Moduli and Conjugate

The *modulus* or absolute value of the complex number $z = x + iy$ is defined as the nonnegative real number and denoted by $|z|$; that is $|z| = \sqrt{x^2 + y^2}$.

The *complex conjugate* of a complex number $z = x + iy$ is defined as the complex number $x - iy$ and denoted by $\overline{z}$; that is $\overline{z} = x - iy$.

**Example 1**

\[
\frac{1 + 3i}{2 - i} = \frac{1 + 3i}{2 - i} \cdot \frac{2 + i}{2 + i} = \frac{1 + 7i}{5} = \frac{1}{5} + \frac{7}{5}i
\]

**Example 2**

If $z$ is a point inside the circle centered at the origin and with radius 3, so that $|z| < 3$, then

\[
|z^3 + 2z^2 - z + 1| \leq |z|^3 + 2|z|^2 - |z| + 1 < 3^3 + 2(3)^2 - 3 + 1 = 43.
\]

**Exercises:**

1. Suppose $z_1 = 1 + i$ and $z_2 = 2 + i$. Evaluate each following
   
   (a) $|3z_1 - 4z_2|$  
   (b) $z_1^3 - 3z_2^2 + 2z_1 - 4$  
   (c) $(\overline{z}_2)^4$ 
   (d) $|3z_2 - 4 - i|^2 \div 2z_1$

2. Find real numbers $x$ and $y$ such that $3x + 2iy - ix + 5y = 7 + 5i$.

3. If $z_1, z_2 \neq 0$, then show that $\left( \frac{z_1}{z_2z_3} \right) = \frac{\overline{z}_1}{\overline{z}_2z_3}$.

4. If $|z| = 3$, then show that $\left| \frac{1}{z^2 + 1} \right| \leq \frac{1}{6}$.