Fuzzy inference is a computer paradigm based on fuzzy set theory, fuzzy if-then-rules and fuzzy reasoning

Applications: data classification, decision analysis, expert systems, times series predictions, robotics & pattern recognition

Different names; fuzzy rule-based system, fuzzy model, fuzzy associative memory, fuzzy logic controller & fuzzy system
- **Structure**
  - Rule base $\leftarrow$ selects the set of fuzzy rules
  - Database (or dictionary) $\leftarrow$ defines the membership functions used in the fuzzy rules
  - A reasoning mechanism $\leftarrow$ performs the inference procedure (derive a conclusion from facts & rules!)

- **Defuzzification**: extraction of a crisp value that best represents a fuzzy set
  - Need: it is necessary to have a crisp output in some situations where an inference system is used as a controller

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Block diagram for a fuzzy inference system
Defuzzification [definition]

“It refers to the way a crisp value is extracted from a fuzzy set as a representative value”

- There are five methods of defuzzifying a fuzzy set A of a universe of discourse Z
  - Centroid of area zCOA
  - Bisector of area zBOA
  - Mean of maximum zMOM
  - Smallest of maximum zSOM
  - Largest of maximum zLOM

Centroid of area $z_{COA}$

\[
 z_{COA} = \frac{\int_{z} \mu_A(z)dz}{\int_{z} \mu_A(z)dz},
\]

where $\mu_A(z)$ is the aggregated output MF.

Bisector of area $z_{BOA}$

This operator satisfies the following:

\[
 \int_{\alpha}^{\beta} \mu_A(z)dz = \int_{z_{BOA}}^{\beta} \mu_A(z)dz,
\]

where $\alpha = \min\{z; z \in Z\}$ & $\beta = \max\{z; z \in Z\}$. The vertical line $z = z_{BOA}$ partitions the region between $z = \alpha, z = \beta, y = 0 \& y = \mu_A(z)$ into two regions with the same area.
- **Mean of maximum** $z_{MOM}$

  This operator computes the average of the maximizing $z$ at which the MF reaches a maximum $\mu^*$. It is expressed by:

  \[
  z_{MOM} = \frac{\int_{z'} z \, dz}{\int_{z'} \, dz},
  \]

  where $z' = \{ z ; \mu_A(z) = \mu^* \}$

  By definition: if $\mu_A(z)$ has a single maximum at $z = z^*$

  then $z_{MOM} = z$

  However: if $\max_z \mu_A(z) = [z_1, z_2]$ then $z_{MOM} = \frac{z_1 + z_2}{2}$

- **Smallest of maximum** $z_{SOM}$

  Amongst all $z$ that belong to $[z_1, z_2]$, the smallest is called $z_{SOM}$

- **Largest of maximum** $z_{LOM}$

  Amongst all $z$ that belong to $[z_1, z_2]$, the largest value is called $z_{LOM}$
Various defuzzification schemes for obtaining a crisp output