Basic Objects of Geometry
Undefined Terms

- Point
- Line
- Plane

There is no definition of point, line and plane, but there are some characteristics of those terms.
Point (1)

- A Point has no size and no dimension
- There are countless points on any one line
- A point has an exact location on a graph, shape or in “space”
Point (2)

- A point is indicated by a pencil scratch marks on a paper
- A point is named with an italicized uppercase letter placed next to it
  \[ A \bullet \]
- A series of points are make up lines, segments, rays, and planes
A line goes in opposite directions and never, never, never ends.

A line has an infinite set of points that extends in both directions.

A line is perfect straight and has no thickness.
Line (2)

- A line is named by any one italicized lowercase letter or by naming any two points on the line.

- If the line is named by using two points on the line, a small symbol of a line (with two arrows) is written above the two letters. For example, this line could be referred to as line $\overline{AB}$ or $\overline{BA}$ or $\overline{g}$.
Plane (1)

- A Plane (no, not the one that flies!) is a flat surface that goes on forever in all directions.

- Imagine sitting on a row boat in the middle of the ocean. No matter which way you look...all you see is water...forever.

- Or, imagine a floor that extends in all directions as far as you can see.
Plane (2)

A *plane* is an infinite set of points extending in all directions along a perfectly flat surface. It is infinitely long and infinitely wide.

A *plane* is a flat surface that has no thickness or boundaries.
A plane is named by a single uppercase letter and is often represented as a four-sided figure, as in planes $U$ or named by three uppercase letters, plane ABC.
Collinear and Coplanar

- Some points are collinear if they are on a same line

- Some points are coplanar if they are on a same plane
Relation Between Two Objects

Two Points ?
Point and Line ?
Point and Plane ?
Two Lines ?
Line and Plane ?
Two Planes ?
Relation Between Two Objects (1)

Two Points
- Coincide/same
- Different

Point and Line
- Point lies on the line or the line passes through the point
- Point is outside of the line
Relation Between Two Objects (2)

Point and Plane

- Point lies on the plane
- Point doesn’t lie on the plane

Two Lines

- Lies on the same plane: coincides, parallel, cuts each other
- Doesn’t lie on the same plane (in space geometry)
Relation Between Two Objects (3) 

Line and Plane

- The line lies on the plane
- The line is parallel to the plane
- The line cuts the plane

Two Planes (in a space geometry)

- Two planes are coincide
- The planes are parallel
- Two planes cut each other
Postulates and Theorems (I)

- Geometry begins with assumptions about certain things that are very difficult, if not, it is impossible to prove and to flow on to things that can be proven.

- The assumptions that geometry’s logic is based upon are called *postulates*. Sometimes, they referred as *axioms*. The two words mean essentially the same thing.
Postulates and Theorems (2)

Postulate 1: A line contains at least two points.
Postulate 2: A plane contains a minimum of three non-collinear points.
Postulate 3: Through any two points there can be exactly one line.
Postulate 4: Through any three non-collinear points there can be exactly one plane.
Postulate 5: If a line contains two points lie in a plane, then the line lie on the same plane.
Postulate 6: When two planes intersect, their intersection is a line.
Postulates and Theorems (3)

From the six postulates it is possible to prove these theorems.

**Theorem 1:** If two lines intersect, then they intersect in exactly one point.

**Theorem 2:** If a line intersect an outside plane, then their intersection is a point.
Postulates and Theorems (4)

From the six postulates it is possible to prove these theorems.

**Theorem 3:** If a point lies outside a line, then exactly one plane contains the line and the point.

**Theorem 4:** If two lines intersect, then exactly one plane contains both lines.
Describing What You See

Point \( P \) is on line \( m \).
Line \( m \) contains \( P \).
Line \( m \) passes through \( P \).

Lines \( \ell \) and \( m \) intersect in \( T \).
Point \( T \) is the intersection of lines \( \ell \) and \( m \).
Point \( T \) is on line \( m \). Point \( T \) is on line \( \ell \).
Describing What You See

Line $x$ and point $R$ are in $\mathcal{N}$.
Point $R$ lies in $\mathcal{N}$.
Plane $\mathcal{N}$ contains $R$ and line $x$.
Line $y$ intersects $\mathcal{N}$ at $R$.
Point $R$ is the intersection of line $y$ with $\mathcal{N}$.
Lines $y$ and $x$ do not intersect.

$\overrightarrow{AB}$ is in $\mathcal{P}$ and $Q$.
Points $A$ and $B$ lie in both $\mathcal{P}$ and $Q$.
Planes $\mathcal{P}$ and $Q$ both contain $\overrightarrow{AB}$.
Planes $\mathcal{P}$ and $Q$ intersect in $\overrightarrow{AB}$.
$\overrightarrow{AB}$ is the intersection of $\mathcal{P}$ and $Q$. 
Describe the figure below!
Segments and Rays

Much of geometry deals with parts of lines, that are segment and ray
Ray

- A ray is part of a line, but it has one endpoint and the other end keeps going. A ray has an infinite number of points on it.
- Laser beams is a good example of rays.
- When you refer to a ray, you always name the endpoint first.
**Line Segment**

- A *line segment* is a finite portion of a line and is named for its two endpoints. In the preceding diagram is segment. It has an infinite number of points on it.

![Diagram of line segment ST between points Q and R]

- A ruler is an example of line segments.
- Line segments are also named with two italicized uppercase letters, but the symbol above the letters has no arrows.
- Notice the bar above the segment’s name. Technically, refers to points $S$ and $T$ and all the points in between them. $ST$, without the bar, refers to the distance from $S$ to $T$. You’ll notice that is a portion of line QR.
Line Segment

- Each point on a line or a segment can be paired with a single real number, which is known as that point’s coordinate. The distance between two points is the absolute value of the difference of their coordinates.

\[ m \overrightarrow{AB} = 2 \text{ cm} \]

\[ AB = 2 \text{ cm} \]
Figure A

\[ AB = 2 \text{ in.}, \text{ or } m\overline{AB} = 2 \text{ in.} \]

Figure B

\[ MN = 5 \text{ units}, \text{ or } m\overline{MN} = 5 \text{ units} \]
Two segments are **congruent** if and only if they have equal measures, or lengths.

\[ AC = DC \]

3.2 cm = 3.2 cm

You use “is equal to” with numbers.

\[ \overline{AC} \cong \overline{DC} \]

You use “is congruent to” with figures.

When drawing figures, you show congruent segments by making identical markings.
The Precision of Measurement

- The precision of any measurement depends on the smallest unit available on the measuring tool.

Find the length of \( \overline{CD} \) using each ruler.

**a.**

The ruler is marked in centimeters. Point \( D \) is closer to the 3-centimeter mark than to 2 centimeters. Thus, \( \overline{CD} \) is about 3 centimeters long.

**b.**

The long marks are centimeters, and the shorter marks are millimeters. There are 10 millimeters for each centimeter. Thus, \( \overline{CD} \) is about 28 millimeters long.
Distance Between Two Points

**Number Line**

\[ PQ = |b - a| \text{ or } |a - b| \]

**Coordinate Plane**

The distance \( d \) between two points with coordinates \((x_1, y_1)\) and \((x_2, y_2)\) is given by

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. \]
Midpoint of a Segment

<table>
<thead>
<tr>
<th>Words</th>
<th>The midpoint $M$ of $PQ$ is the point between $P$ and $Q$ such that $PM = MQ$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>Number Line</td>
</tr>
<tr>
<td></td>
<td>The coordinate of the midpoint of a segment with endpoints that have coordinates $a$ and $b$ is $\frac{a+b}{2}$.</td>
</tr>
<tr>
<td>Models</td>
<td><img src="image1.png" alt="Diagram of Number Line" /></td>
</tr>
</tbody>
</table>
Use the Distance Formula to find the distance between each pair of points.

23. \( J(0, 0), K(12, 9) \)

24. \( L(3, 5), M(7, 9) \)

25.

26.
Exercise

Find the coordinates of the midpoint of a segment having the given endpoints.

35. \( R(-4, 2), S(3, -1) \)

36. \( U(2, 3), T(-4, -4) \)

37. \( A(8, 4), B(12, 2) \)

38. \( C(9, 5), D(17, 4) \)
**Exercise**

**GEOGRAPHY** For Exercises 46–49, use the following information. The geographic center of Massachusetts is in Rutland at \((42.4^\circ, 71.9^\circ)\), which represents north latitude and west longitude. Hampden is near the southern border of Massachusetts at \((42.1^\circ, 72.4^\circ)\).

46. If Hampden is one endpoint of a segment and Rutland is its midpoint, find the latitude and longitude of the other endpoint.

47. Use an atlas or the Internet to find a city near the location of the other endpoint.

48. If Hampden is the midpoint of a segment with one endpoint at Rutland, find the latitude and longitude of the other endpoint.

49. Use an atlas or the Internet to find a city near the location of the other endpoint.
Construction

Copy a Segment

**Step 1** Draw a segment $XY$. Elsewhere on your paper, draw a line and a point on the line. Label the point $P$.

**Step 2** Place the compass at point $X$ and adjust the compass setting so that the pencil is at point $Y$.

**Step 3** Using that setting, place the compass point at $P$ and draw an arc that intersects the line. Label the point of intersection $Q$. Because of identical compass settings, $PQ \cong XY$. 
Construction

Copy an Angle

**Step 1**
Draw an angle like \( \angle P \) on your paper. Use a straightedge to draw a ray on your paper. Label its endpoint \( T \).

**Step 2**
Place the tip of the compass at point \( P \) and draw a large arc that intersects both sides of \( \angle P \). Label the points of intersection \( Q \) and \( R \).

**Step 3**
Using the same compass setting, put the compass at \( T \) and draw a large arc that intersects the ray. Label the point of intersection \( S \).
**Step 4**
Place the point of your compass on \( R \) and adjust so that the pencil tip is on \( Q \).

**Step 5**
Without changing the setting, place the compass at \( S \) and draw an arc to intersect the larger arc you drew in Step 3. Label the point of intersection \( U \).

**Step 6**
Use a straightedge to draw \( \overline{TU} \).
CONSTRUCTION

Bisect an Angle

**Step 1**
Draw an angle and label the vertex as \( A \). Put your compass at point \( A \) and draw a large arc that intersects both sides of \( \angle A \). Label the points of intersection \( B \) and \( C \).

**Step 2**
With the compass at point \( B \), draw an arc in the interior of the angle.

**Step 3**
Keeping the same compass setting, place the compass at point \( C \) and draw an arc that intersects the arc drawn in Step 2.

**Step 4**
Label the point of intersection \( D \). Draw \( \overrightarrow{AD} \). \( \overrightarrow{AD} \) is the bisector of \( \angle A \). Thus, \( m\angle BAD = m\angle DAC \) and \( \angle BAD \cong \angle DAC \).