Fraction

for

Junior High School

By

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Preface

The author has been lucky to have a chance to write this Modul. This Modul means to facilitate the candidate of teachers to learn deeply about the concepts of fraction and its teaching. It has supporting factors both for the teachers and the students of Junior High School in their effort to achieve the competences of mathematics.

This module is supporting teaching resources for teaching:

*Psikologi Pembelajaran Matematika Program PPG*

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Hope that by publication of this Modul, the PPG program will be carried out smoothly.

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INTRODUCTION

The new paradigm of teaching needs the teachers to facilitate their students to develop their competences in mathematics. The teachers also need to develop their understanding of the nature of school mathematics and the nature the students learn mathematics. The nature of school mathematics let the students to learn patterns and relationships, develop mathematical investigations, solve problems and communicate their mathematics knowledge. In learning mathematics the students need to have a good motivation and collaboration with their mates. The teacher should facilitate individual student and give the chance for their students to learn mathematics contextually.

In general, the purpose of publishing this book is to support the government effort in the the expansion of education, the improvement of quality and relevance; and the improvement of efficiency and effectiveness. Many efforts become meaningfully prominent when the government launches the compulsory 9-year basic education; in which the program is aimed at enrolling all children 7-15 age bracket at the basic education institutions. In addition to still lacking of school facilities and lacking of text books contribute to low quality of secondary educations.

Specifically, this Modul is organized to improve intelligence, knowledge, personality, and skill to be independent and continue to higher level of education. Specifically, the content standard of mathematics for junior high school outlined that of facilitating the students:
1. to comprehend the mathematical concepts, explain the relationship between the concepts, apply the concepts and solve the problem.
2. to learn pattern and characteristics of mathematics, practice manipulation to generalize, organize evidences, and explain mathematics ideas.
3. to comprehend mathematical problems, develop mathematical model, figure out and obtain the solution of problems.
4. to communicate mathematical ideas using symbols, tables, diagrams, and other media.
5. to have curiosity to pursue mathematics concepts, develop awareness of mathematics emerges in daily life, and apply mathematics to solve problems.

There are some hints for the teacher or candidate of the teachers to use this modul in the case of method as the following:

Mathematical World Orientation

In the first start, it is not so easy for the teacher to develop and manipulate the concrete material as a mathematical world orientation. It seemed that there are some gaps between teachers’ habit in doing formal mathematics and informal mathematics. Some teachers seemed uncertain whether initiating to introduce concrete model to their students or waiting until their students find for themselves. However most of the teachers believed that mathematical world orientation is important step to offer the students a motive and a solution strategy.

Material Model

For the material models, the teacher tried to identify the role of visual representation in setting up the relationship among fraction concepts, its relations and operations. To some extent the teachers need to manipulate the concrete model in such a way that they represent their and the students’ knowledge of fractions. However it seemed that it was not automatically that the teachers find the supporting aspects of transition toward
sophisticated mathematics. Most of the teacher understood that there were problems of intertwining between informal activities and formal mathematics.

Building Mathematical Relationship

In building mathematical relationship, the teachers perceived that the students need to develop their mathematical attitude as well as mathematical method. It needed for the teacher to facilitate students’ questions, students’ interactions and students’ activities. Uncovering the pattern from material model and trying to connect with mathematical concepts are important aspects. There were possibilities for the students to find out various mathematical methods. To compare some mathematical method lead the students have a clear picture of the problems they faced.

Formal Notation

The teachers perceived that the formal notions of fractions, its relations and operations come up in line with inclination of sharing ideas of fraction concepts through small group discussion. The students will find their interest when they get a clear understanding of formal notions of fractions. The teachers believed that the ultimate achievement of students is that they feel to have the mathematical concepts they found. More than this, the students will have an important capacity to a more sophisticated solution for mathematical problems.

Authors
Chapter I
The Concept of Fraction

Standard Competency:
Understanding the characteristics of arithmetical operation of numbers and its application to solve mathematical problems

Basic Competency
Understanding the concepts of fraction, its type, their relation each other as well as their operations.

Strategy for Teaching Fractions:

1. Ensure that students have mastered the prerequisite skills for the tasks to be learned as part of your advance organizer presentation, demonstrate the “Big Idea” and interconnected relationships between division, fractions, decimals, and percents (e.g., physically divide a whole into five equal pieces)
2. Introduce skill instruction with succinct and unambiguous demonstration of the task to be learned (e.g., solve several problems while the student observes)
3. Introduce instruction using concrete materials (i.e., manipulatives) before proceeding to semi-concrete materials (e.g., pictorial representations) before proceeding to abstract problems (e.g., numerical representation)
4. Ensure that your teaching examples include sufficient practice opportunities to produce task mastery
5. Ensure that your teaching examples include variations of all problem types to avoid students making incorrect generalizations (e.g., that all fractions represent parts of a whole)
6. Provide systematic instruction on discriminating among different problem types that is designed to enable students to know which solution to employ (e.g., teach students to attend to the operation sign, perhaps by circling it, so that they will select the correct computational algorithm)
7. Provide guided practice (following teacher-led demonstrations) before assigning independent work (i.e., have teacher and student work several problems together)

What will you learn in this chapter?

- Introduction of Fraction
- Fraction Calculation
- Scientific Form
• The Rounded of Decimal Fraction

Problem 1:

The word ocean comes from Greece okeanos means river. Approximately 71% of earth is ocean. More than a half of sea has more than 3000 m deep. The total mass of sea water in the earth is approximately \(1.4 \times 10^{21}\) kg, or 0.023% of earth total mass.

71%, \(1.4 \times 10^{21}\), and 0.023% are fraction form that you will learn in this chapter.

A. Fractions
1. Definition

A fraction is number expressed in the form of \(\frac{a}{b}\), where \(a\) and \(b\) are integers and \(b \neq 0\), and \(b\) is not a factor of \(a\). The number \(a\) is called the numerator, and \(b\) is called the denominator.

Problem:

Notice the following picture carefully.

How you can get the fraction from the whole into the part?

Example
1. Which one of the following numbers is fraction?
   a. \(\frac{1}{12}\)  b. \(\frac{3}{2}\)  c. \(\frac{6}{3}\)

Solution
a. \( \frac{1}{12} \) is fraction, because it matches with the definition.

b. \( \frac{3}{2} \) is fraction, because it matches with the definition.

c. \( \frac{6}{3} \) is not fraction, because 3 is a factor of 6.

2. See the picture below. How many parts are shadowed?

Solution
The circle is divide into six equal parts.
The shadowed areas of the circle are 2 of 6 parts.

Hence, the shadowed parts is \( \frac{2}{6} \).

Note
6 is the product of \( 1 \times 6 \), \( 2 \times 6 \), and \( 6 \times 1 \). Hence, the factors of 6 are 1, 2, 3, and 6.

Exercise
1. Which one of the following numbers is fraction?
   a. \( \frac{1}{7} \)  b. \( \frac{3}{125} \)  c. \( \frac{250}{25} \)  d. \( \frac{33}{11} \)  e. \( \frac{100}{4} \)

2. How many parts are shadowed from the picture below?

3. Write your name completely. What is the proportion of vocal words compare with the whole words on your name?

2. Type of Fraction

Problem:
Notice the following picture carefully.
Indicate the fractions that you get from the picture.
a. Proper Fraction

Proper fraction is a fraction where numerator is less than denominator. For example
\[ \frac{11}{12}, \frac{23}{47}, \text{and} \frac{3}{6}. \]

b. Improper Fraction

Improper fraction is a fraction where numerator is more than denominator. For example
\[ \frac{5}{3}, \frac{22}{7}, \text{and} \frac{314}{100}. \]

**Example:**
\[
\frac{20}{16} = \frac{20 \div 2}{16 \div 2} = \frac{10}{8} = \frac{10 \div 2}{8 \div 2} = \frac{5}{4}
\]

c. Mixed Fraction

Teacher’s Guide:

Let the students to learn the following mix number \( 1 \frac{3}{4} \) as simple fraction?

In a small group discussion, let the students discuss the meaning of fraction.
The teacher may use ICEBERG approach to encourage the students construct their own concept of mix fraction, as follow:

Mixed fraction is a fraction that consists of integer number $a$, $b$, and $c$ such that $\frac{a}{c} = a + \frac{b}{c}$, where $\frac{b}{c}$ is a pure fraction. For example $1\frac{2}{3}, 5\frac{8}{11}, 21\frac{3}{7}$.

Mixed fraction may be convert to an improper fraction. Otherwise, we can convert improper fractions to mixed fractions or whole numbers.

As an illustration, consider the following picture carefully:
i). To Convert an Improper Fraction to Mixed Fraction

We can convert improper fractions to mixed fractions or whole numbers by dividing the numerator by the denominator and expressing any remainder as a fraction of the denominator.

Example

Convert improper fraction $\frac{43}{7}$ to mixed fraction.

Solution

Divide 43 by 7 such as follow

$\frac{43}{7} \Rightarrow 7)43$

\[
\begin{align*}
43 & \quad \rightarrow \\
42 & \quad - \\
1 & \quad \text{remainder}
\end{align*}
\]

Hence, $\frac{43}{7} = 6 \frac{1}{7}$ an integer, pure fraction
Exercise
Convert the following improper fractions to mixed fraction.
1. $\frac{8}{3}$  2. $\frac{15}{4}$  3. $\frac{20}{3}$  4. $\frac{35}{3}$  5. $\frac{49}{6}$

ii). To Convert a Mixed Fraction to Improper Fraction

We can use the following formula to convert a mixed fraction to improper fraction.

$$\frac{a}{c} = \frac{(a \times c) + b}{c}, \text{ where } c \neq 0$$

Example

Write down the following mixed fractions in the form of improper fraction.
1. $2 \frac{1}{4}$  2. $3 \frac{2}{7}$

Solution

1. $2 \frac{1}{4} = \frac{(2 \times 4) + 1}{4} = \frac{8 + 1}{4} = \frac{9}{4}$

2. $3 \frac{2}{7} = \frac{(3 \times 7) + 2}{7} = \frac{21 + 2}{7} = \frac{23}{7}$

Exercise
Convert the mixed fraction below into an improper fraction.
1. $2 \frac{1}{2}$  2. $3 \frac{1}{4}$  3. $5 \frac{1}{6}$  4. $7 \frac{2}{3}$  5. $10 \frac{11}{12}$
3. Equivalent Fractions

Equivalent fractions are the fractions with the same location on the line-numbers. See the picture below.

\[ \frac{1}{4} \text{ is equivalent to } \frac{2}{8}, \text{ and this may be written as } \frac{1}{4} = \frac{2}{8}. \]

\[ \frac{1}{2} \text{ is equivalent to } \frac{2}{4} \text{ and } \frac{4}{8}, \text{ and this may be written as } \frac{1}{2} = \frac{2}{4} = \frac{4}{8}. \]

\[ \frac{3}{4} \text{ is equivalent to } \frac{6}{8}, \text{ and this may be written as } \frac{3}{4} = \frac{6}{8}. \]

We can get equivalent fractions by multiplying, or dividing, the numerator (top) and denominator (bottom) of a fraction by the same number \( \neq 0 \).

Example

Determine two equivalent fractions of \( \frac{3}{4} \).

Solution

Multiplied \( \frac{3}{4} \) by the number \( \neq 0 \), e.g. 2 or 3

\[ \frac{3}{4} \text{ is equivalent to } \frac{3 \times 2}{4 \times 2} = \frac{6}{8}. \]

\[ \frac{3}{4} \text{ is equivalent to } \frac{3 \times 3}{4 \times 3} = \frac{9}{12}. \]

Hence, \( \frac{3}{4} \) is equivalent to \( \frac{6}{8} \) and \( \frac{9}{12} \).

Exercise
1. Write two equivalent fractions of each of the followings
   a. \( \frac{5}{8} \)  
   b. \( \frac{31}{43} \)

2. Fill in the blank with the correct number to make equivalent fractions.
   a. \( \frac{1}{2} = \ldots \frac{3}{4} = \ldots \frac{9}{10} = \ldots \)
   b. \( \frac{7}{28} = \ldots \frac{14}{42} = \ldots = \frac{140}{\ldots} \)

**Simplifying Fraction**

Simplifying fraction involves dividing the top and bottom by their common factors until this is no longer possible.

A fraction \( \frac{a}{b} \), where \( b \neq 0 \) can be simplified by dividing the numerator and the denominator by \( \text{GCF (the Greatest Common Factor)} \) of \( a \) and \( b \).

Express with your own word to show the following calculation:

Source: green-planet-solar-energy.com

Example

Write \( \frac{9}{36} \) in the simplest form.
Solution

The GCF of 9 and 36 is 9.

\[ \frac{9}{36} \rightarrow \text{Divide the numerator and the denominator by 9.} \]

\[ \frac{9}{36} \div \frac{9}{36} = \frac{1}{4} \]

Therefore, the simplest form of fraction \( \frac{9}{36} \) is \( \frac{1}{4} \).

Exercise

Write down the following fractions in the simplest form.

1. \( \frac{2}{4} \)  
2. \( \frac{6}{9} \)  
3. \( \frac{3}{9} \)  
4. \( \frac{15}{30} \)  
5. \( \frac{12}{36} \)

Story Problems

1. "They" are found on Mars, Venus and Earth. As far as is known there are fifty of "them." If twenty-two are on Mars, and eighteen are on Earth, what fraction of "them" are on Venus?

2. Ryan and Kyle had been friends for almost 6 years. Now there was friction between them. There had been some verbal exchanges, but they had not gotten physical yet. The argument was over who should get the last piece of the cake Kaylee made for her birthday party. When Kaylee made the cake she used a ratio of half of a cup of sugar to every two and a third cups of flour. If she used sixteen and a third cups of flour, how many cups of sugar did she use?

3. Nicky's uncle wants to paint his apartment in Venice, which is five hundred square meters. He will have to cover one-fifth of that space with a drop cloth to prevent the paint from getting on the floor. How many square meters of drop cloth will he need?
4. Daniel made a basket of fire starters for his father. He made them by dipping pinecones in wax. The wax was different colors. When he finished he put them in a pretty basket with a big red bow on it. He made forty-five fire starters. One-fifth of them were red and the rest of them were green. How many were green?

5. Brittany and her father went to Paulo’s Pizzeria for pizza with everything except anchovies. The pizza was divided into six slices. Brittany’s father ate two-thirds of the pizza. Brittany ate the rest. How many slices of pizza did Brittany eat?
Chapter II
Comparing and Ordering Fractions

Standard Competency:
Understanding the characteristics of arithmetical operation of numbers and its application to solve mathematical problems

Basic Competency:
Comparing and ordering the fractions

Strategy for Teaching Fractions:

1. Ensure that students have mastered the prerequisite skills for the tasks to be learned as part of your advance organizer presentation, demonstrate the “Big Idea” and interconnected relationships between division, fractions, decimals, and percents (e.g., physically divide a whole into five equal pieces)
2. Introduce skill instruction with succinct and unambiguous demonstration of the task to be learned (e.g., solve several problems while the student observes)
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4. Ensure that your teaching examples include sufficient practice opportunities to produce task mastery
5. Ensure that your teaching examples include variations of all problem types to avoid students making incorrect generalizations (e.g., that all fractions represent parts of a whole)
6. Provide systematic instruction on discriminating among different problem types that is designed to enable students to know which solution to employ (e.g., teach students to attend to the operation sign, perhaps by circling it, so that they will select the correct computational algorithm)
7. Provide guided practice (following teacher-led demonstrations) before assigning independent work (i.e., have teacher and student work several problems together)
What will you learn in this chapter?

- Comparing the Fractions
- Ordering the Fractions

Teacher’s Guide:

Compare the following fraction: \( \frac{2}{3} \) and \( \frac{1}{2} \.

Which is the bigger?

In a small group discussion, let the students discuss how to compare \( \frac{2}{3} \) and \( \frac{1}{2} \).

The teacher may use ICEBERG approach to encourage the students construct their own ways, as follow:

Proper fractions are located in between 0 and 1 of the line-numbers. Following is the method to get fractions from line-numbers.

1. Divide the segment line of 0 to 1 in three equal parts, and mark on it the fractions \( \frac{1}{3} \) and \( \frac{2}{3} \).

2. Compare the fractions 0, \( \frac{1}{3} \), \( \frac{2}{3} \), and \( \frac{3}{3} = 1 \).
3. You can conclude that \(0 < \frac{1}{3} < \frac{2}{3} < 1\).

Thus, the location of \(\frac{2}{3}\) on the line-numbers is as follow:

Example

Mark the location of \(\frac{5}{8}\) on the line-numbers

Solution

Divide the space between 0 to 1 into eight equal parts

Make a comparison between \(\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \frac{6}{8}, \frac{7}{8}\) and \(\frac{8}{8} = 1\).

You get \(\frac{1}{8} < \frac{2}{8} < \frac{3}{8} < \frac{4}{8} < \frac{5}{8} < \frac{6}{8} < \frac{7}{8} < 1\).

Thus, the location of \(\frac{5}{8}\) on the line-numbers is as follow:

Express with your own word to show the following calculation:

\[
\begin{align*}
\text{Compare } \frac{3}{4} & \text{ and } \frac{2}{3} \\
\text{The LCM of 3 and 4 is 12} \\
\frac{3 \times 3}{4 \times 3} &= \frac{9}{12} & \frac{2 \times 4}{3 \times 4} &= \frac{8}{12} \\
\frac{9}{12} &> \frac{8}{12}
\end{align*}
\]

Source: enchantedlearning.com

Exercise
Mark on it the following fractions in the line-numbers

1. \( \frac{3}{7} \)  2. \( \frac{1}{9} \)  3. \( \frac{2}{5} \)  4. \( \frac{3}{10} \)  5. \( \frac{5}{11} \)

A. Comparing the Fractions

To compare two fractions with the same denominator, you just need to compare the numerators.

See the following figure.

You see that the number of shadowed areas on the model of \( \frac{3}{4} \) is more than the number of shadowed area on the model of \( \frac{1}{4} \).

Thus, \( \frac{1}{4} \) \( \frac{3}{4} \).

Example

Put the sign < or > on \( \frac{3}{7} \ldots \frac{18}{7} \) to get correct statement.

Solution

The fractions have the same denominators, so you just need to compare the numerators.

The numerator of \( \frac{3}{7} \) is 3.

The numerator of \( \frac{18}{7} \) is 18.

It's clear that 3 < 8. So, \( \frac{3}{7} < \frac{18}{7} \).

Express with your own word to describe the following:
Exercise

Determine the value of fractions indicated by the following diagrams. Then, find out the greater fraction!

1. [Diagram]

2. [Diagram]
To compare two fractions with different denominators, you need to express the fractions in the same denominators, and then compare the numerator. To have the same denominators, we need to find the Least Common Multiple (LCM) of the denominators.

Example

Which fraction is greater: $\frac{1}{5}$ and $\frac{3}{7}$?

Solution

LCM of 5 and 7 is 35.

$\frac{1}{5}$ is equivalent to $\frac{7}{35}$

$\frac{3}{7}$ is equivalent to $\frac{15}{35}$

We can conclude that $\frac{7}{35} < \frac{15}{35}$.

Thus, $\frac{1}{5} < \frac{3}{7}$.

Note:
LCM is the smallest positive common multiple

Exercise

Fill in the blank the sign of $\ =, <, or >$, to compare the following each two fractions.

1. $\frac{2}{7} ... \frac{2}{3}$  
2. $\frac{3}{9} ... \frac{4}{12}$  
3. $\frac{3}{10} ... \frac{2}{11}$  
4. $\frac{13}{15} ... \frac{4}{6}$  
5. $\frac{49}{81} ... \frac{21}{27}$

B. Ordering the Fractions

Deciding the bigger or the smaller of two or more fractions may be called as ordering the fractions.
To order two or more fractions with similar denominators, you just need to order the numerators. However, if you have fractions with different denominators, how did you decide what order they should be in? You need first to determine the equivalent fractions.

Source: eduplace.com

Example

Arrange the following fractions in order, with the smallest on the left, \( \frac{13}{15}, \frac{9}{10}, \frac{11}{20}, \) and \( \frac{3}{5} \).

Solution

The denominators of the fractions are 5, 10, 15, and 20.

The LCM of 5, 10, 15, and 20 is 60.

You find that:

\( \frac{13}{15} \) is equivalent to \( \frac{52}{60} \).

\( \frac{9}{10} \) is equivalent to \( \frac{54}{60} \).
\[
\frac{11}{20} \text{ is equivalent to } \frac{33}{60}.
\]
\[
\frac{3}{5} \text{ is equivalent to } \frac{36}{60}.
\]

Hence, if we put the fractions in order, from the least to the greatest, we get:
\[
\frac{11}{20}, \frac{3}{5}, \frac{13}{15}, \frac{9}{10}.
\]

Exercise

Arrange the following fractions in order, with the smallest on the left

1. \(\frac{11}{17}, \frac{14}{16}, \text{and } \frac{14}{27}\)
2. \(\frac{8}{9}, \frac{6}{7}, \text{and } \frac{7}{8}\)
3. Express with your own word to show the following calculation:

Source: titirangi.school.nz
Chapter III
Decimals

Standard Competency:
Understanding the characteristics of arithmetical operation of numbers and its application to solve mathematical problems

Basic Competency:
Understanding decimals, its type and its characteristics.

Strategy for Teaching Fractions:
1. Ensure that students have mastered the prerequisite skills for the tasks to be learned as part of your advance organizer presentation, demonstrate the “Big Idea” and interconnected relationships between division, fractions, decimals, and percents (e.g., physically divide a whole into five equal pieces)
2. Introduce skill instruction with succinct and unambiguous demonstration of the task to be learned (e.g., solve several problems while the student observes)
3. Introduce instruction using concrete materials (i.e., manipulatives) before proceeding to semi-concrete materials (e.g., pictorial representations) before proceeding to abstract problems (e.g., numerical representation)
4. Ensure that your teaching examples include sufficient practice opportunities to produce task mastery
5. Ensure that your teaching examples include variations of all problem types to avoid students making incorrect generalizations (e.g., that all fractions represent parts of a whole)
6. Provide systematic instruction on discriminating among different problem types that is designed to enable students to know which solution to employ (e.g., teach students to attend to the operation sign, perhaps by circling it, so that they will select the correct computational algorithm)
7. Provide guided practice (following teacher-led demonstrations) before assigning independent work (i.e., have teacher and student work several problems together)

What will you learn in this chapter?
- Understanding decimals

Teacher’s Guide:
Write 0.35 into the simple form of fraction?
In a small group discussion, let the students discuss how to write 0.35 into the simple form of fraction.

The teacher may use ICEBERG approach to encourage the students construct their own ways, as follow:

We can use a decimal point (.) to represent the fraction. Just as places to the left of the decimal represent units, tens, hundreds, and so on, those to the right of the decimal represent places for tenths (\(\frac{1}{10}\)), hundredths (\(\frac{1}{100}\)), thousandths (\(\frac{1}{1000}\)), and so forth.

Express with your own word to describe the following:

Source: enchantedlearning.com
Example

\[
\frac{8}{10} = (0 \times 1) + (8 \times \frac{1}{10}), \text{ can be written as } 0,8
\]

\[
\frac{47}{100} = (0 \times 1) + (0 \times \frac{1}{10}) + (4 \times \frac{1}{100}) + (7 \times \frac{1}{1000}), \text{ can be written as } 0,047
\]

\[
2 \frac{3}{100} = (2 \times 1) + (0 \times \frac{1}{10}) + (3 \times \frac{1}{100}), \text{ can be written as } 2,03
\]

The fraction 0,8 is written as one decimal point. The fraction 0,047 is written as three decimal point. And the fraction 2,03 is written as two decimal point.

Example

Write \( \frac{3}{9} \) as decimal!

Solution

You can write the decimal form of the fraction by dividing the numerator by the denominator.

\[
\frac{3}{9} \rightarrow 9)3,0
\]

\[
\begin{align*}
2,7 - \\
30 - \\
27 - \\
3
\end{align*}
\]

The process of division of 3 by 9 results that \( \frac{3}{9} = 0,333... \)
Exercise

Write the following fractions as decimals:

a. \( \frac{4}{25} \)  
b. \( \frac{6}{8} \)

You can also represent the decimals into proper fractions. How to do that?

Example

Represent 0,775 into the simplest proper fraction.

Solution

The decimal 0,775 can be written as \( (0 \times 1) + (7 \times \frac{1}{10}) + (7 \times \frac{1}{100}) + (5 \times \frac{1}{1000}) \).

Therefore, 0,775 = \( \frac{775}{1000} \).

The Greatest Common Factor of 775 and 1000 is 25.

Simplify the fraction by dividing the numerator and the denominator by 25.

You get, \( \frac{775}{1000} = \frac{775 \div 25}{1000 \div 25} = \frac{31}{40} \)

Hence, the simplest proper fraction form of 0,775 is \( \frac{31}{40} \).
Exercise

1. Write the following decimals as proper fractions!
   a. 5,15  b. 0,124

2. Write the following fractions as three decimal point!
   a. \( \frac{1}{7} \)  b. \( \frac{3}{32} \)
Chapter IV
Percent and Permill

Standard Competency:
Understanding the characteristics of arithmetical operation of numbers and its application to solve mathematical problems

Basic Competency:
Understanding percent and permill, their type and their characteristics

Strategy for Teaching Fractions:
1. Ensure that students have mastered the prerequisite skills for the tasks to be learned as part of your advance organizer presentation, demonstrate the “Big Idea” and interconnected relationships between division, fractions, decimals, and percents (e.g., physically divide a whole into five equal pieces)
2. Introduce skill instruction with succinct and unambiguous demonstration of the task to be learned (e.g., solve several problems while the student observes)
3. Introduce instruction using concrete materials (i.e., manipulatives) before proceeding to semi-concrete materials (e.g., pictorial representations) before proceeding to abstract problems (e.g., numerical representation)
4. Ensure that your teaching examples include sufficient practice opportunities to produce task mastery
5. Ensure that your teaching examples include variations of all problem types to avoid students making incorrect generalizations (e.g., that all fractions represent parts of a whole)
6. Provide systematic instruction on discriminating among different problem types that is designed to enable students to know which solution to employ (e.g., teach students to attend to the operation sign, perhaps by circling it, so that they will select the correct computational algorithm)
7. Provide guided practice (following teacher-led demonstrations) before assigning independent work (i.e., have teacher and student work several problems together)

What will you learn in this chapter?
- Percent
- Permill
A. Percent

Teacher’s Guide:

Two third of the Hemisphere consist of water.
State into the percentage the amount of water?

In a small group discussion, let the students discuss how state into the percentage the amount of water?

The teacher may use ICEBERG approach to encourage the students construct their own ways, as follow:

Word of percent comes from the word *per cent*, means per hundred. Percent is a fraction where the denominator is hundred or per hundred fraction. The notation of percent is %.

\[ a\% = \frac{a}{100} \], and \( a \) % is read a percent.

Express with your own word to show the following data:
Example

\[ 5\% = \frac{5}{100} = \frac{1}{20} ; 75\% = \frac{75}{100} = \frac{3}{4} \]

Example

1. About \(\frac{2}{3}\) of earth surface is covered by water. What is the percentage of the water?

Solution

About \(\frac{2}{3}\) of earth surface is covered by water.

\[ \frac{2}{3} \times 100\% = \frac{200}{3}\% . \]

You get 66.67\%.

Therefore, 66.67 % of earth surface is covered by water.

2. Aris has two type of books i.e. text book and story books. If Ari’s story books is 40% and his total books is 120 books, how many Ari’s school books?
Solution

The total books is 120 books, and 40% of total books is story books.

Thus, Ari’s story books is \(\dfrac{40}{100} \times 120\) books = 48 books.

Therefore, Ari’s school books is 120 – 48 = 72 books.

Exercise

1. Write the following fractions as percent form!
   a. \(\dfrac{1}{8}\)  
   b. \(\dfrac{13}{20}\)

2. Write down the following percentages as simple fractions.
   a. 32 %  
   b. 65 %

3. A 24 meters long hedge has 4 meters been painted. What the percentage of the hedge has not been painted yet?

4. There are 70% students of Junior High School present on the Independent Day celebration. If the total number of students is 40, how many students do not participated in the celebration?

5. I am the simplest form of a certain fraction. My numerator and denominator are prime numbers having 2 as their differences. The sum of my numerator and denominator is 12. What number am I?

B. Per mill

Permil is a fraction where the denominator is thousand or per thousand fraction. The notation of percent is \(\% \).
\[ a_{\%} = \frac{a}{1000}, \text{ and a } \% \text{ is read a permil.} \]

Express with your own word to indicate the following symbol:

Example:

Example: \[ 5\% = \frac{5}{1000} = \frac{1}{200}; 150\% = \frac{150}{1000} = \frac{3}{20} \]

Example

1. Write \[ \frac{1}{4} \] in per thousand.

2. Write \[ 750\% \] in a simple fraction.

Solution

1. \[ \frac{1}{4} = \frac{1}{4} \times 1000\% = 250\% \]

2. \[ 750\% = \frac{750}{1000} = \frac{750 \div 250}{1000 \div 250} = \frac{3}{4} \]

Exercise

1. Write the following fractions into per thousand.
2. Write down the following per thousand as a simple fraction.

a. \( \frac{1}{16} \)  b. \( \frac{3}{20} \)  c. \( \frac{21}{4} \)  d. \( \frac{11}{20} \)  e. \( \frac{3}{10} \)

a. 240‰  b. 850‰  c. 380‰  d. 780‰  e. 125‰
Chapter V
Working with Fractions

Standard Competency:
Understanding the characteristics of arithmetical operation of numbers and its application to solve mathematical problems

Basic Competency:
Working with fractions i.e. adding, subtracting, multiplying and dividing them

Strategy for Teaching Fractions:
1. Ensure that students have mastered the prerequisite skills for the tasks to be learned as part of your advance organizer presentation, demonstrate the “Big Idea” and interconnected relationships between division, fractions, decimals, and percents (e.g., physically divide a whole into five equal pieces)
2. Introduce skill instruction with succinct and unambiguous demonstration of the task to be learned (e.g., solve several problems while the student observes)
3. Introduce instruction using concrete materials (i.e., manipulatives) before proceeding to semi-concrete materials (e.g., pictorial representations) before proceeding to abstract problems (e.g., numerical representation)
4. Ensure that your teaching examples include sufficient practice opportunities to produce task mastery
5. Ensure that your teaching examples include variations of all problem types to avoid students making incorrect generalizations (e.g., that all fractions represent parts of a whole)
6. Provide systematic instruction on discriminating among different problem types that is designed to enable students to know which solution to employ (e.g., teach students to attend to the operation sign, perhaps by circling it, so that they will select the correct computational algorithm)
7. Provide guided practice (following teacher-led demonstrations) before assigning independent work (i.e., have teacher and student work several problems together)

What will you learn in this chapter?
- Adding Like Fractions
- Adding Unlike Fractions
- Subtracting Like Fractions
- Subtracting Unlike Fractions
A. Adding and Subtracting the Fractions

1. Adding the Fractions

1) Adding the Like Fractions

To add two like fractions, you just need to add the numerator while the denominator remains unchanged.

\[
\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}, \text{ where } c \neq 0
\]

Example

Calculate the following addition of fractions.

1. \( \frac{3}{8} + \frac{2}{8} \)
2. \( \frac{4}{9} + \left( -\frac{3}{9} \right) \)
3. \( 3\frac{1}{7} + 5\frac{3}{7} \)

Solution

1. \( \frac{3}{8} + \frac{2}{8} = \frac{3 + 2}{8} = \frac{5}{8} \)
2. \( \frac{4}{9} + \left( -\frac{3}{9} \right) = \frac{4 + (-3)}{9} = \frac{1}{9} \)
3. \[ \frac{3}{7} + \frac{5}{7} = \frac{22}{7} + \frac{38}{7} = \frac{22 + 38}{7} = \frac{60}{7} = 8\frac{4}{7} \]

Note:

Mixed fraction \( \frac{ab}{c} \) can be written as an improper fraction \( \frac{(a \times c) + b}{c} \).

2) Adding Unlike Fractions

To add two unlike fractions, you have to convert the fractions to like fractions first by using the Least Common Multiple (LCM) of the denominators and then add the numerators.

Example

Calculate the following addition of fractions!

1. \( \frac{3}{4} + \left( -\frac{1}{7} \right) \)
2. 1,37 + 2,18

Penyelesaian

1. \( \frac{3}{4} + \left( -\frac{1}{7} \right) = \frac{21}{28} + \left( -\frac{4}{28} \right) = \frac{17}{28} \)

The LCM of 4 and 7 is 28

2. 1,37 + 2,18 = \( \frac{123}{100} + \frac{218}{100} = \frac{355}{100} = (3 \times 1) + (5 \times \frac{1}{10}) + (5 \times \frac{1}{100}) = 3,55 \)

Exercise

Find the result of each of the following addition.

1. \( 2\frac{1}{2} + 12\frac{2}{7} \)
2. $0.03 + 1.2 + 10.08$

**B. Subtracting the Fractions**

Teacher’s Guide:

Agus has bought Tart Cake. He cut it into eight similar parts. He and his three younger brothers will each take one part of it. And he will save the rest. How many parts of the Cake Agus will save?

In a small group discussion, let the students discuss how many parts of the Cake Agus will save?

The teacher may use ICEBERG approach to encourage the students construct their own ways, as follow:

1. **Subtracting Like Fractions**
Subtracting the fractions is the opposite of adding fractions. To subtract like fractions, you just need to subtract the numerator while the denominator remain unchanged.

\[ \frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}, \text{ where } c \neq 0 \]

Express with your own word to show the following calculation:

\[ \frac{14}{3} - \frac{10}{3} = \frac{4}{3} \]

Simplified answer... \( 1 \frac{1}{3} \)

Source: helpwithfractions.com

Example

Calculate the subtraction fractions below.

1. \( \frac{4}{7} - \frac{3}{7} \)
2. \( \frac{5}{9} - \left( -\frac{1}{9} \right) \)
3. \( 5 \frac{6}{8} - 3 \frac{3}{8} \)

Solution

1. \( \frac{4}{7} - \frac{3}{7} = \frac{4 - 3}{7} = \frac{1}{7} \)

2. \( \frac{5}{9} - \left( -\frac{1}{9} \right) = \frac{5 - (-1)}{9} = \frac{5 + 1}{9} = \frac{6}{9} \)

3. \( 5 \frac{6}{8} - 3 \frac{3}{8} = \frac{46}{8} - \frac{27}{8} = \frac{19}{8} = 2 \frac{3}{8} \)

Exercise

Find the result of the following subtraction.

1. \( \frac{2}{27} - \frac{18}{27} \)
2. \( \frac{2}{52} - \left( -\frac{24}{52} \right) \)
3. $\frac{5}{7} - 2 \frac{6}{7}$

4. $\frac{15}{30} - \left( -\frac{37}{30} \right)$

5. $5 \frac{3}{5} - 3 \frac{1}{5}$

2. Subtracting the Unlike Fractions

To subtract unlike fractions, you have to convert the fractions to like fractions first by using the **Least Common Multiple (LCM)** of the denominators and then subtract the numerators.

Example

Calculate the following subtraction fractions.

1. $2 \frac{3}{8} - 1 \frac{5}{7}$

2. $6,28 - 0,37$

Solution

1. $2 \frac{3}{8} - 1 \frac{5}{7} = \frac{19}{8} - \frac{12}{7} = \frac{133}{56} - \frac{96}{56} = \frac{37}{56}$

   The **LCM** of 8 and 7 is 56.

2. $6,28 - 0,37 = \frac{628}{100} - \frac{37}{100} = \frac{591}{100} = (5 \times 1) + \left( 9 \times \frac{1}{10} \right) + \left( 1 \times \frac{1}{100} \right) = 5,91$
Exercise

1. Find the results of the following subtraction!
   a. \( \frac{2122}{2} - 12 \frac{2}{7} \)
   b. \( 2.93 - 0.13 \)

2. Mr. Amran had 3.87 litres of gasoline on his motor tank. About 0.19 litres have been spent by Mr. Amran to go to his office. How much of the gasoline on his motor tank remained?

B. Multiplying and Dividing the Fractions

1. Multiplying the Fractions

   The multiplication of the fractions \( \frac{a}{b} \) and \( \frac{c}{d} \), where \( b \neq 0 \) and \( d \neq 0 \), is
   \[
   \frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}, \quad b \neq 0, \quad d \neq 0
   \]

   Here are some properties of multiplication the fractions.
1. **Commutative**
   
   \[ a \times b = b \times a, \text{ where } a \text{ and } b \text{ are fractions.} \]

2. **Associative**
   
   \[ (a \times b) \times c = a \times (b \times c), \text{ where } a, b \text{ and } c \text{ are fractions.} \]

3. **Distributive**
   
   \[ a \times (b + c) = (a \times b) + (a \times c), \text{ where } a, b \text{ and } c \text{ are fractions.} \]

**Example**

Find the results of the following multiplication:

1. \[ \frac{7}{12} \times \frac{2}{17} \]
2. \[ -3 \frac{1}{5} \times \left( -7 \frac{2}{11} \right) \]
3. \[ 0,35 \times 1,42 \]

**Solution**

1. \[ \frac{7}{12} \times \frac{2}{17} = \frac{7 \times 2}{12 \times 17} = \frac{14}{204} = \frac{7}{102} \]
2. \[ -3 \frac{1}{5} \times \left( -7 \frac{2}{11} \right) = -\frac{16}{5} \times \left( -\frac{79}{11} \right) = \frac{16 \times (-79)}{5 \times 11} = \frac{1264}{55} \]
3. \[ 0,35 \times 1,42 = \frac{35}{100} \times \frac{142}{100} = \frac{35 \times 142}{100 \times 100} = \frac{4970}{10000} = 0.497 \]

**Exercise**

Calculate the following multiplication fractions:

1. \[ \frac{21}{43} \times \frac{17}{57} \]
2. \[ 9 \times \frac{7}{15} \]
3. \[ 5 \frac{3}{5} \times \left( -2 \frac{1}{3} \right) \]
4. \[ 2,41 \times 0,37 \]
5. \[ -5,17 \times \left( -5 \frac{1}{9} \right) \]

**2. Dividing the Fractions**
Teacher’s Guide:

Fifteen kilogram of rice will be put into some smaller containers size \( \frac{3}{4} \) kg. How many containers does it need?

In a small group discussion, let the students discuss how many containers does it need?

The teacher may use ICEBERG approach to encourage the students construct their own ways, as follow:

The division of fraction \( \frac{a}{b} \) by \( \frac{c}{d} \) where \( b \neq 0, c \neq 0 \) and \( d \neq 0 \), is

\[
\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}, \quad \text{where} \quad b \neq 0, c \neq 0 \text{ and } d \neq 0
\]

Find the results of the following divisions.

1. \( 6 : \frac{1}{8} \)

2. \( \frac{1}{8} : 6 \)

3. \( \frac{3}{2} : \frac{1}{5} \)
4. $9 : \frac{2}{5}$

5. $0,05 : 0,31$

Solution

1. $6 : \frac{1}{8} = \frac{6}{1} : \frac{1}{8} = \frac{6 \times 8}{1} = \frac{6 \cdot 8}{1} = 48$

2. $\frac{1}{8} : 6 = \frac{1}{8} \times \frac{1}{6} = \frac{1}{48}$

3. $\frac{3}{2} : \frac{1}{2} = \frac{3}{2} \times \frac{1}{1} = \frac{3}{2} = 7 \frac{1}{2}$

4. $9 : \frac{2}{5} = 9 : \frac{17}{5} = \frac{9 \times 5}{17} = \frac{9 \times 5}{17} = \frac{45}{17} = 2 \frac{11}{17}$

5. $0,05 : 0,31 = \frac{5}{100} : \frac{31}{100} = \frac{5 \times 100}{100 \times 31} = \frac{5 \times 100}{100 \times 31} = \frac{500}{3100} = \frac{500 : 100}{3100 : 100} = \frac{5}{31}$

Exercise

1. Find the results of the following divisions.
   
   a. $\frac{1}{3} : \frac{1}{4}$

   b. $\frac{3}{5} : (-12)$

   c. $27 : 0,25$

   d. $3 : \frac{1}{3}$

   e. $4 \frac{1}{7} : (-0,28)$
2. The ticket price on “Galaksi” theatre on Tuesday-Monday is Rp 15,000.00. Special offer on Monday is that $\frac{5}{6}$ of the ticket price on Tuesday-Monday. Pipit brought Rp 50,000.00 and she wants with 4 of her friends to see a movie on Monday. Has Pipit enough money to see the movie with her friends?

3. Mrs. Rosita bought 5 cakes that will be given to her children. Each of her children will get $\frac{1}{4}$ cakes. How many children Mrs. Rosita has?

C. Exponent of the Fractions

The exponent form of the fractions can be written as $\left(\frac{a}{b}\right)^n$.

\[
\left(\frac{a}{b}\right)^n = \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \ldots \times \frac{a}{b} = \frac{a \times a \times a \times \ldots \times a}{b \times b \times b \times \ldots \times b} = \frac{a^n}{b^n}
\]

Here are some properties of the exponent of the fractions.

1. $\left(\frac{a}{b}\right)^m \times \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m+n} = \frac{a^{m+n}}{b^{m+n}}$

2. $\left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{m-n} = \frac{a^{m-n}}{b^{m-n}}$, where $m > n$
Example

Find the results of the following operations.

1. \( \left( \frac{1}{3} \right)^3 \times \left( \frac{1}{3} \right)^2 \)
2. \( \frac{\left( \frac{1}{2} \right)^3}{\left( \frac{1}{2} \right)} \)
3. \( \left( \frac{1}{3} \right)^2 \)^3

Solution

1. \( \left( \frac{1}{3} \right)^3 \times \left( \frac{1}{3} \right)^2 = \left( \frac{1}{3} \right)^{3+2} = \left( \frac{1}{3} \right)^5 = \frac{1 \times 1 \times 1 \times 1 \times 1}{3 \times 3 \times 3 \times 3 \times 3} = \frac{1}{243} \)

2. \( \frac{\left( \frac{1}{2} \right)^3}{\left( \frac{1}{2} \right)} = \left( \frac{1}{2} \right)^{5-2} = \left( \frac{1}{2} \right)^3 = \frac{1 \times 1 \times 1}{2 \times 2 \times 2} = \frac{1}{8} \)

3. \( \left( \frac{1}{3} \right)^2 \)^3 = \( \left( \frac{1}{3} \right)^{2 \times 3} = \left( \frac{1}{3} \right)^6 = \frac{1 \times 1 \times 1 \times 1 \times 1 \times 1}{3 \times 3 \times 3 \times 3 \times 3 \times 3} = \frac{1}{729} \)

Exercise

Find the result of each of the following.

1. \( \left( \frac{1}{5} \right)^2 \times \left( \frac{1}{5} \right)^3 \)
2. \( \frac{(0,3)^5}{(0,3)^2} \)
3. \[ \left( - \frac{1}{4} \right)^3 = 2 \]

4. Express with your own word to show the following calculation:

\[
\begin{align*}
d^m \times d^n &= d^{m+n} \\
\frac{d^m}{d^n} &= d^{m-n} \quad (a \neq 0) \\
(d^m)^n &= d^{mn} \\
(ab)^n &= a^n b^n \\
\left( \frac{a}{b} \right)^n &= \frac{a^n}{b^n} \\
d^0 &= 1 \quad (a \neq 0) \\
d^{-n} &= \frac{1}{d^n} \quad (a \neq 0)
\end{align*}
\]
Chapter VI
Scientific Notation of Fraction and Rounding-up the Decimals

Standard Competency:
Understanding the characteristics of arithmetical operation of numbers and its application to solve mathematical problems

Basic Competency:
Expressing the scientific notation of fractions

Strategy for Teaching Fractions:
1. Ensure that students have mastered the prerequisite skills for the tasks to be learned as part of your advance organizer presentation, demonstrate the “Big Idea” and interconnected relationships between division, fractions, decimals, and percents (e.g., physically divide a whole into five equal pieces)
2. Introduce skill instruction with succinct and unambiguous demonstration of the task to be learned (e.g., solve several problems while the student observes)
3. Introduce instruction using concrete materials (i.e., manipulatives) before proceeding to semi-concrete materials (e.g., pictorial representations) before proceeding to abstract problems (e.g., numerical representation)
4. Ensure that your teaching examples include sufficient practice opportunities to produce task mastery
5. Ensure that your teaching examples include variations of all problem types to avoid students making incorrect generalizations (e.g., that all fractions represent parts of a whole)
6. Provide systematic instruction on discriminating among different problem types that is designed to enable students to know which solution to employ (e.g., teach students to attend to the operation sign, perhaps by circling it, so that they will select the correct computational algorithm)
7. Provide guided practice (following teacher-led demonstrations) before assigning independent work (i.e., have teacher and student work several problems together)

What will you learn from this chapter?
- Scientific Notation
- Rounding-up the Decimals
A. Scientific Notation

Scientific notation is a short way to write a very large or a very small number to make it more efficient in its writing.

\[ 10^{-1} = \frac{1}{10} = 0.1 \]
\[ 10^{-3} = \frac{1}{1,000} = 0.001 \]
\[ 10^{-6} = \frac{1}{1,000,000} = 0.000001 \]

Here are some rules of scientific notation.

1. For numbers bigger than 10, the scientific notation is \( a \times 10^n \), with \( 1 \leq a < 10 \) and \( n \) is the real number.
2. For numbers between 0 and 1, the scientific notation is \( a \times 10^{-n} \), with \( 1 \leq a < 10 \) and \( n \) is the real number.

Example

Write down the number below in the scientific notation.
1. 2.732  2. 0.000253

Solution:

a. 2.732 is bigger than 10. So, use \( a \times 10^n \) rule with \( 1 \leq a < 10 \) with the real number of \( n \). You have, \( a = 2.732 \) and \( n = 3 \). So the scientific notation of 2.732 is \( 2.732 \times 10^3 \).

b. 0.000253 is the number between 0 and 1. So use \( a \times 10^{-n} \) rule with \( 1 \leq a < 10 \) with the real number of \( n \). So the scientific notation of 0.000253 is \( 2.53 \times 10^{-4} \).
Exercise

Write down the following number as a scientific form!

1. 278.000    2. 0,000008654

B. Rounding-up the Decimals

The purpose of rounding-up the decimals is to approximate a number by replacing it with a simpler number, one with fewer digits or one with zeros for its ending digits.

There are two rules in rounding-up the decimals
1. If the number is ≥ 5 it should be rounded-up.
2. If the number is less than 5 it should not be rounded-up.

Examples

Do the following task!

1. Round 6,321 off up to two decimal places!
2. Round 7,461 off up to one decimal place!

Solution:

1. 6,321 has three places of decimal. Because of 1<5 so the round off is 6,32. So the round off 6,321 until two places of decimal is 6,32.

2. 7,461 has three places of decimal.
   - The last number of 7,461 is 1, Because of 1<5 so the round off is 7,46.
The last number of 7.46 is 6. Because of 6>5 so the round off is 7.5
So the round off 7.461 until one place of decimal is 7.5

Exercises

Do the following task!

1. Round 78.3646 off until two places of decimal.
2. Round 4.5562396 off until one place of decimal!

References

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