SIFAT UJIAN : OPEN BOOKS

Problem 1 (30)
Find the principal root of \((-8 + 8i\sqrt{3})^{1/4}\).

Problem 2 (40)
Give two examples of complex function which satisfy the Cauchy-Riemann condition but they do not have a derivative at a point.

Problem 3 (30)
Evaluate \(\int_C f(z)\,dz\), where \(C\) is an unit circle with positive orientation and \(f(z) = (z + 1)/z\).
1) Principal root of \((-8 + 8i\sqrt{3})^{\frac{1}{4}}\) is ...?

\[-8 + 8i\sqrt{3} = 16 \cdot e^{\frac{2\pi i}{3} + 2\pi n}, \quad n = 0, \pm 1, \pm 2, \ldots\]

\[(-8 + 8i\sqrt{3})^{\frac{1}{4}} = (16 \cdot e^{\frac{2\pi i}{3} + 2\pi n})^{\frac{1}{4}} = 2 \cdot e^{\left(\frac{\pi}{6} + \frac{n\pi}{2}\right)i}, \quad n = 0, \pm 1, \pm 2, \ldots\]

\[n = 0 \rightarrow 2 \cdot e^{\frac{\pi i}{6}} = 2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right) = 2 \left(\frac{\sqrt{3}}{2} + \frac{1}{2} i\right) = \sqrt{3} + i\]

\[n = 1 \rightarrow 2 \cdot e^{\frac{7\pi i}{6}} = 2 \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}\right) = 2 \left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) = -1 - \sqrt{3} i\]

\[n = 2 \rightarrow 2 \cdot e^{\frac{13\pi i}{6}} = 2 \left(\cos \frac{13\pi}{6} + i \sin \frac{13\pi}{6}\right) = 2 \left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) = -1 + \sqrt{3} i\]

\[n = 3 \rightarrow 2 \cdot e^{\frac{19\pi i}{6}} = 2 \left(\cos \frac{19\pi}{6} + i \sin \frac{19\pi}{6}\right) = 2 \left(\frac{\sqrt{3}}{2} - \frac{1}{2} i\right) = \sqrt{3} - i\]
2. Memahami C-R untuk fungsi terlihat

\[ f(x) = \begin{cases} \frac{\alpha}{2}(x-\alpha)^2 & \text{for } 1 + \alpha \\ \alpha x & \text{for } 0 \leq x < 1 + \alpha \end{cases} \]

\[ f'(\alpha) = \infty \]

\[ f(x) = 1 + x^2 \quad \Rightarrow \quad f'(\alpha) = 0 \quad \Rightarrow \quad C-R \]

\[ f(x) = \alpha x = x \]

\[ U_x = 1, \quad V_y = 0 \quad \Rightarrow \quad C-R \]

\[ f(x) = x^3 + \alpha (1-x)^3 \]

\[ U_x = 3x^2, \quad V_y = -3(1-x)^2 \quad \Rightarrow \quad C-R \quad \Rightarrow \quad (0,0) \]

\[ U_y = 0, \quad V_x = 0 \]

\[ f(x) = x^2 + \alpha y^3 \]

\[ U_x = 3x^2, \quad V_y = 3y^2 \]

\[ U_y = 0, \quad V_x = 0 \quad \Rightarrow \quad C-R \quad \Rightarrow \quad (0,0) \]

\[ f'(0) \text{ adan} \]
\[ \int_{\theta}^{2\pi} \frac{e^{i\theta} + 1}{e^{i\theta}} \, d\theta = \int_{0}^{\pi} \frac{e^{i\theta} + 1}{e^{i\theta}} \, e^{i\theta} \, d\theta \]

\[ = \int_{0}^{\pi} (e^{i\theta} + 1) \, d\theta \]

\[ = [e^{i\theta} + i\theta]_{0}^{\pi} \]

\[ = (e^{i\pi} + i\pi) - (1 + 0) \]

\[ = (e^{i\pi} + \pi i) - (1 + 0) \]

\[ = (1 + 0 + \pi i) - 1 \]

\[ = \pi i \]
\[ \int \frac{2e^{i\theta} + 2}{2e^{i\theta}} \, d(2e^{i\theta}) = \left[ 2e^{i\theta} + 2i\theta \right]^{2\pi}_0 = (2e^{i\pi} + 2\pi) - (0 + 0) = -2 + 4\pi i - 2 = 4\pi i \]