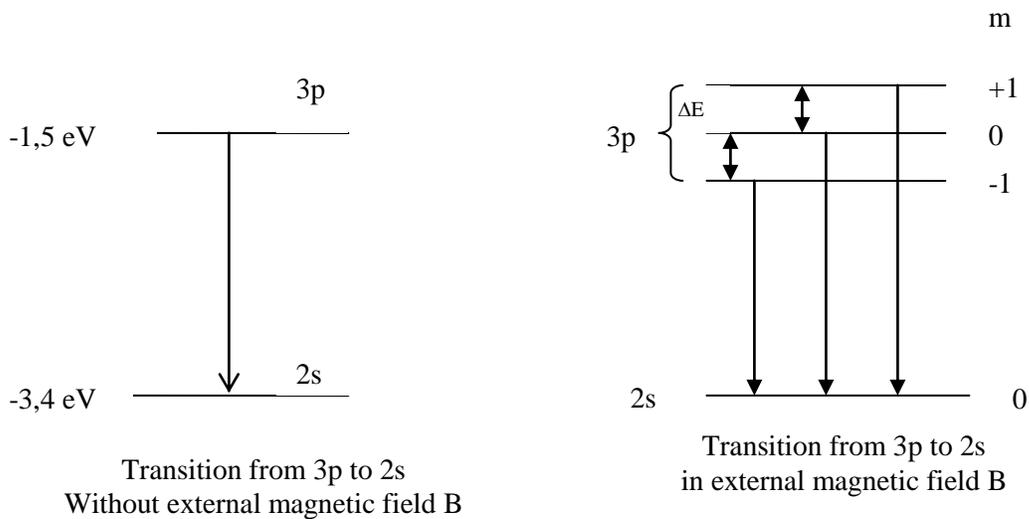

Figure 5.2. Energy Transition in Hydrogen Atom

The shift of energy state $\Delta E = (g_l \mu_B B)$, because m=+1 or m=-1. Without the attendance of external magnetic field, the transition energy is:

$$E_3 - E_2 = (-1,5 - (-3,4))eV = 1,9eV \quad \text{1 spektal line.}$$

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In external magnetic field, there are 3 different of energy:

$$1,9eV + \Delta E = E_1$$
$$1,9eV = E_2 \qquad \text{3 spectral lines.} \qquad (5.12)$$
$$1,9eV - \Delta E = E_3$$

In ground state, it doesn'n split because its magnetic quantum number m=0.

Further more, it is presented the transition which permitted from subshell 4d to 3p. Not all transition are permitted. The rule selection of transition is:

$$\Delta l = \pm 1$$
$$\Delta m = 0, \ \pm 1 \qquad (5.13)$$

The attendance of external magnetic field $\vec{B}$ cause the energy hydrogen atom system:

$$\Delta_m E = \left(g_l \mu_B B \right) m = \Delta_1 E m \qquad (5.14)$$

In this case: $\Delta_1 E = g_l \mu_B B$

In the attendance of external magnetic field, it will be 3 energy transition as follow:

$$0,65 \ eV + \Delta_1 E$$
$$0,65 \ eV \qquad (5.16)$$
$$0,65 \ eV - \Delta_1 E$$

In spectral lines, it only can be sawn 3 spektral lines which have different wave length. The 3 lines ae from 9 transition from subshel 4d to 3p. It means that single line in emission spectrum of hydrogen atom will split to be *triplet* in the effect of homogeneus external magnetic field. If the magnetic field is large enough, the splitting can be observed. This phenomenon is called Zeeman effect.

m

+2
+1
0
-1
-2

+1
0
-1

-0,85 eV ——— 4d   m=-2, -1, 0, +1, +2

0,65 eV

3p
-1,5 eV ———   m=-1, 0, +1

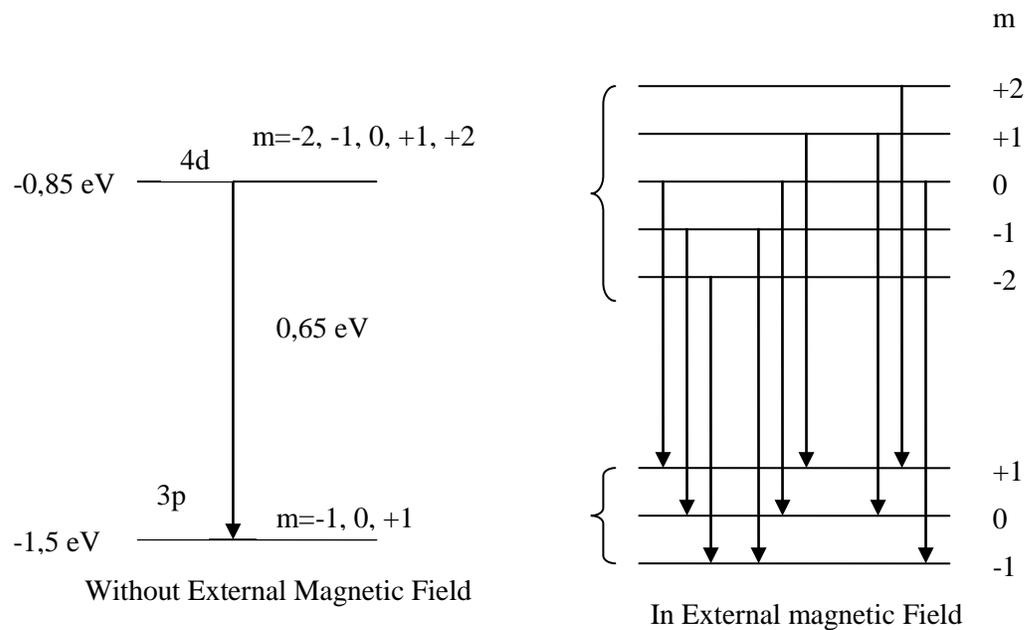Without External Magnetic Field

In External magnetic Field

Figure 5.3. Transition from Subshell 4d to 3p

Energy shift of hydrogen atom is from potential energy that is obtained from interaction of dipole moment and external magnetic field. The energy shift is quantificated. It means that the direction of angular meomentum $\vec{L}$ is quantificated in space that is effected by magnetic field $\vec{B}$. In free space, there is no specific orientation, all orientations are the same condition, and $\vec{L}$ has arbitrary position in space.

If in the space is effected by external magnetic field $\vec{B}$ so the change of space characteristic is occurred. Initial symmetry of spherical symmetry changes to the lower symmetry. Because interaction between maka terjadi perubahan sifat ruang tersebut. Simetri awal yang berupa simetri bola berubah $\vec{\mu}_L$ and $\vec{B}$ produce total energy shift of hydrogen atom which is quantificated cause the angular momentum of hydrogen atom have certain orientation.

For example l=2:

o   The magnitude of angular momentum: $\langle L \rangle = \sqrt{l(l+1)}\hbar = \hbar\sqrt{6}$
o   Component L in zdirection: $L_z = m\hbar = -2\hbar, \ -\hbar, \ 0, \ +\hbar, \ +2\hbar$

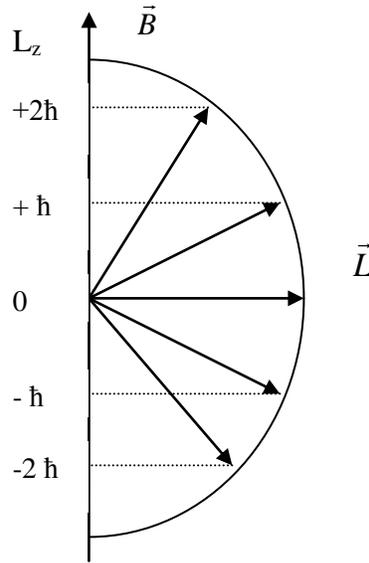Quantification of angular momentum in space can be drawn as follow:

Figure 5.4. Space Quantification of Angular Momentum in Magnetic Field

The attendance of homogeneus magnetic field in space will quantificate orientation of angular momentum in space. Qunatification of angular momentum in space is recorded in the shift of total energy in atom. Angular momentum $\vec{L}$ also do spin motion from z axis. In the effect of magnetic induction $\vec{B}$, electron in hydrogen atom which has magnetic dipole moment $\vec{\mu}_L$ will get force moment:

$$\vec{\tau} = \vec{\mu}_L x\vec{B} = -\frac{g_l \mu_B}{\hbar} \vec{L}x\vec{B} \tag{5.17}$$

Precision frequency (Larmor frequency) can be written:

$$\omega_L = g_l \frac{\mu_B}{\hbar} B \tag{5.18}$$

In vector notation Larmor frequency can be written:

$$\vec{\omega}_L = g_l \frac{\mu_B}{\hbar} \vec{B} \tag{5.19}$$

Larmor frequency has practical usage to determine magnetic induction based on the measurement of frequency.

G. Problem

1. Hydrogen atom conduct transition from 3p to 2s. Determine the wave length and frequency of photon that emitted transition is occurred! Further more hydrogen atom is placed in external magnetic field B of 1T. Determine the energy shift of atom! Determine also its wave length shift!

2. If it is known orbital quantum number of hydrogen atom $l = 3$, determine:
    a. Magnitude of angular momentum.
    b. Component of angular momentum L in z axis direction z.
    c. Gambarkan kuantisasi ruang momentum sudut.

3. Draw transition from subshell 2p to 1s in hydrogen for case:
    a. Without magnetic fied.
    b. In external magnetic field

4. Determine the difference percentation between angular momentum L and L_z for quantum state p, d, and f.

5. Hydrogen atom has principal quantum number $n = 4$. Determine set of orbital quantum number (l) and magnetic quantum number (m).

H. Reference

Yusman Wiyatmo. 2008. Fisika Atom dalam Perspekstif Klasik, Semiklasik, dan Kuantum. Yogyakarta: Pustaka pelajar