Suppose that there is one of your students’ parents asking this question to you:
"My son said to me that he is often asked to work with two or three other students in his mathematics class. He insists that he works harder than the others most of the time. Shouldn’t each student do his or her own work? Why should he be working with other students?"
What would be your response? Please take a moment to think about it before reading further.

**Empirically Grounded Theoretical Framework**

There is no doubt that social interaction plays a major role in students learn (Gillies, 2003b; Webb, 1992), yet, in many classrooms, students are often the passive recipients of knowledge rather than being active in its creation. Unfortunately, students are often placed in classroom situations where they have little opportunities to reap the benefits from interacting with others. Studying directly from the teacher through memorizing and focusing only on the course contents cannot develop students in all aspects. Thus, students have not yet been facilitated to develop to their full potentials and have not cultivated the desired characteristics such as curiosity, analysis thinking, logical solution, disciplines and honesty. Moreover, teachers usually give lectures focusing on the contents and the subjects, but not the students. This kind of teaching cannot help students learn to face problems and solve them in their real lives (The Office of the National Education Committee, 2002). Good teaching and learning needs to be able to draw more students’ attention, serve different groups of students, and emphasizes more on skill practice, thinking process, and situational management.

Cooperative learning was designed and implemented in order to develop social strategies and social attitudes in students, and to improve social relations within and between groups. Cooperative learning can be perceived as an instructional method that provides opportunities to teachers to aim both students' cognitive and social skills. If used in an appropriate way, students with a wide range of academic skills can collaborate together on a task that probably too complex for them to complete individually. It fits the new paradigm in teaching asserting that knowledge should be jointly constructed by students and teachers. Cooperative learning also allows students to develop the skills
needed for success in high school and beyond. It is deemed highly desirable because of its tendency to reduce peer competition and isolation, and to promote academic achievement and positive interrelationships. Research has shown that explaining and justifying one’s understanding of a concept are helpful for concept attainment. Listening to and comparing different strategies when solving a problem can help students revisit their own thinking and consider different ideas that may be helpful in solving other problems.

What is cooperative learning?
Cooperative learning has its roots in the theories of social interdependence, cognitive development, and behavioral learning. Cooperative learning was relatively unknown and unused from 1940s to 1970s. During this time, there was considerable cultural resistance to the use of cooperative learning, based first on the social Darwinism that promoted interpersonal competition with slogans such as, “It’s a dog-cat-dog world” and “survival of the fittest.” In the late 1960s, after competition began to be criticized, the cultural resistance switched to rugged individualism, that is, the view that strong individuals were built by isolating each student and having students learn by themselves without interacting with classmates. Individualistic learning was then challenged by social scientists who pointed out the essential role of peer interaction and relationships in socialization and learning (Hartup, 1976; Johnson, 1980). It was not until the 1980s that cooperative learning began to be widely accepted. The modern use of cooperative learning primarily began in 1966 with the training of teachers at the University of Minnesota in the effective instructional use of small groups (Johnson, 1970; Johnson & Johnson, 1974).

Several definitions of cooperative learning have been formulated. Davidson (1990) notes that it is difficult to precisely define cooperative learning because of the large variety of learning settings that are regarded as facilitating cooperative learning and the differences among them. However, the one most widely used in higher education is probably that of David and Roger Johnson of the University of Minnesota. According to Johnson and Johnson (1999), cooperative learning is the instructional practice in which students help each other to learn in small groups towards a common goal. Rich (1993) and Sharan (1980) state that it is a generic term that is used to describe an instructional arrangement for teaching academic and collaborative skills to small, heterogeneous groups of students. On the basis of information in Artzt and Newman (1990) and Sutton (1992), there are four necessary conditions that together constitute a cooperative-learning setting:

- Students learn in small groups with two to six members in a group.
The learning tasks in which students are engaged require that the students mutually and positively depend on one another and on the group’s work as a whole.

The learning environment offers all members of the group an equal opportunity to interact with one another regarding the learning tasks and encourages them to communicate their ideas in various ways, for example, verbally.

Each member of the group has a responsibility to contribute to the group work and is accountable for the learning progress of the group.

To be cooperative, a learning setting should ensure the existence of all these conditions. Contrary to common belief, forming groups in the classroom is not sufficient to create a genuine cooperative learning setting. Of the four conditions, it is considered that the third to be particularly significant (Bishop 1985; Clement 1991; Jaworski 1992).

**Five elements of cooperative learning**

Cooperative learning is not simply a synonym for students working in groups. A learning exercise only qualifies as cooperative learning to the extent that the five listed elements are present (Johnson et al., 1994).

1. Positive interdependence

   Positive interdependence is linking students together so one cannot succeed without the others. Group members have to know that they sink or swim together (Johnson et al., 1998). When students clearly understand positive interdependence, they see the importance of working as a team and realize that they are responsible for contributing to the group’s effort. Group goals and tasks, therefore must be designed and communicated to students in ways that make them believe they sink or swim together.

   When positive interdependence is solidly structured, it highlights that (a) each group member’s efforts are required and indispensable for group success, and (b) each group member has a unique contribution to make to the joint effort because of his or her resources or role and task responsibilities. If there is no positive interdependence, there is no cooperation.

2. Face-to-face interaction

   Although it is an important element of cooperative learning, positive interdependence alone does not generate the degree and intensity of interaction required in cooperative learning activities. First, team members need to think that success of the team depends on the contributions by each member. Next, they need to think that ongoing interactions, particularly face-to-face interactions, are required for success. Face-to-
face interaction involves students working in environmental situations that promote eye contact and social space so that students can engage in discussions. The interaction involves (a) orally explaining how to solve problems, (b) teaching one’s knowledge to other, (c) checking for understanding, (d) discussing concepts being learnt, and (e) connecting present with past learning.

3. Individual & group accountability

Individual accountability is the belief by each individual that he or she will be accountable for his or her performance and learning. Individual accountability suggests that each person is responsible to the group and must be a contributing member - not someone who lets others do all of the work. Johnson & Johnson (1999) describe the need for both group and individual accountability. The group must be accountable for achieving its goals and each member must be accountable for contributing his or her share of the work. Individual accountability exists when the performance of each individual is assessed and the results are given back to the group and the individual in order to ascertain who needs more assistance, support, and encouragement in learning. The purpose of cooperative learning groups is to make each member a stronger individual in his or her own right. Students learn together so that they subsequently can gain greater individual competency. There are some steps in achieving individual and group accountability, namely (a) keeping the size of the group small (the smaller the size of the group, the greater the individual accountability may be), (b) giving an individual test to each student, (c) randomly examining students orally by calling on one student to present his or her group’s work to the teacher (in the presence of the group) or to the entire class, (d) observing each group and recording the frequency with which each member-contributes to the group’s work, (e) assigning one student in each group the role of checker. The checker asks other group members to explain the reasoning and rationale underlying group answers, and (f) having students teach what they learnt to someone else. After participating in a cooperative lesson, group members should accomplish the same kinds of tasks by themselves. They learn to do something together so that they can do it more easily when they are alone. Individual accountability is the structural element required to discourage and lower the likelihood of free riders or social loafing.

4. Interpersonal & small-group skills

Unskilled group member cannot cooperate effectively. Effective cooperation is based on skilled teamwork as well as on task work. Students, therefore, must be taught the
interpersonal and small-group skills needed for high-quality cooperation and be motivated to use them. To coordinate efforts to achieve mutual goals, participants must (a) get to know and trust each other, (b) communicate accurately and unambiguously, (c) accept and support each other, and (d) resolve conflicts constructively (Johnson, 2009). Social skills that must be taught are leadership, decision making, trust-building, communication, conflict-management skills. Not only do social skills promote higher achievement, but they also contribute to building more positive relationships among group members. Putnam, Rynders, Johnson, and Johnson (1989) demonstrated that, when participants were taught social skills, observed, and given individual feedback as to how frequently they engaged in the skills, their relationships became more positive.

5. Group processing

Group processing is a time after the cooperative learning task is finished when team members analyze their own and their group’s abilities to work collaboratively. Group processing occurs when group members (a) reflect on which member actions were helpful and unhelpful and (b) make decisions about which actions to continue or change. The purpose of group processing is to clarify and improve the effectiveness with which members carry out the processes necessary to achieve the group’s goals. Archer-Kath et al. (1994) discovered that group processing with individual feedback was more effective than was group processing with whole-group feedback in increasing participants’ (a) achievement motivation, actual achievement, uniformity of achievement among group members, and influence toward higher achievement within cooperative groups, (b) positive relationships among group members and between participants and the teacher, and (c) participants’ self-esteem and positive attitudes toward the subject area.

These five elements can be structured to promote team work and collaborative skills. They can be facilitated in various ways, for example, by (a) asking students to be responsible for certain duties (e.g., record keeper, spokesperson, encourager), (b) providing limited materials thus necessitating sharing, (c) providing bonus points for demonstrating collaborative behaviors, (d) asking students to self-evaluate after task completion, (e) assigning a group grade to the math activity, and (f) arranging the environment so students interact in small groups (Johnson et al., 1994).
Variations of cooperative learning

Although the basic principles of cooperative learning do not change, there are several variations of the model. The leading developers of cooperative learning include Robert Slavin, Roger and David Johnson, and Spencer Kagan, all of whom have a slightly different approaches and emphases (Metzke & Berghoff, 1999). Johnson and Johnson (1975) focus on developing a specific structure that can be incorporated with a variety of curriculums, with an emphasis on integrating social skills with academic tasks. Kagan’s work focuses on the use of many different structures to help facilitate active learning, team building, and group skills. Slavin’s work utilizes methods from both Johnson and Johnson and Kagan, and has resulted in the development of specific cooperative learning structures. Various cooperative learning methods have been developed over the years and put into practice in the classroom. Some of the most extensively researched and widely used methods include Student Teams-Achievement Divisions (STAD), Teams-Games-Tournaments (TGT), Jigsaw II, Team accelerated Instruction (TAI) and Cooperative Integrated Reading, and Composition (CIRC) (Slavin, 1995). Each of these methods has its own characteristics and relevance to different curriculum areas and students in different key stages of learning. For example, while STAD, TGT and Jigsaw II can be adapted for use across most subjects and grade levels, TAI is specifically designed for mathematics in Grade 3-6 and CIRC for reading and writing instruction in Grade 2-8. There are some other popular cooperative learning methods, which include Group Investigation, Learning Together, Complex Instruction, and Structured Dyadic Methods.

Benefits of cooperative learning

The research supporting cooperative learning is boundless. Many research projects have been done over the past forty years on the use of cooperative learning across age groups, ability levels, and cultural backgrounds. Researchers list numerous positive outcomes associated with this innovative style of teaching. The results generally suggest that cooperative learning develops higher-order thinking skills (Mathews et. Al, 1995), enhances motivation, improves interpersonal relations (Nastasi & Clements, 1991) and peer relations (Slavin, 1985). Most important, it exploits the diversified abilities of pupils to enhance their cognitive and social performance. A benefit of cooperative learning, therefore, is to provide students who have math disabilities and social interaction difficulties, an instructional arrangement that fosters the application and practice of mathematics and collaborative skills within a natural setting (i.e., group activity). Thus,
Cooperative learning has been used extensively to promote mathematics achievement of students (Slavin, Leavey, & Madden, 1984; Slavin, Madden, & Leavey, 1984). Cooperative efforts result in participants striving for mutual benefit so that all group members:

- Gain from each other’s efforts. (your success benefits me and my success benefits you)
- Recognize that all group members share a common fate. (we all sink or swim together here)
- Know that one’s performance is mutual caused by oneself and one’s team members. (we cannot do it without you)
- Feel proud and jointly celebrate when a group member is recognizing for achievement. (we all congratulate you and your accomplishment)

Cooperative learning activities can be used to supplement textbook instruction by providing students with opportunities to practice newly introduced or to review skills and concepts. Teachers can use cooperative learning activities to help students make connections between the concrete and abstract level of instruction through peer interactions and carefully designed activities. Researchers report that, regardless of the subject matter, students working in small groups tend to learn more of what is taught and retain it longer than when the same content is presented in other instructional formats (Davis, 1993).

Teacher’s role in cooperative learning

Teacher has four main roles in cooperative learning (Johnson et al. 1998), namely:

1. Make pre-instructional decisions
   - Specify academic and social skills objectives: Every lesson has both academic and interpersonal and small group skills objectives.
   - Decide on group size: Learning groups should be small (groups of two or three members, four at the most)
   - Decide on group composition: Assign students to group randomly or select groups yourself. Usually you will wish to maximize the heterogeneity in each group.
   - Assign roles: Structure student-student interaction by assigning roles such as Reader, Recorder, Encourager of Participation and Checker for Understanding.
   - Arrange the room: Group members should be “knee to knee and eye to eye” but arranged so they all can see the teacher at the front of the room.
• Plan materials: Arrange materials to give a “sink or swim together” message. Give only one paper to the group or give each member part of the material to be learnt.

2. Explain task and cooperative structure

• Explain the academic task: Explain the task, the objectives of the lesson, the concepts and principles students need to know to complete the assignment and the procedures they are follow.
• Explain the criteria for success: Student work should be evaluated on a criteria-referenced basis. Make clear your criteria for evaluating students’ work.
• Structure positive interdependence: Students must believe they “sink or swim together.” Always establish mutual goals (students are responsible for their own learning and the learning of all other group members). Supplement, goal interdependence with celebration or reward, resource, role, and identity interdependence.
• Structure intergroup cooperation: Have groups check with and help other groups. Extend the benefits of cooperation to the whole class.
• Structure individual accountability: Each student must feel responsible for doing his or her share of the work and helping the other group members. Ways to ensure accountability are frequent oral quizzes of group members picked at random, individual test, and assigning a member the role of Checker for Understanding.
• Specify expected behaviors: The more specific you are about the behaviors you want to see in the groups, the more likely students will do them. Social skills may be classified as forming (staying with the group, using quiet voices), formulating (summarizing, elaborating), and fermenting (criticizing ideas, asking for justification). Regularly teach the interpersonal and small group skills you wish to see used in the learning groups.

3. Monitor and intervene

• Arrange face-to-face promotive interaction: Conduct the lesson in ways that ensure students promote each other’s success face-to-face.
• Monitor students’ behavior: This is the fun part. While students are working, you circulate to see whether they understand the assignment and the material, give immediate feedback and reinforcement, and praise good use of group skills. Collect observation data on each group and student.
• Intervene to improve taskwork and teamwork: Provide taskwork assistance (clarify, reteach) if students do not understand the assignment. Provide teamwork assistance if students are having difficulties in working together productively.

4. Assess and process

• Evaluate student learning: Assess and evaluate the quality and quantity of student learning. Involve students in the assessment process.

• Process group functioning: Ensure each student receives feedback, sets an improvement goal, and participates in a team celebration. Have groups routinely list three things they did well in working together and one thing they will do better tomorrow. Summarize as a whole class. Have groups celebrate their success and hard work.

Not only concerning about the role that teacher plays in conducting cooperative learning model, a safe classroom where students can interact and construct their conceptual understanding has to be addressed. A safe classroom is one where every student belongs and is cared by the teacher and the other students. This does not happen by accident or by simply telling students to care about each other. Relationships are carefully built so that students have a positive learning environment. Some ways in which relationships are developed are (Johnson et al., 1998):

1. Keep groups small. The relationship between two students working together is different from three, which is different from four. Start with pairs and venture to threes when they are successful. Hold off on fours (usually) until students are skillful at keeping everyone included in a positive way.

2. Every time the group meets, members should check on each other and see how they are. Teachers structure this with their first instructions: “Check your group, ask members how they are, and see if they are ready to have a good day.” This allows group members to unload any excess baggage and ask for needed help. It also teaches the social skill of being concerned about the people you work with.

3. You can extend the relationships with a sharing question. Whip around the group and everyone share. (their favorite food and why, their favorite song and why, where they would like to go in all the world, etc.)

4. Although base groups stay together for a long period of time, formal and informal groups allow students to develop relationships with all the class members. By the end of the year, or earlier if possible, every student should have done some meaningful
work with every other students. The more students work with each other and have success, the more included every student will feel and will be.

5. Teach students how to work successfully together by starting with small tasks that they can achieve in a short amount of time and gradually moving to longer and more difficult tasks. For examples one math problem, one paragraph written together, one idea explored.

6. Emphasize the learning in the task for everyone. Simply asking for a finished product is not enough. Ask for a product that everyone in the group understands and can explain. The product is not important, the explaining is.

7. While monitoring the groups, routinely ask individuals within the group to explain the work. If they have difficulty, send the group back to work, come back and ask again later.

8. Teach students to be positive with each other by emphasizing the positive words you want to hear in the group, and reinforcing and encouraging their use.

9. Include the five elements of cooperative learning in each formal lesson: positive interdependence (a group learning goal), individual accountability (checking on individual learning or helping), face-to-face promotive interaction (positive exchanges), teaching social or teamwork skills (giving support, summarizing, etc), and processing (reflecting on what the group does well and making a commitment to continuous improvement).

Most common mistakes teachers make and what to do about them

Cooperative learning is a well documented pedagogical practice that promotes academic achievement and socialization, yet many teachers struggle with implementing it in their classes. Johnson, et al. (1998) list the most common mistakes teachers make with cooperative learning and what to do about them as follows:

1. **Group size too large.** It takes a lot of skill for students to manage a group of 4 or more. Instead, keep group size small: 2 or 3 at best. Smaller groups are more effective and take less time. In twos, no one is left out. Threes take more skill, but provide more resources. It takes careful planning for someone not to be left out of a four-some.

2. **Not preparing students to work in cooperative groups.** Explain to students why you are using cooperative learning, do a short cooperative learning activity, then have them explain how it can help them. Initially, do short get-acquainted and review activities.
3. **Not teaching students appropriate interaction skills.** Ask students to contribute to a class list of appropriate group behaviors. Display and continually remind students to use them. Add to the list as needed. For example, stay on task, contribute ideas, help others learn, encourage everyone to participate, listen with care, show respect for others.

4. **Letting students choose their own groups.** We would all choose our friends to work with if given the choice, it is safer. However, it takes a lot of skill to work with friends and not get off-task. In addition, students need to develop positive working relationships with all class members. Make teacher assigned or random groups so students get to know and work with divergent, and all, class members. Change groups often enough so no-one gets stuck for long periods with a difficult class member.

5. **Not doing cooperative activities often enough for students to develop cooperative skills.** Have students do something cooperative at each class session to reinforce positive cooperative habits. If nothing else, have them share what they learnt with a partner.

6. **Not planning cooperative lessons with care.** Many teachers confuse group work with cooperative learning. They put students in groups, tell them to work together, and wonder why the groups are not successful. But cooperative learning groups have five essential elements (positive interdependence, individual accountability, face-to-face promotive interaction, teaching social skills, and processing) built carefully into every lesson to teach the students to learn well together. Learn how to include them in each cooperative lesson.

7. **Assuming that students can handle complex tasks before being taught how to do simple ones successfully together.** Students will not know how to learn together unless you teach them how. Start with short in-class activities and progress to longer and more significant ones as your students show success. For example, do two problems together and you are not done until all group members can explain how to do them. Have frequent class discussions on what helps the groups do well.

8. **Emphasizing paper or project completion as a group goal.** With completion as the only goal, there is nothing to stop one student from doing the work and the others from “hitchhiking.” Instead, assignments should have a cooperative learning goal: a paper or project that everyone helped with, understands, and can explain; mastery of the learning objective by all members; or learning improvement by all members.
9. **An unclear learning goal.** A clear group learning goal is an easily measured learning goal so students can determine both group and individual learning success. For instance, you are finished when every member in your group can explain the work or pass a quiz.

10. **Assuming that students will magically figure out how to work successfully together.** Teacher must teach students how to coordinate their work with others and keep everyone included in the learning. It can be done by helping them see the need, showing them exactly what to do, having them practice under your eagle eyes, then giving them feedback and coaching until their cooperative skills are automatic.

11. **Not building positive student relationships and not understanding the power of success in forging relationships.** Start every group session with a get acquainted or relationship building question. Build in initial success by giving review or easier assignments, and then slowly increase the difficulty of the tasks as students gain confidence in their ability to work together.

12. **Not carefully monitoring the groups while they are working.** This is teaching time. Be among the groups, correcting misconceptions, helping students understand, and reinforcing good teamwork skills. Monitor the groups carefully by observing interactions and encouraging appropriate learning and teamwork skills. Help the groups work toward mastery of every student. Keep individuals on their toes by asking them at random to explain their group’s work.

13. **Giving group grades** (with no thought about its fairness for individual members). Give group grades only when absolutely necessary, absolutely fair for each member, and when you have taught the students how to work together. In the meantime, use cooperative learning for ungraded guided practice to help students perform well on individual assessments of learning. Avoid having students grade each other that can turn into a popularity contest. Assess learning with individual quizzes or papers. Have students assess their own learning by comparing what they can do with a criteria-based evaluation.

14. **Using jigsaw with material that is too difficult for individuals to learn.** The jigsaw technique is one where each student learns part of the material and then teaches it to their group members. If individual students cannot learn the material they need to teach, your students are not ready to do jigsaw with that lesson. Instead, do direct instruction, cooperative guided practice, and check learning with individual quizzes or assignments. Also, never assume that jigsaw by itself is sufficient for student learning.
15. **Not eliciting parent support.** Parents may think that cooperative learning is the same as the dysfunctional groups they may have experienced as students. Teach parents the difference between cooperative learning and group work. Let them experience a cooperative learning group on parent night or send a letter home explaining the differences and what cooperative learning can do for their children.

16. **Assuming that cooperative learning and group work are the same and that doing it well takes no training.** If students could magically learn subject matter without instruction, they would not need teachers. Teachers also cannot do what they have not been taught how to do. Cooperative learning is complex, procedural learning, like learning to play a new sport. Plan on several years of on-going training and practice in cooperative learning to achieve intuitive, wise use. Plan on a lifetime of continuous improvement.

**Challenges incorporating cooperative learning**

Incorporating cooperative learning in science and mathematics classroom is not without challenges. Initially, teachers and students have to face various challenges. The main problems which arise include the followings (Zakaria & Iksan, 2007):

1. Need to prepare extra materials for class use. The need to prepare materials require a lot of work by the teachers, therefore, it is a burden for them to prepare new materials.
2. Fear of the loss of content coverage. Cooperative learning methods often take longer than lectures. Teacher concludes that it is a waste of time.
3. Do not trust students in acquiring knowledge by themselves. Teachers think they must tell their students what and how to learn. Only the teachers have the knowledge and expertise.
4. Lacks of familiarity with cooperative learning methods. Cooperative learning is new to some teachers so they need times to get familiar with the new method. Intensive in-service course can be implemented to overcome the problem.
5. Students lack the skills to work in group. Teachers are often concerned with students’ participation in group activities. They think that students lack the necessary skills to work in group. However, according to Ong and Yeam (2000) teachers should teach the missing skills or review and reinforce the skills that students need.
Cooperative Learning in Mathematics Classroom

In considering how best to prepare students for the challenges of the next century, educators are changing the content of the mathematics curriculum and the ways we teach it. We are moving from a focus on arithmetic and computational skills toward a curriculum that develops students’ abilities to think, reason, and communicate mathematically. Mathematics educators are shifting away from traditional classrooms to reform-oriented mathematics classrooms that focus on students actively engaged in mathematical discourse in cooperative settings (NCTM, 2000). The goal is to help students construct their conceptual understanding of mathematics, not just memorize facts and rules. The teaching of mathematics is likewise changing in order to meet these new goals. Instead of teaching by telling or by demonstration, a blend of instructional methodologies is recommended that includes individual and group work and direct instruction. The focus is to provide frequent opportunities for students to explore and solve problems, individually and with others; and to develop their mathematical skills in the context of this exploration. The teacher is a facilitator of learning, guiding students’ explorations, asking questions that extend their thinking, and encouraging students to communicate their thinking. Direct instruction is provided as the need emerges during this process.

Why does cooperative learning deserve a central place in mathematics instruction? Mathematics instruction offers specific opportunities for cooperative learning with this purpose in view. The content of mathematics allows for specific models of cooperative learning in order to accommodate individual differences between students. Cooperative learning gives students the chance to analyze and evaluate the mathematical thinking and strategies, another key concept NCTM promotes. The interactions with other students can help deepen the level of understanding for all students. This can be done in a non-threatening environment. The communication of mathematical ideas helps to develop reasoning skills and better understanding of arithmetic procedures. Mathematical problems can be situated in real-life contexts and designed in such a way that solutions can be reached along different routes and at different levels. This makes cooperative learning in mathematics different from cooperative learning in other domains, such as languages and world orientation. Each domain has their own opportunities for teaching and learning with regard to individual differences among students.

The study of mathematics is often viewed as an isolated, individualistic, or competitive matter. One works alone and struggles to understand the material or solve the assigned problems. It is not surprising that many students and adults are afraid of
mathematics and develop math avoidance or math anxiety. They often believe that only a few talented individuals can function successfully in the mathematical realm. Small-group cooperative learning addresses these problems in several ways.

1. Small groups provide a social support mechanism for the learning of mathematics. Small groups provide a forum in which students ask questions, discuss ideas, make mistakes, learn to listen to others’ ideas, offer constructive criticism, and summarize their discoveries in writing (NCTM, 1989). Students learn by talking, listening, explaining, and thinking with others. According to the National Council of Teachers of Mathematics (NCTM, 1991), learning environments should be created that promote active learning and teaching; classroom discourse; and individual, small-group, and whole-group learning. Cooperative learning is one example of an instructional arrangement that can be used to foster active student learning, which an important dimension of mathematics is learning and highly endorsed by math educators and researchers. Students can be given tasks to discuss, problem solve, and accomplish.

2. Small-group cooperative learning offers opportunities for success for all students in mathematics (and in general). The group interaction is designed to help all members learn the concepts and problem-solving strategies.

3. Mathematics problems are ideally suited for cooperative group discussion because they have solutions that can be objectively demonstrated. Students can persuade one another by the logic of their arguments. Mathematics problems can often be solved by several different approaches, and students in groups can discuss the merits of different proposed solutions.

4. The field of mathematics is filled with exciting and challenging ideas that invite discussion. Mathematics offers many opportunities for creative thinking, for exploring open-ended situations, for making conjectures and testing them with data, for posing intriguing problems, and for solving non-routine problems. Students in groups can often handle challenging situations that are well beyond the capabilities of individuals at that developmental stage.

**Research Outcomes of Cooperative Learning in Mathematics**

Some studies suggest that students with different levels of ability become more involved in task related interactions as a result of cooperative learning and that students’ attitudes toward school and toward the discipline become more positive. While learning mathematics in certain cooperative-learning settings, students often improve their problem
solving abilities, solve more abstract mathematical problems, and develop their mathematical understanding. With respect to mathematics achievement, some studies show that students’ achievements do not change as a result of learning in a cooperative learning environment, whereas other studies give empirical evidence that cooperative learning may improve students' mathematical achievements.

Reviews by Davidson (1985, 1990) and Webb (1985) focused on mathematics learning, addressing achievement and group interaction and dynamics, respectively. Davidson (1990) reviewed about eighty studies in mathematics comparing student achievement in cooperative learning versus whole-class traditional instruction. In over 40% of these studies, students in the small-group approaches significantly outscored the control students on individual mathematical performance measures. In support of these findings, Webb found a high correlation between students who gave directions in mixed-ability mathematics groups and their own achievement. Davidson also found that the effects of cooperative learning of mathematical skills were consistently positive when there was a combination of individual accountability and some form of team goal or team recognition for commendable achievement. The effects of small-group learning were not significantly different from traditional instruction if the teacher had no prior experience in small-group learning, was not aware of well-established methods, and did little to foster group cooperation or interdependence. For many mathematics teachers, the social benefits of cooperative learning are at least as important as the academic effects. In his review, Davidson concluded that cooperative learning is a powerful tool for increasing self-confidence as a learner and problem solver and for fostering true integration among diverse student populations.

It is aligned with the study conducted by Leikin and Zaslavsky (1999). Students’ learning in traditional settings was compared with the exchange-of-knowledge method, an example of a structured setting that facilitates students’ cooperative learning of mathematics. The learning setting resembles some features of Slavin’s (1987) team assisted individualization program, which fosters students' individual work within larger groups and encourages them to check and help each other when necessary by using given answer sheets. The findings of this study show that the experimental small-group cooperative-learning setting facilitates a higher level of learning activities. Classroom observations indicated an increase in students' activeness.

Artzt (1999) states that one of the greatest benefits of cooperative learning is that it increases students’ skills in communicating mathematics. Leiken and Zaslavsky (1999)
found that using cooperative learning encouraged students to be actively engaged in mathematical learning and to communicate with one another about mathematics. This communication helps them better understand the subject matter. When students are required to explain, elaborate, and defend their positions to others, they may be forced to think more deeply about their ideas. However, students who are listening to the explanations of others are exposed to, and must think about, other approaches to a given task. Observing others and practicing in such settings help learners internalize the concepts that they are attempting to master or understand (Stevens, Slavin, and Farnish 1991). Teachers who require students to reflect on how they answered a problem and explain or elaborate to the other students in the group help the entire group learn more and emphasize mathematical communication skills (NCTM, 2000). They then learn more than students who simply get the correct answers (Stevens, Slavin, and Farnish 1991).

The long-term outcomes of cooperative learning include greater employability and career success (Johnson and Johnson, 1989). Many employers value an employee who has skills in verbal communication, responsibility, initiative, interpersonal interaction, and decision making. All these qualities can be developed by experiencing cooperative learning. Thus, cooperative learning not only helps students with mathematics but also prepares them for life after they graduate.

Facilitating Cooperative Learning in Mathematics

Teachers can use many methods to facilitate cooperative learning. In designing a cooperative learning setting in mathematics, special attention is usually given to the following issues (Hertz-Lazarowitz and Fuks 1987; Kroll, Masingila, and Mau 1992):

1. The structure of the cooperative group

   The structure of a cooperative group is defined by the number of students within a group and by the degree of heterogeneity of a group.
   
   - Number of students within a cooperative group.
   
   The majority of the authors discussing cooperative learning refer to this issue (Artzt and Newman 1990; Davidson 1990; Slavin 1985; Webb 1985; Hertz-Lazarowitz and Fuks 1987). The number of the students in a group depends on the type of the mathematical activity that is intended to take place in the classroom. In general, four is the optimal number of members in a cooperative group. Some researchers recommend that students work in pairs and emphasize that working in pairs facilitates active learning. Others suggest that a group of six students is the best
group size for a cooperative-learning setting. However, all the researchers agree that the number of students in a group should not exceed seven.

- Heterogeneity of a cooperative group
  According to Davidson (1990), heterogeneity of a small group is one of the most important issues when planning a cooperative-learning setting. Students learn better in groups of different ability levels, that is, heterogeneous groups (Davidson 1990; Slavin 1985). Note that students with high ability levels prefer to learn with students having similar ability levels. At the same time, students who have learning difficulties prefer to cooperate with students who are able to help them while learning. In the exchange-of-knowledge method, the heterogeneity of the small groups varies according to the different stages of learning. Students begin their learning in heterogeneous small groups with respect to students' achievement levels, and then high achievers continue their work in homogeneous groups.

2. Students’ interactions in each group
   Students' interactions can also be enhanced by the nature of the task, for example, the specific task can call for an exchange of ideas. The types of interactions depend on the types of learning objectives (Sharan et al. 1980). The learning objectives can be determined for each student individually. Cooperation within the group then is mainly a means for achieving the objectives.

3. Interactions among different groups
   Interactions among the learning groups may or may not take place (Sharan et al. 1980). Students may present the results of their group work to the other groups, or they may finish their group work within the small group without communicating with members of the other groups. Interactions between various groups can be facilitated by some sort of competition.

4. Types of learning tasks and the teacher’s role in the classroom
   The way in which the learning material is presented to the students and the way in which a teacher communicates with students during the group work influence students' learning interactions. In the exchange-of-knowledge setting, the teacher's role is to help students solve problems when they request help. The learning tasks are presented as worked-out examples. This design is intended to focus students' interactions on understanding these examples, by explaining to each other what they already know and by solving new problems similar to the worked-out examples.
5. Assessment and evaluation of the learning process

The teacher's ability to assess learning progress can influence the success of the learning process. The type of assessment depends on the type of learning objectives and setting. Individual objectives demand individual means of assessment, whereas group objectives imply group assessment.

**Cooperative mathematics lesson**

A cooperative mathematics lesson might begin with a meeting of the entire class to provide an overall perspective. This may include a teacher presentation of new material, class discussion, posing problems or questions for investigation, and clarifying directions for the group activities. The class then works in small groups, often in pairs in the elementary grades and groups of four in upper grades. Each group has its own working space, which in the upper grades might include a flip chart or section of the chalkboard. In their groups, students might work together to discuss mathematical ideas, solve problems, look for patterns and relationships in sets of data, and make and test conjectures. Students actively exchange ideas with one another and help each other understand their work. The teacher circulates from group to group, providing assistance and encouragement and asking thought-provoking questions. The teacher's role in conducting cooperative learning is described by Robertson et al as follow:

1. **Planning**

As the teacher plans, several questions specific to cooperative mathematics lessons need to be addressed, including the following:

- What are the important mathematical concepts of this lesson? How will I help students link these goals with previous work and with long-term goals?
- Does the problem or exploration allow for multiple strategies, perspectives, and solutions?
- What opportunities for direct instruction or class discussion of a mathematics concept or skill might arise from students’ exploration?
- What are some possible opportunities for supporting social, as well as mathematical, learning?
- How can I make this lesson interesting, accessible, and challenging for students at all levels of mathematical understanding and proficiency?
20

- How will this lesson provide opportunities for students to make decisions about such things as questions to explore and strategies and tools to use for problem solving?
- What open-ended questions might extend students’ thinking? What questions might be asked to introduce the lesson? What questions might help students during group work? What questions might help students reflect on their experience?
- How can I link assessment with instruction?
- What are appropriate extension activities for groups that finish early or for the next day?

2. Introducing a Lesson

Cooperative mathematics lessons might begin with a problem statement or a question for exploration. Setting a context for the mathematical investigation serves as a motivator and helps students link the exploration to their own lives. It is also important to help students understand the mathematical goals of the lesson and how these goals connect to prior lessons and learning; what is expected of them; and how they will be held accountable. In addition, the teacher might facilitate a discussion about the group work by asking questions, such as What do you want to find out?, What are some possible strategies you might try?, What has helped your group work well in the past?, How will you share the work?, and How will you agree on decisions?

3. Facilitating group work

During group work, the teacher’s role is to encourage students to define the problems they are investigating, to solve interpersonal problems, and to take responsibility for their learning and behavior. The teacher may intervene to refocus a group, to help its members see a problem from another perspective, to ask questions that extend mathematical and social learning, or to assess understanding. The teacher should try not to interrupt the flow of the group work and, instead, should wait for a natural pause in the action. The teacher might ask open-ended questions that require progressively more thought or understanding and avoid giving the impression that there is a right answer. It is important to allow groups time to solve a problem themselves before intervention. Asking key questions to help them resolve the difficulty is more beneficial than solving the problem for them or giving lengthy explanations.
4. Helping students reflect

Reflection on the mathematical and social aspects of group work helps students develop their conceptual understanding as they discuss their experience and hear about the strategies, problems, and successes of others. Asking open-ended questions helps students consider such important issues as the following:

- What strategies did you try? Were they effective? Why?
- What solution do you think is reasonable? Why do you think that? Is there another solution that might work?
- What problems did you have? How did they affect the group? Were you able to solve them? If so, how?
- What are some ways to work that you would recommend to other groups?
- What was something someone in your group did that was particularly helpful to you or the group?

The reflection time should be kept relatively short and include use of a variety of methods such as group reflection, whole-class reflection, and reflection through writing or drawing pictures. If appropriate, the teacher might wish to give feedback regarding both successes and problems he or she observed as groups worked.

**Types of Cooperative Learning**

Although they were mentioned in the beginning of this paper, some of the cooperative learning types will be elaborated here based on the steps that can be implemented in the classroom.

**THINK PAIR SHARE**

The think-pair-share strategy is a cooperative learning technique that allows students to engage in individual and small group thinking. It is applicable across all grade levels and class sizes. The benefit of using think-pair-share strategy includes the positive changes in students’ self-esteem that occur when they listen to one another and respect others’ ideas. Students will have the opportunities to learn from their peers and gain confidence when reporting ideas to the whole class.

There are some steps to this method:

1. Think. This step may require students to be quiet for a few moments and ponder their thoughts about problems given. Students think independently about the problems, forming ideas of their own.
2. Pair. Students are grouped in pairs to discuss their thoughts. This step allows students to articulate their ideas and to consider those of others. Some teachers find it helpful to set a time limit for the think and pair steps. To do so, be sure to give students an idea of how much time they will have. Remember to allow sufficient time during the pair step to allow both students to talk about their thoughts.

3. Share. In this step of the strategy, students can share their ideas in several ways. One way is to have all students stand, and after one student gives response, he or she sits down, as does any other student with the similar response. This continues until everyone is seated. Another way is to move quickly through the class, having students respond quickly, one after the other, or to have a class vote. Often, students are more comfortable presenting ideas to a group with the support of a partner. In addition, students' ideas have become more refined through this three-step process.

NUMBERED HEADS TOGETHER
Numbered Heads Together is a cooperative learning strategy that holds each student accountable for learning the material. Students are seated in groups and each person is given a number (from one to the maximum number in each group). The teacher poses a question and students discuss it in groups to figure out the solution. After that, the teacher randomly calls one number, and students with that number will be the presenter for his or her group. By having students work together in groups, this strategy ensures that each member understands strategies to approach problems asked by the teacher. Since no one knows which number will be called, all team members must be prepared.

The steps of Numbered Heads Together:
1. Divide the students into groups of four and give each one a number from one to four.
2. Pose a question or a problem to the class
3. Have students gather to think about the question and to make sure everyone in their group understands and can give an answer.
4. Ask the question and call out a number randomly.
5. The students with that number raise their hands, and when called on, the student answers for his or her team.

TEAM-ASSISTED INDIVIDUALIZATION (TAI)
Team Assisted Individualization is sometimes called Team Accelerated Instruction is a cooperative learning strategy developed at Johns Hopkins University by a team led by Bob
Slavin and Nancy Madden. TAI is cited as a “top rated” program, for elementary mathematics, by the Best Evidence Encyclopedia. TAI was designed to counter the heterogeneity of the classroom.

The principal components of TAI are given as follows:

1. Four or five-member teams are formed based on teacher assignment. Teacher assigns teams so that they are made up of low, average, and high achievers, boys and girls, and a representation of all ethnic groups (proportionally). Reassignment using this method takes place every four weeks.

2. Students are pretested at the beginning of the lesson. The teacher administers a diagnostic mathematics test to assess the level at which the students should start.

3. Based on the diagnostic test, each student is given materials and sits with his or her team. Teams follow these steps during math time:
   - Work in pairs or triads.
   - Students find the materials they are to work on and take them to their area.
   - If students are in pairs, they give their answer sheets to their partners. Students give answer sheets to the person sitting to the left of them if they are in triads.
   - Students read instruction sheets on their own and ask for help as needed from teammates or teacher.
   - Students start the first skills sheet, doing the first four problems.
   - Students ask the person with the answers to check their work. If all four problems are correct, the student can start the next skills sheet. However, if any of the four are incorrect, the student must complete the next four. The student must continue this process until four problems are correct.
   - If a student gets four correct on the last skills sheet, he/she takes quiz A. The partner checks the quiz, and if the student gets eight or more correct, he/she can take the final test. If not, the teacher is asked help the student with difficulties and the teacher assigns skills sheets to work on to remediate the problem. After completing the skills sheets, the student can take quiz B, and if he/she gets at least eight items correct, the final test can be taken.

4. During math classes, the teacher takes single students or groups of 2-10 students who are at the same level for 5-15 minutes to work individually on problem areas and get ready for the next unit.
5. The team score is calculated by taking the average number correct on all final tests taken by the team and adding that to the average number of units finished by team members times ten. Three levels are created: Superteam, Greatteam, and Goodteam. Certificates are given to teams meeting the Super or Greatteam levels.

TEAMS GAMES TOURNAMENTS

Teams-Games-Tournament is one of the team learning strategies designed by Robert Slavin for review and mastery learning of material. Slavin has found that TGT increased basic skills, students’ achievement, positive interactions between students, acceptance of mainstreamed classmates and self-esteem.

1. Select a instructional topic and present it to the students

2. Develop a list of questions on the topic. Number them. Cut out small pieces of paper and number them so that the total matches the number of questions that you have developed for the topic to measure understanding (e.g. if you have 35 questions, create small pieces of paper with numbers 1-35 on them). Give a set of questions to one student in each group who reads the questions as their corresponding numbers are drawn from the pile.

   Variation: have students place any numbers for which they were unable to come up with the correct answers in a small bag. Collect those numbers and use them to guide what you will reteach.

3. Team Game - place students in heterogeneous groups of 4-5 by ability and have them review material during this “team” phase by selecting a number from the pile. Groups must be equal in size. Give each group a “Letter Identity” (e.g. Group A) and each student a Number Identity (e.g. Student 1). Students must answer the question that matches the number they selected from the pile. For example, if a student selects #22 from the pile that student is challenged to answer that question #22. If he or she cannot come up with an answer, a teammate can “steal” the question. Teams share knowledge during this phase of the lesson. (i.e. teach their teammates).

4. Tournament- place students in new groups made up of individuals from each of the "Team Review" tables (step 2). All “Students 1s” go to Table 1 (these might be lower achieving students) while all “Student 2s” (higher achieving) go to Table 2. In the "Game" phase, students are placed in homogeneous groups with students of similar ability and compete against one another. For every question a student answers
correctly, he or she earns a point. One person at each “tournament table” must keep scores for every individual at the "Game" table.

5. Students return to their Team Game tables and report their scores. Team scores are compared and the winning team earns a reward.

6. Students take an assessment. The scores for each Team (e.g. A, B, C...) are compiled and averaged. Offer “bonus points” for the team that earns the highest average and/or “improvement points” to the team that improves its average the most over previous assessments.

JIGSAW II

Jigsaw is a cooperative learning strategy that emphasizes learning by providing students an opportunity to actively help each other build comprehension. Jigsaw was originally designed by Elliot Aronson and his colleagues (1978). In Aronson’s Jigsaw method, students are assigned to six-member teams to work on academic material that has been broken down into sections. Next, members of different teams who have studied the same sections meet in expert groups to discuss their sections. Then the students return to their teams and take turns teaching their teammates about their sections. Since the only way students can learn sections other than their own is to listen carefully to their teammates, they are motivated to support and show interest in one another’s work.

Slavin (1994) developed a modification of Jigsaw at Johns Hopkins University and then incorporated it in the Student Team Learning program. In this method, called Jigsaw II, students work in four-or five-member teams as in TGT and STAD. Each group member is responsible for becoming an expert on one section of the assigned material and then teaching it to the other members of the team. Jigsaw is a well-established method for encouraging group sharing and learning of specific content. This technique can be used as an instructional activity across several days and is best to use when there is a large amount of content to teach. Jigsaw helps students learn cooperation as group members share responsibility for each other's learning by using critical thinking and social skills to complete an assignment. Subsequently, this strategy helps to improve listening, communication, and problem-solving skills.

Teachers can use the following steps when developing the jigsaw strategy for a class:

1. Introduce the technique and the topic to be studied.
2. Assign each student to a "home group" of 3-5 students who reflect a range of reading abilities.
3. Determine a set of reading selections and assign one selection to each student.
4. Create "expert groups" that consist of students across "home groups" who will read the same selection.
5. Give all students a framework for managing their time on the various parts of the jigsaw task.
6. Provide key questions to help the "expert groups" gather information in their particular area.
7. Provide materials and resources necessary for all students to learn about their topics and become "experts".
8. Discuss the rules for reconvening into "home groups" and provide guidelines as each "expert" reports the information learned.
9. Prepare a summary chart or graphic organizer for each "home group" as a guide for organizing the experts' information report.
10. Remind students that "home group" members are responsible to learn all content from one another.

During this process teachers should:
1. Circulate to ensure that groups are on task and managing their work well;
2. Ask groups to stop and think about how they are checking for everyone's understanding and ensuring that everyone's voice is heard; and
3. Monitor the comprehension of the group members by asking questions and rephrasing information until it is clear that all group members understand the points.

**Students Teams Achievement Divisions (STAD)**

STAD is a cooperative learning strategy in which small groups of learners with different levels of ability work together to accomplish a shared learning goal. Students are assigned to four or five member learning teams that are mixed in performance level, gender, and ethnicity.

The steps of the strategy are:
1. Separate students in to the team of high, middle, and low ability ranks as well as mixed gender and cultural status.
2. Determine which material you will assign to the teams and present the lesson objective.
3. Present the material that the students will work on. Work some problems on the board to ensure that all students understand the process.
4. Give the teams a worksheet with 20-30 problems on it. The teams will work on the problems together.
5. Once the worksheets are finished, the students will return to their own desks, and take an individual quiz on the material just covered.
6. Scoring (this is where a spreadsheet is most helpful): Each student's average is already entered. The quiz score is then posted, then the average score is subtracted from the quiz score (“difference”). This is the "improvement" score and is used in computing the team score.
7. The "winning" team is the one that earned the overall greatest improvement. Rewards can be given immediately, or a cumulative scoreboard can be used for the grading period, or other designated length of time.

LEARNING TOGETHER
David Johnson and Roger Johnson at the University of Minnesota developed the Learning Together models of cooperative learning (Johnson and Johnson, 1998). The methods they have researched involve students working on assignment sheets in four- or five-member heterogeneous groups. The groups hand in a single sheet and receive praise and rewards based on the group product. Their methods emphasize team-building activities before students begin working together and regular discussions within groups about how well they are working together. Numerous relatively brief experiments have shown positive effects of these approaches.

GROUP INVESTIGATION
Group Investigation, developed by Sharan and Sharan (1992) at the University of Tel-Aviv, is a general classroom organization plan in which students work in small groups using cooperative inquiry, group discussion, and cooperative planning and projects. In this method, students form their own two- to six-member groups. After choosing subtopics from a unit being studied by the entire class, the groups further break their subtopics into individual tasks and carry out the activities necessary to prepare group reports. Each group then makes a presentation or display to communicate its findings to the entire class. A study in Israel by Sharan & Shachar (1988) found positive effects of Group Investigation on achievement in language and literature.

LEARNING CYCLE

27
Learning cycle incorporates the following stations:

1. Experiencing. Students must be actively involved in their own learning. They must engage in activities which engender their mathematical thinking. These activities may involve physical action with materials but will involve mental action. Learning must involve 'doing' in order to be effective.

2. Discussing. Reactions and observations arising from the experiences need to be shared with fellow learners and other members of the community and talked about in order for them to be evaluated and, perhaps, validated against the taken-as-shared knowledge of the learner's community. Explanation, justification and negotiation of meaning through communication will help the learner establish this knowledge.

3. Generalizing. Learners need to develop for themselves, through individual construction and interaction with their communities, hypotheses which indicate the current state of their understanding. These hypotheses, or generalizations, will then be tested for viability through their application to other problematic situations or further communicative discourse. It is these generalizations which form the basis for the learner's next experience.

4. Applying. Planning how to use the new or revised learning and actually applying it to other contextual situations will not only validate it as viable knowledge (or suggest rejection of it as non-viable) but will also provide the learner with another experience which could be used to commence yet another cycle.

Template for Cooperative Learning Activities

**Purpose:** What is the activity intended to do for the participants?

<table>
<thead>
<tr>
<th>Time</th>
<th>Instructor Activities</th>
<th>Learner Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>... min</td>
<td>Provide information</td>
<td>Work interactively on</td>
</tr>
<tr>
<td></td>
<td>Guide activity</td>
<td>Objective</td>
</tr>
<tr>
<td>10 min</td>
<td>Complete accountability</td>
<td>Achieve content</td>
</tr>
<tr>
<td></td>
<td>Debrief activity</td>
<td>objective &amp; Discuss</td>
</tr>
<tr>
<td></td>
<td></td>
<td>aids to process</td>
</tr>
</tbody>
</table>

**Activity – name of the activity**

**Organizer:** A statement about how the activity bridges to the content.

**Objective:** A clear objective that can be achieved by the end of the activity.

**Pre Assessment:** Can the learner already accomplish this skill?

**Time:** How many minutes the activity will take. This will vary with the groups.
Techniques / Equipment: All equipment and materials that will be needed. All instructor actions. Monitor and support participation and the exchange of ideas.

Process: Type and size of the groups
- Steps in the activity
- ...
- ...

Group Success / Assessment: Everyone in the group has to be able to explain the objective and how it was reached.

Accountability: How each person will be individually accountable for their part of the process.

Debrief: The group processes for how the activity went for each of them and how others helped them in their group.

Summary: The instructor’s statement that wraps up the process.

Example:

Math Class Applications of Think-Pair-Share

General math: Describe the meaning of the terms area and perimeter and show how to compute them for a rectangle.

Algebra I: Is the slope computed for a given line always the same, no matter which two points on that line are used to calculate that slope? Illustrate with examples.

Geometry: Clarify the relationships among these figures: rectangle, square, rhombus, parallelogram.

Think-Pair-Share can be used for discussion of concepts or procedures, for problem solving, or for practice. It can also be used spontaneously to clear up ideas given during a presentation. The teacher asks the students to think about what has been presented, and then, in discussion with a partner, students either clarify it or pose a question. Clarifications or questions are then stated to the whole class.

Here is an example of a guess-my-rule task well suited for Think-Pair-Share. The teacher asks students working first individually and then with a partner to find possible rules for the following function tables:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$x$</th>
<th>$g(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>3</td>
<td>10</td>
</tr>
</tbody>
</table>
Think-Pair-Share

**Purpose:** To ensure maximum discussion within a group.

<table>
<thead>
<tr>
<th>Time</th>
<th>Instructor Activities</th>
<th>Learner Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 min</td>
<td>Present <em>(insert case, situation or question here)</em> Guide activity</td>
<td>Share information on and feelings about <em>(insert case, situation or question here)</em></td>
</tr>
<tr>
<td>10 min</td>
<td>Collect and comment on information</td>
<td>Discuss aids to process</td>
</tr>
<tr>
<td></td>
<td>Debrief activity</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Share information and feelings</td>
<td></td>
</tr>
</tbody>
</table>

**Activity – Think-Pair-Share**

**Organizer:** Increases the discussion on *(content topic)*.

**Objective:** Share information on and feelings about *(insert case, situation or question here)*.

**Time:** 20 minutes

**Techniques / Equipment:** Details of a case, situation or question. Monitor and encourage participation.

**Process:** Individually:

- Think about the situation and its implications.

*(insert case, situation or question here)*

In pairs:

- Discuss the situation and your thoughts around the situation.

**Group Success:** Both people can explain the point of view of the other.

**Accountability:** Share the information and personal feelings about *(situation)* with others.

**Debrief:** Identify how discussing the situation added to how well you could identify your feelings about the situation. Identify how sharing information added to how well you know the material.
References


C. Webb, & Schmuck (Eds.) *Learning to Cooperate, Cooperative to Learn* (pp. 5-15). New York: Plenum.


Gillies, R. (2003a). The behaviours, interactions, and perceptions of junior high school students during small-group learning. *Journal of Educational Psychology*, 95, 137-147


Robertson, L., Davidson, N., & Dees, R.L. (-----). *Cooperative Learning to Support Thinking, Reasoning, and Communication in Mathematics.*


