Problem

Interplanar separation. (Problem 2.1 in Kittel.)
Consider a plane hkl in a crystal lattice.

(a). Prove that the reciprocal lattice vector \( G = hb_1 + kb_2 + lb_3 \) is perpendicular to this plane.

(b). Prove that the distance between two adjacent parallel planes of the lattice is \( d(hkl) = \frac{2\pi}{|G|} \).

(c). Show for a simple cubic lattice that \( d^2 = \frac{a^2}{(h^2 + k^2 + l^2)} \).
Problem #4

**Hexagonal space lattice.** (Problem 2.2 in Kittel.) The primitive translation vectors of the hexagonal space lattice may be taken as:

\[ a_1 = \frac{\sqrt{3}a}{2}x + \frac{a}{2}y, \quad a_2 = -\frac{\sqrt{3}a}{2}x + \frac{a}{2}y, \quad a_3 = cz \] (20)

(a). Show that the volume of the primitive cell is \( \sqrt{3}a^2c/2 \).

(b). Show that the primitive translations of the reciprocal lattice are:

\[ b_1 = \frac{2\pi}{\sqrt{3}a}x + \frac{2\pi}{a}y, \quad b_2 = -\frac{2\pi}{\sqrt{3}a}x + \frac{2\pi}{a}y, \quad b_3 = \frac{2\pi}{c}z \] (21)

so that the lattice is its own reciprocal, but with a rotation of axes.

(c). Describe and sketch the first Brillouin zone of the hexagonal space lattice.
Reciprocal lattice:
Diffraction pattern of the crystal lattice
Diffraction data:
Reciprocal lattice X diffraction pattern of the unit cell content
The reciprocal lattice

- A diffraction pattern is not a direct representation of the crystal lattice
- The diffraction pattern is a representation of the **reciprocal lattice**

We have already considered some reciprocal features -

Miller indices were derived as the reciprocal (or inverse) of unit cell intercepts.
Reciprocal Lattice vectors

Any set of planes can be defined by:
(1) their orientation in the crystal (hkl)
(2) their d-spacing

The orientation of a plane is defined by the direction of a normal (vector product)
Defining the reciprocal

Take two sets of planes:

Draw directions normal:

These lines define the orientation but not the length

We use $\frac{1}{d}$ to define the lengths

These are called reciprocal lattice vectors $G_1$ and $G_2$

Dimensions = $1/\text{length}$
The **K** vector

We define incident and reflected X-rays as $k_o$ and $k$ respectively, with moduli $1/\lambda$. 

Then we define vector $K = k - k_o$. 
As $\mathbf{k}$ and $\mathbf{k}_0$ are of equal length, $1/\lambda$, the triangle $O, O', O''$ is isosceles.

The angle between $\mathbf{k}$ and $-\mathbf{k}_0$ is $2\theta_{hkl}$ and the hkl plane bisects it.

The length of $\mathbf{K}$ is given by:

$$|\mathbf{K}| = 2|\mathbf{k}|\sin\theta_{hkl} = \frac{2\sin\theta_{hkl}}{\lambda}$$
The Laue condition

\( K \) is perpendicular to the \((hkl)\) plane, so can be defined as:

\[
K = \left[ \frac{2 \sin \theta_{hkl}}{\lambda} \right] \hat{n}
\]

where \( \hat{n} \) is a vector of unit length

\( G \) is also perpendicular to \((hkl)\) so

\[
\hat{n} = \frac{G_{hkl}}{|G_{hkl}|}
\]

\[
\Rightarrow K = \frac{2}{\lambda |G_{hkl}|} \sin \theta_{hkl} G_{hkl}
\]

and

\[
|G_{hkl}| = \frac{1}{d_{hkl}} \quad \text{from previous}
\]

\[
\Rightarrow K = \frac{2d_{hkl} \sin \theta_{hkl}}{\lambda} G_{hkl}
\]

But Bragg: \( 2d \sin \theta = \lambda \)

So

\[ K = G_{hkl} \quad \text{the Laue condition} \]
What does this mean?! 

Laue assumed that each set of atoms could radiate the incident radiation in all directions.

Constructive interference only occurs when the scattering vector, \( K \), coincides with a reciprocal lattice vector, \( G \).

This naturally leads to the Ewald Sphere construction.