1.1 REAL NUMBER SYSTEM

1.1 Real Number System
Calculus is based on real number system and its properties.
The simplest number system: **natural numbers**, which are 1, 2, 3, ...

- The set of natural numbers is usually denoted as \( \mathbb{N} \). So \( \mathbb{N} = \{1, 2, 3, 4, \ldots\} \)

If we add all of their negatives and 0, then we get **integer numbers**, which includes \( \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \)

- The set of all integers numbers is with \( \mathbb{Z} \). Thus, \( \mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} \)

How about if we want to measure the length or weight of an object? We need **rational numbers**, such as 2.195, 2.4, dan 8.7.

- **Rational number**: a number that can be written as \( \frac{a}{b} \) where \( a \) and \( b \) are integers and \( b \neq 0 \).

Then, integers are also rational numbers, i.e. 3 is a rational number because it can be written as \( \frac{2}{6} \).

- The set of all rational numbers are denoted by \( \mathbb{Q} \):

\[
\mathbb{Q} = \left\{ \frac{a}{b} \mid a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0 \right\}
\]
What about the length of hypotenuse of this triangle?

![Triangle Diagram](image)

Using **irrational numbers** this thing would be easy. Other examples of irrational numbers are $\sqrt{3}$, $\sqrt{7}$, $e$ and $\pi$.

- The set of all rational and irrational numbers with their negatives and zero is called **real numbers**, & it is denoted as $\mathbb{R}$.

- Relation between those four set $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ an be defined as

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

- Rational/irrational number: can be written as decimal. What’s the difference?
  - Rational number:
    - $\frac{1}{2} = 0,5$
    - $\frac{13}{11} = 1,181818 \ldots$
    - $\frac{3}{8} = 0,375$
    - $\frac{3}{7} = 0,428571428571428571 \ldots$
o Irrational number:

\[ \sqrt{2} = 1,4142135623 \ldots \]
\[ \pi = 3,1415926535 \ldots \]

Difference: repeated decimal is a rational number. ie. \( x = 0.136136136 \ldots \)

1.2 Number Operations
Given \( x, y \in \mathbb{R} \), then we already knew such operations: addition \((x + y)\) & multiplication \((x \cdot y \text{ or } xy)\).

- The properties of those two operation in \( \mathbb{R} \) are:
  1) Commutative: \( x + y = y + x \) & \( xy = yx \).
  2) Assosiative: \( x + (y + z) = (x + y) + z \) & \( x(yz) = (xy)z \).
  3) Distributive: \( x(y + z) = xy + xz \).
  4) Identity elements:
     - for addtion: 0 because \( x + 0 = x \).
     - for multiplication: 1 because \( x \cdot 1 = x \).
  5) Invers:
     - Every \( x \in \mathbb{R} \) has an additive invers (disebut juga negatif)
       - \( x \) such that \( x + (-x) = 0 \).
     - Every \( x \in \mathbb{R}, x \neq 0 \) has a multiplicative invers (disebut juga balikan)
       \( x^{-1} \) such that \( x \cdot x^{-1} = 1 \).

1.3 Order
Non zero real numbers can be divided into 2 different sets: positive real and negative real numbers. Based on this fact, we introduce ordering relation \(<\) (read “less than”) defined as:

\( x < y \) jika dan hanya jika \( y - x \) positif.
• $x < y$ has the same meaning with $y > x$.

• Properties of Order:
1) **Trichotomous**: for $\forall x, y \in \mathbb{R}$, exactly one of $x < y$ atau $x = y$ atau $x > y$ holds.
2) Transitive: If $x < y$ and $y < z$ then $x < z$.
3) Addition: $x < y \iff x + z < y + z$
4) Multiplication:
   
   \[
   \text{If } z \text{ positif then } x < y \iff xz < yz
   \]
   
   \[
   \text{If } z \text{ negatif then } x < y \iff xz > yz
   \]

### 1.3 Inequalities

- Inequality is an open sentence that uses $<, >, \leq$ or $\geq$ relation.
- The solution of an inequality is all real numbers that satisfies the inequality, which usually in the form of an interval atau union of intervals.

Some common intervals:

<table>
<thead>
<tr>
<th>Penulisan Interval</th>
<th>Penulisan Himpunan</th>
<th>Dalam Garis Bilangan</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(a, b)$</td>
<td>${x \in \mathbb{R}</td>
<td>a &lt; x &lt; b}$</td>
</tr>
<tr>
<td>$[a, b]$</td>
<td>${x \in \mathbb{R}</td>
<td>a \leq x \leq b}$</td>
</tr>
<tr>
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<tr>
<td>$(a, b]$</td>
<td>${x \in \mathbb{R}</td>
<td>a &lt; x \leq b}$</td>
</tr>
<tr>
<td>$(-\infty, b)$</td>
<td>${x \in \mathbb{R}</td>
<td>x &lt; b}$</td>
</tr>
<tr>
<td>$(-\infty, b]$</td>
<td>${x \in \mathbb{R}</td>
<td>x \leq b}$</td>
</tr>
<tr>
<td>$(a, \infty)$</td>
<td>${x \in \mathbb{R}</td>
<td>x &gt; a}$</td>
</tr>
<tr>
<td>$[a, \infty)$</td>
<td>${x \in \mathbb{R}</td>
<td>x \geq a}$</td>
</tr>
<tr>
<td>$(-\infty, \infty)$</td>
<td>$\mathbb{R}$</td>
<td></td>
</tr>
</tbody>
</table>
How to solve the inequalities?

- We can add the same number to both sides of inequality.
- We can multiply a positive number to both sides of inequality.
- We can multiply a negative number to both sides of inequality, and the order relation is inversed.

Example of Inequalities
1) \(2x - 7 < 4x - 2\)
2) \(-5 \leq 2x + 6 < 4\)
3) \(x^2 - x - 6 < 0\)
4) \(3x^2 - x - 2 > 0\)
5) \(\frac{2x - 5}{x - 2} \leq 1\)

Contoh 1
Find the solution of \(2x - 7 < 4x - 2\).

Solution: \(2x - 7 < 4x - 2\)
\[\Leftrightarrow 2x < 4x + 5\]
\[\Leftrightarrow -2x < 5\]
\[\Leftrightarrow x > -\frac{5}{2}\]

Solution: interval \((-\frac{5}{2}, \infty) = \left\{x \mid x > -\frac{5}{2}\right\}\)