

TIME VALUE OF MONEY

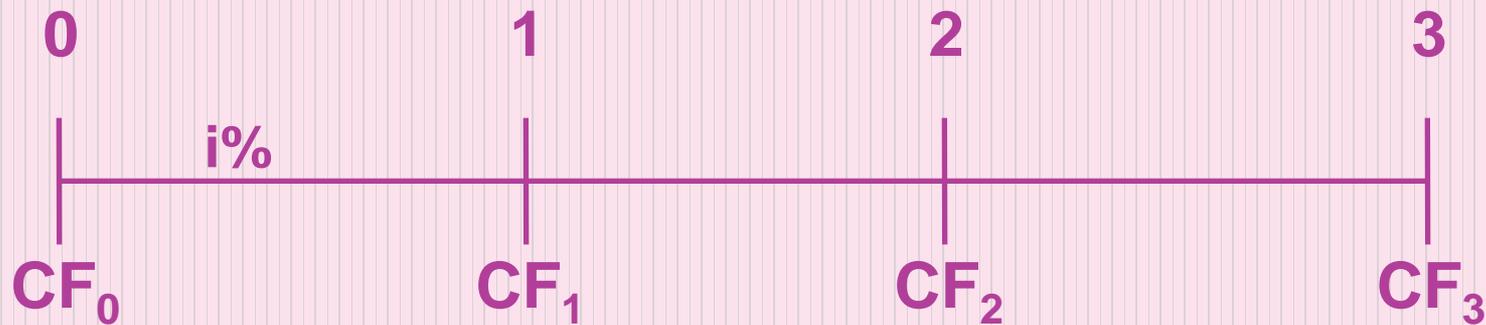
1. Time value of money concepts
2. Future value
3. Present value
4. Annuity
5. Perpetuity
6. Effective Annual Rate (EAR)

Muniya Alteza
m_alteza@uny.ac.id

Basic Concepts

- Financial decisions often involve situations in which someone pays money at one point in time and receives money at some later time.
- Dollars paid or received at two different points in time are different.
- This difference is recognized and accounted for by time value of money analysis.
- A dollar that we have today is worth more than a dollar to be received in the future because, if you had it now, you could invest it, earn interest and end up with more than one dollar in the future.

Time lines



- An important tool used in time value of money analysis.
- Show the timing of cash flows.
- Tick marks occur at the end of periods, so Time 0 is today; Time 1 is the end of the first period (year, month, etc.) or the beginning of the second period.

Future Value

- Compounding is the arithmetic process of determining the final value of a cash flow or series of cash flows when compound interest is applied → the process of going from today's values, or present values (PVs) to future values (FVs).
- Future value= initial amount + interest earned
- Future value can be calculated as follows:

$$FV_n = PV (1+i)^n$$

If you use time value tables so the formula $(1+i)^n$ is FVIF

$$FV_n = PV (FVIF, i, n)$$

FV_n = Future Value at the end of n years

PV = Present Value

i = interest rate for each year

n = number of years in the compounding process

Solving for FV: The arithmetic method

- After 1 year:
$$FV_1 = PV (1 + i) = \$100 (1.10)$$
$$= \$110.00$$
- After 2 years:
$$FV_2 = PV (1 + i)^2 = \$100 (1.10)^2$$
$$= \$121.00$$
- After 3 years:
$$FV_3 = PV (1 + i)^3 = \$100 (1.10)^3$$
$$= \$133.10$$
- After n years (general case):
$$FV_n = PV (1 + i)^n$$

Present Value

- Discounting is finding the PV of a cash flow or series of cash flows when compound interest is applied (the reverse of compounding).
- The PV shows the value of cash flows in terms of today's purchasing power.
- Formula:

$$PV_n = FV \left[\frac{1}{(1+i)^n} \right]$$

If you use time value tables so the formula $[1/(1+i)^n]$ is PVIF

$$PV_n = FV (PVIF, i, n)$$

PV_n = Present Value at the end of n years

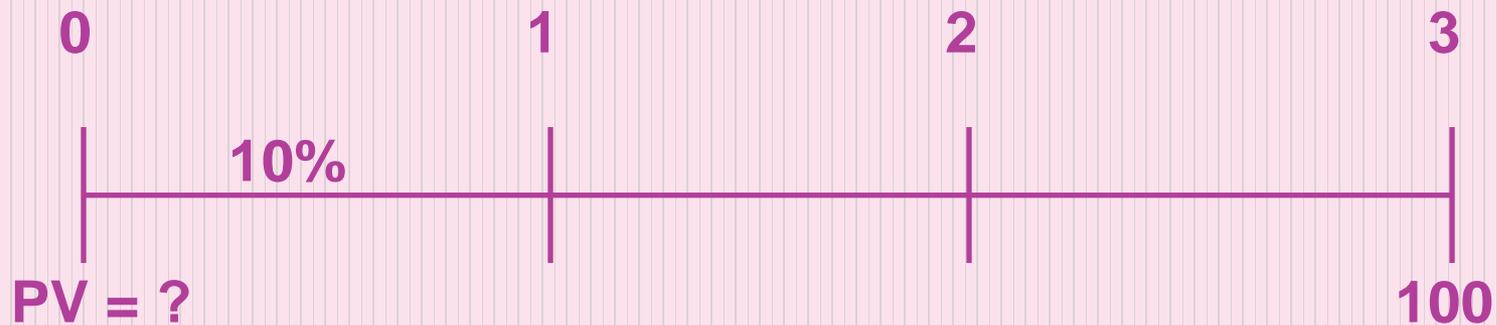
= Future Value

i = interest rate for each year

n = number of years in the discounting process

Example:

- What is the present value (PV) of \$100 due in 3 years, if I/YR = 10%?



Solving for PV: The arithmetic method

- Solve the general FV equation for PV:

$$PV = FV_n / (1 + i)^n$$

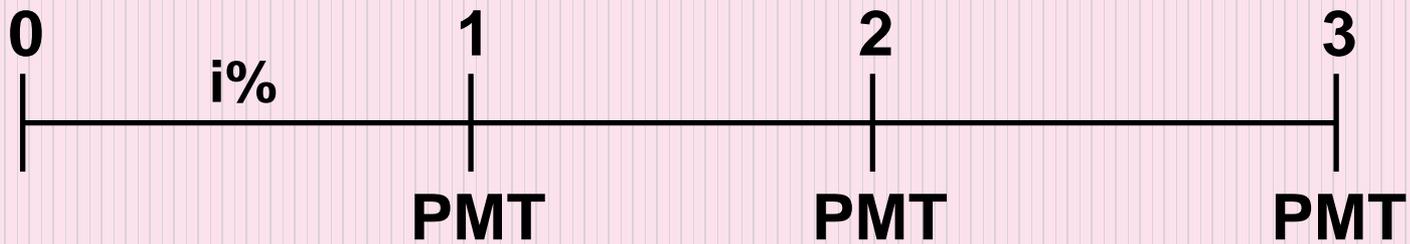
$$\begin{aligned} PV &= FV_3 / (1 + i)^3 \\ &= \$100 / (1.10)^3 \\ &= \$75.13 \end{aligned}$$

ANNUITY

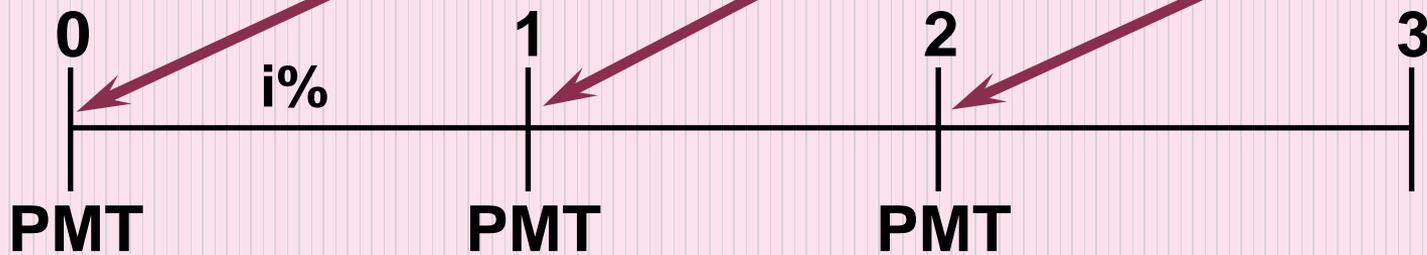
- Annuity is a series of payments of an equal amount at fixed intervals for a specified number of periods.
- Kinds of annuities:
 1. Ordinary (deferred) annuity is an annuity whose payments occur at the end of each period.
 2. Annuity due is an annuity whose payments occur at the beginning of each period.

Illustration

Ordinary Annuity



Annuity Due



Future Value of Ordinary Annuity

- Equation for solving the future value of ordinary annuity :

$$FVA_n = PMT \sum_{t=1}^n (1+i)^{n-t}$$

- If we use the time value of money tables, the term $\sum_{t=1}^n (1+i)^{n-t}$ is called Future Value Interest Factor Annuity (FVIFA) and the formula is:

$$FVA_n = PMT(FVIFA, i, n) \quad \rightarrow \quad FVA_n = PMT \left[\frac{(1+i)^n - 1}{i} \right]$$

- Notations:

FVA_n = Future Value Ordinary Annuity at the end of n years

PMT = payment for each year

i = interest rate for each year

n = number of years

Present Value of Ordinary Annuity

- Equation for solving the present value of ordinary annuity :

$$PVA_n = PMT \sum_{t=1}^n \left(\frac{1}{1+i} \right)^{n-t}$$

- If we use the time value of money tables, the term $\sum_{t=1}^n \left(\frac{1}{1+i} \right)^{n-t}$ is called Present Value Interest Factor Annuity (PVIFA) and the formula is:

$$PVA_n = PMT(PVIFA, i, n) \quad \rightarrow \quad PVA_n = PMT \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right]$$

- Notations:

PVA_n = Present Value Ordinary Annuity at the end of n year

PMT = payment for each year

i = interest rate for each year

n = number of years

Annuities Due

- Equation for solving Future Value Annuity Due is:
$$FVA(\text{due}) = PMT (FVIFA, i, n)(1+i)$$
- Equation for solving Present Value Annuity Due is:

$$PVA(\text{due}) = PMT(PVIFA, i, n)(1+i)$$

- Notations:

FVA(due) = Future Value Annuity Due at the beginning of n years

PVA(due) = Present Value Annuity Due at the beginning of n years

PMT = payment for each year

i = interest rate

n = number of years

PERPETUITIES

- Most annuities call for payments to be made over some finite period of time. However, some annuities go on indefinitely or perpetually→ these are called perpetuity.
- Perpetuity is a stream of equal payments expected to continue forever.
- The present value of a perpetuity is found by applying this equation:

$$PV (\text{perpetuity}) = PMT/i$$

- PMT = payment
- i = interest rate

Uneven Cash Flow Streams

- The definition of an annuity involve payments that are equal in every period.
- Other important financial decision s involve uneven or nonconstant cash flow for example: common stocks typically pay an increasing stream of dividends over time, and fixed asset investments such as new equipment normally do not generate constant cash flow.
- Uneven cash flow stream is a series of cash flows in which the amount varies from one period to the next.
- We have to differentiate between the term payment and cash flow.

Semiannual and Other Compounding Periods

- In all of our examples thus far, we have assumed that interest is compounded once a year or annually.
- However, many contracts call for more frequent payments for example, mortgage and auto loans call for monthly payments and most bonds pay interest semi annually.
- When compounding occurs more frequently than once a year, this fact must be recognized.
- The steps should be done are:
 - (1) Convert the stated interest rate to a periodic rate
Periodic rate = Stated rate/ Number of payments per year
 - (2) Convert the number of years to number of periods
Number of periods = Number of years x Periods per year

Equations for Solving Future Value (Semiannual & Other Periods)

- Future Value:

$$FV_n = PV (1 + i_{\text{nom}/m})^{m \cdot n}$$

If we use the time value of money tables, the formula is:

$$FV_n = PV (FVIF, i_{\text{nom}/m}, m \cdot n)$$

Classifications of Interest Rates

- Nominal rate (i_{NOM}) – also called the quoted or state rate. An annual rate that ignores compounding effects.
 i_{NOM} is stated in contracts. Periods must also be given, e.g. 8% Quarterly or 8% Daily interest.
- Periodic rate (i_{PER}) – amount of interest charged each period, e.g. monthly or quarterly.
 $i_{\text{PER}} = i_{\text{NOM}} / m$, where m is the number of compounding periods per year. $m = 4$ for quarterly and $m = 12$ for monthly compounding.
- Effective (or equivalent) annual rate (EAR = EFF%) – the annual rate of interest actually being earned, taking into account compounding.

When is Each Rate Used?

- i_{NOM} written into contracts, quoted by banks and brokers. Not used in calculations or shown on time lines.
- i_{PER} Used in calculations and shown on time lines. If $m = 1$, $i_{\text{NOM}} = i_{\text{PER}} = \text{EAR}$.
- EAR used to compare returns on investments with different payments per year. Used in calculations when annuity payments don't match compounding periods.

Effective/ Equivalent Annual Rate (EAR)

- We can determine the effective annual rate, given the nominal rate and the number of compounding periods per year by solving this equation:

$$\text{EAR} = (1 + i_{\text{nom}/m})^m - 1$$

i_{nom} = nominal/ quoted interest rate

m = the number of compounding periods per year

Loan Amortization

- Amortized loan is a loan that is repaid in equal payments over its life (monthly, quarterly or annually)
- Amortization schedule:
A table showing precisely how a loan will be repaid. It gives the required payment for each payment date and a breakdown of the payment, showing how much is repayment of principal.
- The payment for each payment date can be solved by this equation:

$$PMT = \frac{PVA}{PVIFA, i, n}$$

| Year | Beginning Amount (1) | Payment (2) | Interest (3) | Repayment of Principal (2)-(3) =(4) | Remaining Balance (1)-(4) =(5) |
|------|-------------------------|----------------|-----------------|--|-----------------------------------|
| 1 | | | | | |
| 2 | | | | | |

Example:

- Construct an amortization schedule for a \$1,000, 10% annual rate loan with 3 equal payments.
- Payment (PMT) = $\$1,000 / (\text{PVIFA}, 10\%, 3)$
= $\$1,000 / 2.4869$
= \$402

| Year | Beginning Amount (1) | Payment (2) | Interest (3) | Repayment of Principal (2)-(3) =(4) | Remaining Balance (1)-(4) =(5) |
|-------|-------------------------|----------------|-----------------|--|-----------------------------------|
| 1 | \$1,000 | \$402 | \$100 | \$302 | \$698 |
| 2 | 698 | 402 | 70 | 332 | 366 |
| 3 | 366 | 402 | 37 | 366 | 0 |
| Total | | 1,206.34 | 206.34 | 1,000 | - |

Note: Interest is calculated by multiplying the loan balance at the beginning of the year by the interest rate