Congruent Polygons
Congruency

Two geometric figures are *congruent* if they have exactly the same size and shape.
Identify Congruent Figure

Each of the red figures is congruent to the other red figures. None of the blue figures is congruent to another blue figure.
Identify Congruent Figure

- When two figures are congruent, there is a correspondence between their angles and sides such that corresponding angles are congruent and corresponding sides are congruent.

- In other words, they have matching angles and matching sides.
Naming & Comparing Polygons

♥ List vertices in order, either clockwise or counterclockwise.

♥ When comparing 2 polygons, begin at corresponding vertices; name the vertices in order and go in the same direction.

♥ By doing this you can identify corresponding parts.

- D corresponds to I
- AE corresponds to PH
Name Corresponding Parts

- **Name all the angles that correspond:**
  - D corresponds to I
  - C corresponds to J
  - B corresponds to K
  - A corresponds to P
  - E corresponds to H

- **Name all the segments that correspond:**
  - DC corresponds to IJ
  - CB corresponds to JK
  - BA corresponds to KP
  - AE corresponds to PH
  - ED corresponds to HI

- How many corresponding angles are there? 5
- How many corresponding sides are there? 5
How many ways can you name pentagon DCBAE?

Do it.

Pick a vertex and go clockwise

- DCBAE
- CBAED
- BAEDC
- AEDCB
- EDCBA

Pick a vertex and go counterclockwise

- DEABC
- CDEAB
- BCDEA
- ABCDE
- EABCD
Polygon Congruence Postulate

If each pair of corresponding angles is congruent, and each pair of corresponding sides is congruent, then the two polygons are congruent.
Congruence Statements

- Given: These polygons are congruent.
- Remember, if they are congruent, they are EXACTLY the same.
- That means that all of the corresponding angles are congruent and all of the corresponding sides are congruent.
- DO NOT say that “all the sides are congruent” and “all the angles are congruent”, because they are not.

ABCD ≅ EFGH
Prove: $\triangle LX M \cong \triangle YXM$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$XY \approx XL$</td>
<td>Given</td>
</tr>
<tr>
<td>$LM \approx YM$</td>
<td>Given</td>
</tr>
<tr>
<td>$LM = YM$</td>
<td>Given</td>
</tr>
<tr>
<td>$XM = XM$</td>
<td>Reflexive Property</td>
</tr>
<tr>
<td>$\angle LX M \approx \angle YXM$</td>
<td>Given</td>
</tr>
<tr>
<td>$\angle L = \angle Y$</td>
<td>Given</td>
</tr>
<tr>
<td>$\angle XMY \approx \angle SML$</td>
<td>Right angles</td>
</tr>
<tr>
<td>$\triangle LX M \approx \triangle YXM$</td>
<td>Polygon Congruence Postulate</td>
</tr>
</tbody>
</table>
Proving Triangles Congruent
**SSS - Postulate**

If all the sides of one triangle are congruent to all of the sides of a second triangle, then the triangles are congruent. (SSS)
Example #1 – SSS – Postulate

Use the SSS Postulate to show the two triangles are congruent. Find the length of each side.

![Diagram of triangles ABC and MNO]

\[ AC = 5 \]
\[ BC = 7 \]
\[ AB = \sqrt{5^2 + 7^2} = \sqrt{74} \]
\[ MO = 5 \]
\[ NO = 7 \]
\[ MN = \sqrt{5^2 + 7^2} = \sqrt{74} \]

\[ \triangle ABC \cong \triangle MNO \]
Definition – Included Angle

\( \angle K \) is the angle between JK and KL. It is called the included angle of sides JK and KL.

What is the included angle for sides KL and JL?

\( \angle L \)
If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent. (SAS)
## Example #2 – SAS – Postulate

Given:  
- N is the midpoint of LW  
- N is the midpoint of SK  

Prove: \( \triangle LNS \cong \triangle WNK \)

<table>
<thead>
<tr>
<th>N is the midpoint of LW</th>
<th>Given</th>
</tr>
</thead>
<tbody>
<tr>
<td>N is the midpoint of SK</td>
<td></td>
</tr>
<tr>
<td>( LN \cong NW, \ SN \cong NK )</td>
<td>Definition of Midpoint</td>
</tr>
<tr>
<td>( \angle LNS \cong \angle WNK )</td>
<td>Vertical Angles are congruent</td>
</tr>
<tr>
<td>( \triangle VLNS \cong \triangle VWNK )</td>
<td>SAS Postulate</td>
</tr>
</tbody>
</table>
JK is the side between \( \angle J \) and \( \angle K \). It is called the *included side* of angles J and K.

What is the included side for angles K and L?

KL
ASA - Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the triangles are congruent. (ASA)

\[ \triangle VJKL \cong \triangle VZXY \] by ASA
Example #3 – ASA – Postulate

Given: \( HA \parallel KS \)

\[ AW \cong WK \]

Prove: \( VHAW \cong VSKW \)

\begin{align*}
|HA \parallel KS, \quad AW \cong WK| & \quad \text{Given} \\
\angle HAW & \cong \angle SKW \quad \text{Alt. Int. Angles are congruent} \\
\angle HWA & \cong \angle SWK \quad \text{Vertical Angles are congruent} \\
VHAW & \cong VSKW \quad \text{ASA Postulate}
\end{align*}
Identify the congruent triangles (if any). State the postulate by which the triangles are congruent.

\[ \triangle ABC \cong \triangle VST \] by SSS

\[ \triangle VPN \cong \triangle VWU \] by SAS

Note: \( \triangle VHI \) is not SSS, SAS, or ASA.
Example #4—Paragraph Proof

Given: \( \text{VMAT} \) is isosceles with vertex bisected by \( \text{AH} \).

Prove: \( MH \cong HT \)

- Sides \( MA \) and \( AT \) are congruent by the definition of an isosceles triangle.
- Angle \( MAH \) is congruent to angle \( TAH \) by the definition of an angle bisector.
- Side \( AH \) is congruent to side \( AH \) by the reflexive property.
- Triangle \( MAH \) is congruent to triangle \( TAH \) by SAS.
- So, side \( MH \) is congruent to side \( HT \).
Example #5 – Column Proof

Given:

- \( QM \parallel PO, \quad QM \perp MO \)
- \( QM \cong PO \)
- \( MO \) has midpoint \( N \)

Prove:

- \( QN \cong PN \)

<table>
<thead>
<tr>
<th>Given</th>
<th>Prove</th>
</tr>
</thead>
<tbody>
<tr>
<td>( QM \parallel PO, \quad QM \perp MO )</td>
<td>( QN \cong PN )</td>
</tr>
<tr>
<td>( QM \cong PO )</td>
<td></td>
</tr>
<tr>
<td>( PO \perp MO )</td>
<td>A line ( \perp ) to one of two ( \parallel ) lines is ( \perp ) to the other line.</td>
</tr>
<tr>
<td>( m\angle QMN = 90^\circ )</td>
<td>Perpendicular lines intersect at 4 right angles.</td>
</tr>
<tr>
<td>( m\angle PON = 90^\circ )</td>
<td></td>
</tr>
<tr>
<td>( \angle QMN \cong \angle PON )</td>
<td>Substitution, Def of Congruent Angles</td>
</tr>
<tr>
<td>( MO \cong ON )</td>
<td>Definition of Midpoint</td>
</tr>
<tr>
<td>( \overline{QMN} \cong \overline{VPN} )</td>
<td>SAS</td>
</tr>
<tr>
<td>( QN \cong PN )</td>
<td>Proven</td>
</tr>
</tbody>
</table>
Warning: No SSA Postulate

There is no such thing as an SSA postulate!

NOT CONGRUENT
Warning: No AAA Postulate

There is no such thing as an AAA postulate!

NOT CONGRUENT
The Congruence Postulates

- SSS correspondence
- ASA correspondence
- SAS correspondence
- AAS correspondence
- SSA correspondence
- AAA correspondence
Name That Postulate

(when possible)

SAS

ASA

SSS

SSA
Name That Postulate (when possible)

ASA

AAA

SAS

SSA
Name That Postulate

(when possible)

Reflexive Property

SAS

Vertical Angles

SAS

Vertical Angles

SAS

Reflexive Property

SSA

SSA
Give name the Postulate
(when possible)
Give name the Postulate

(when possible)
Let’s Practice

Indicate the additional information needed to enable us to apply the specified congruence postulate.

For ASA: \( \angle B \cong \angle D \)

For SAS: \( \overline{AC} \cong \overline{FE} \)

For AAS: \( \angle A \cong \angle F \)
Indicate the additional information needed to enable us to apply the specified congruence postulate.

For ASA:

For SAS:

For AAS:
Properties of Congruent Triangles

- Reflexive:
  - Every triangle is congruent to itself.

- Symmetric:
  - If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$

- Transitive:
  - If $\triangle ABC \cong \triangle DEF$, and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$
Summary

Conditions for Triangles to be Congruent

A. Three Sides Equal

B. Two Sides and Their Included Angle Equal

C. Two Angles and One Side Equal

D. Two Right-angled Triangles with Equal Hypotenuses and Another Pair of Equal Sides
Summary

- Triangles may be proved congruent by
  Side – Side – Side (SSS) Postulate
  Side – Angle – Side (SAS) Postulate
  Angle – Side – Angle (ASA) Postulate
  Angle – Angle – Side (AAS) Postulate
  Hypotenuse – Leg (HL) Postulate

- Parts of triangles may be shown to be congruent by Congruent Parts of Congruent Triangles are Congruent (CPCTC)
Congruent Figures

1. Two figures having the same shape and the same size are called **congruent figures**.
   E.g. The figures $X$ and $Y$ as shown are congruent.

2. If two figures are congruent, then they will fit exactly on each other.
The figure on the right shows a symmetric figure with $l$ being the axis of symmetry. Find out if there are any congruent figures.

**Solution**

The line $l$ divides the figure into 2 congruent figures, i.e. $\text{and}$ $\text{are congruent figures.}$

Therefore, there are two congruent figures.
The Meaning of Congruence

Find out by inspection the congruent figures among the following.

A

B

C

D

E

F

G

H

Solution

B, D; C, F
B) Transformation and Congruence

- When a figure is translated, rotated or reflected, the image produced is congruent to the original figure. When a figure is enlarged or reduced, the image produced will **NOT** be congruent to the original one.
In each of the following pairs of figures, the red one is obtained by transforming the blue one about the fixed point $x$. Determine

(i) which type of transformation (translation, rotation, reflection, enlargement, reduction) it is,
(ii) whether the two figures are congruent or not.

(a)

(i) Reflection
(ii) Yes
The Meaning of Congruence

(b) (i) Translation
(ii) Yes

(c) (i) Enlargement
(ii) No
The Meaning of Congruence

(d)

(i) Rotation
(ii) Yes

(e)

(i) Reduction
(ii) No
C) Congruent Triangles

- When two triangles are congruent, all their **corresponding sides** and **corresponding angles** are equal.

E.g. In the figure, if $\triangle ABC \cong \triangle XYZ$,

then

\[
\angle A = \angle X, \quad AB = XY, \\
\angle B = \angle Y, \quad \text{and} \quad BC = YZ, \\
\angle C = \angle Z, \quad CA = ZX.
\]
Name a pair of congruent triangles in the figure.

Solution

From the figure, we see that $\triangle ABC \cong \triangle RQP$. 

$\triangle ABC \cong \triangle RQP$. 

The Meaning of Congruence

Quick Example

Given that $\triangle ABC \cong \triangle XYZ$ in the figure, find the unknowns $p$, $q$ and $r$.

Solution

$\therefore$ For two congruent triangles, their corresponding sides and angles are equal.

$\therefore p = 6 \text{ cm} \quad , \quad q = 5 \text{ cm} \quad , \quad r = 50^\circ$
Write down the congruent triangles in each of the following.

(a) \( \triangle ABC \cong \triangle XYZ \)

(b) \( \triangle PQR \cong \triangle STU \)
The Meaning of Congruence

Extra Example IV

Find the unknowns (denoted by small letters) in each of the following.

(a) $\triangle ABC \cong \triangle XYZ$

(b) $\triangle MNP \cong \triangle IJK$

Solution

(a) $x = 14$ \hspace{1cm} z = 13

(b) $j = 35^\circ$ \hspace{1cm} i = 47^\circ
A) Three Sides Equal

- If $AB = XY$, $BC = YZ$ and $CA = ZX$, then $\triangle ABC \cong \triangle XYZ$.

【Reference: SSS】
Determine which pair(s) of triangles in the following are congruent.

**Solution**

In the figure, because of SSS,

(I) and (IV) are a pair of congruent triangles;

(II) and (III) are another pair of congruent triangles.
Each of the following pairs of triangles are congruent. Which of them are congruent because of SSS?

**Solution**

B
B) Two Sides and Their Included Angle Equal

- If $AB = XY$, $\angle B = \angle Y$ and $BC = YZ$,
  then $\triangle ABC \cong \triangle XYZ$.

  【Reference: SAS】
Determine which pair(s) of triangles in the following are congruent.

In the figure, because of SAS,

(I) and (III) are a pair of congruent triangles;

(II) and (IV) are another pair of congruent triangles.
In each of the following figures, equal sides and equal angles are indicated with the same markings. Write down a pair of congruent triangles, and give reasons.

(a) \( \triangle ABC \cong \triangle CDA \) (SSS)

(b) \( \triangle ACB \cong \triangle ECD \) (SAS)

Fulfill Exercise Objective

- Identify congruent triangles from given diagram and give reasons.
C) Two Angles and One Side Equal

1. If \( \angle A = \angle X \), \( AB = XY \) and \( \angle B = \angle Y \), then \( \triangle ABC \cong \triangle XYZ \).

【Reference: ASA】
Two Angles and One Side Equal

2. If \( \angle A = \angle X \), \( \angle B = \angle Y \) and \( BC = YZ \),
then \( \triangle ABC \cong \triangle XYZ \).

【Reference: AAS】
Determine which pair(s) of triangles in the following are congruent.

**Solution**

In the figure, because of ASA,

(I) and (IV) are a pair of congruent triangles;

(II) and (III) are another pair of congruent triangles.
In the figure, equal angles are indicated with the same markings. Write down a pair of congruent triangles, and give reasons.

\[ \triangle ABD \cong \triangle ACD \ (ASA) \]

**Solution**

Identify congruent triangles from given diagram and given reasons.

**Fulfill Exercise Objective**

- Identify congruent triangles from given diagram and given reasons.
Determine which pair(s) of triangles in the following are congruent.

Conditions for Triangles to be Congruent

**Quick Example**

Determine which pair(s) of triangles in the following are congruent.

![Diagrams of triangles](image)

(I) and (II) are a pair of congruent triangles;
(III) and (IV) are another pair of congruent triangles.

**Solution**

In the figure, because of AAS,

(I) and (II) are a pair of congruent triangles;
(III) and (IV) are another pair of congruent triangles.
In the figure, equal angles are indicated with the same markings. Write down a pair of congruent triangles, and give reasons.

\[ \triangle ABD \cong \triangle CBD \quad (AAS) \]
Conditions for Triangles to be Congruent

Key Concept 5

D) Two Right-angled Triangles with Equal Hypotenuses and Another Pair of Equal Sides

- If ∠C = ∠Z = 90°, AB = XY and BC = YZ,
  then △ABC ≅ △XYZ.

【Reference: RHS】
Determine which of the following pair(s) of triangles are congruent.

In the figure, because of RHS,
(I) and (III) are a pair of congruent triangles;
(II) and (IV) are another pair of congruent triangles.

Solution

In the figure, because of RHS,
(I) and (III) are a pair of congruent triangles;
(II) and (IV) are another pair of congruent triangles.
In the figure, \( \angle DAB \) and \( \angle BCD \) are both right angles and \( AD = BC \). Judge whether \( \triangle ABD \) and \( \triangle CDB \) are congruent, and give reasons.

**Solution**

Yes, \( \triangle ABD \cong \triangle CDB \) (RHS)

**Fulfill Exercise Objective**

- Determine whether two given triangles are congruent.
Exercises

1. Prove that any point at perpendicular bisector of a line segment is equidistant to both ends of the line segment
2. Prove that the intersection of three perpendicular bisector of a triangle is a center of outside circle the triangle
3. Prove that any point at angle bisector of a angle is equidistant to both rays of the angle
4. Prove that the intersection of three angle bisector of a triangle is a center of inside circle of the triangle