

C. DIFERENSIASI BAKU TRIGONOMETRI

1. $y = \sin x$	maka	$dy/dx = \cos x$
2. $y = \cos x$		$dy/dx = -\sin x$
3. $y = \operatorname{tg} x$		$dy/dx = \sec^2 x$
4. $y = \operatorname{cotg} x$		$dy/dx = -\operatorname{cosec}^2 x$
5. $y = \sec x$		$dy/dx = \sec x \cdot \operatorname{tg} x$
6. $y = \operatorname{cosec} x$		$dy/dx = -\operatorname{cosec} x \operatorname{ctg} x$
7. $y = \sinh x$		$dy/dx = \cosh x$
8. $y = \cosh x$		$dy/dx = \sinh x$

Bila $U = f(x)$ dapat diturunkan, maka

$$\frac{d \sin U}{dx} = \cos U \frac{du}{dx} \rightarrow \text{rumus no.2 s/d 8 identik}$$

contoh 1.

Hitunglah $\frac{dy}{dx}$ dari $y = \cos^3 5x$

$$\begin{aligned} \text{Penyelesaian : } \frac{dy}{dx} &= 3(\cos^2 5x) \frac{d \cos 5x}{dx} \rightarrow \text{rumus no.2} \\ &= 3(\cos^2 5x)(-\sin 5x) \frac{d5x}{dx} \\ &= -15 \sin 5x \cos^2 5x \end{aligned}$$

contoh 2.

Hitunglah $\frac{dy}{dx}$ dari $y = \operatorname{ctg} 2x \operatorname{cosec} 2x$

$$\text{Penyelesaian : } \rightarrow \text{ingat } y = U.V \text{ maka } \frac{dy}{dx} = U \frac{dv}{dx} + V \frac{du}{dx}$$

$$\begin{aligned} \frac{dy}{dx} &= \operatorname{ctg} 2x \frac{d \operatorname{cosec} 2x}{dx} + \operatorname{cosec} 2x \frac{d \operatorname{ctg} 2x}{dx} \\ &= \operatorname{ctg} 2x (- \operatorname{cosec} 2x \cdot \operatorname{ctg} 2x) 2 + \operatorname{cosec} 2x (- \operatorname{cosec}^2 2x) 2 \\ &= \operatorname{ctg}^2 2x (- \operatorname{cosec} 2x) \cdot 2 + \operatorname{cosec} 2x (- \operatorname{cosec}^2 2x) 2 \\ &= -2 \operatorname{cosec} 2x (\operatorname{ctg}^2 2x + \operatorname{cosec}^2 2x) \end{aligned}$$

Karena $\text{ctg}^2 \alpha = \text{cosec}^2 \alpha - 1$, maka :

$$\begin{aligned}
 &= -2 \text{ cosec } 2x [(\text{cosec}^2 2x - 1) + \text{cosec}^2 2x] \\
 &= -2 \text{ cosec } 2x (2 \text{ cosec}^2 2x - 1) \\
 &= 2 \text{ cosec } 2x - 4 \text{ cosec}^3 2x
 \end{aligned}$$

INGAT !

$\sin^2 \alpha + \cos^2 \alpha = 1$ $1 + \text{ctg}^2 \alpha = \text{cosec}^2 \alpha$ $1 + \text{tg}^2 \alpha = \text{sec}^2 \alpha$
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D. DIFERENSIASI FUNGSI IMPLISIT

$y = x^2 - 4x + 2 \rightarrow$ fungsi eksplisit dari x

$x^2 - 4x - y = 2 \rightarrow$ fungsi implisit dari x

contoh :

jika $x^2 + y^2 - 2x - 6y + 5 = 0$, tentukan $\frac{dy}{dx}$ di titik $x = 3, y = 2$

Penyelsaian :

$$x^2 + y^2 - 2x - 6y + 5 = 0$$

$$2x + 2y \frac{dy}{dx} - 2 - 6 \frac{dy}{dx} = 0$$

$$(2y - 6) \frac{dy}{dx} = 2 - 2x$$

$$\frac{dy}{dx} = \frac{2 - 2x}{2y - 6} = \frac{1 - x}{y - 3}$$

$$\therefore \text{ di } (3, 2) \rightarrow \frac{dy}{dx} = \frac{1 - 3}{2 - 3} = \frac{-2}{-1} = 2$$

E. DIFERENSIASI LOGARITMIK LEBIH DARI DUA FAKTOR

Jika $y = \frac{U.V}{W}$ dimana $U = f(x)$; $V = g(x)$; $W = h(x)$

Maka untuk mencari turunan pertamanya adalah dengan logaritma dengan bilangan dasar e

${}^e \log y = {}^e \log \frac{U.V}{W}$ dimana $\ln a.b = \ln a + \ln b$

$\ln a/b = \ln a - \ln b$

dirubah menjadi $\ln y = \ln U + \ln V - \ln W$

sehingga $\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{U} \cdot \frac{du}{dx} + \frac{1}{V} \cdot \frac{dv}{dx} - \frac{1}{W} \cdot \frac{dw}{dx}$

$\frac{dy}{dx} = y \left(\frac{1}{U} \cdot \frac{du}{dx} + \frac{1}{V} \cdot \frac{dv}{dx} - \frac{1}{W} \cdot \frac{dw}{dx} \right)$

jadi jika

$y = \frac{U.V}{W}$ maka $\frac{dy}{dx} = \frac{U.V}{W} \left(\frac{1}{U} \cdot \frac{du}{dx} + \frac{1}{V} \cdot \frac{dv}{dx} - \frac{1}{W} \cdot \frac{dw}{dx} \right)$

INGAT SIFAT-SIFAT LOG

$\ln x = {}^e \log x$

Jadi sifat log juga berlaku untuk ln, al :

$$\ln 1 = 0$$

$$\ln e = 1$$

$$\ln a.x = \ln a + \ln x$$

$$\ln a/x = \ln a - \ln x \rightarrow x \neq 0$$

$$\ln x^n = n \ln x$$

$$a = e^{\ln a}$$

Syarat : x dan a bilangan positif

N bilangan rasional

F. DIFERENSIASI FUNGSI EKSPONEN

$$\frac{d}{dx} e^x = e^x; \frac{d}{dx} e^{k \cdot x} = k \cdot e^{kx}$$

$$\frac{d}{dx} e^u = e^u; \frac{du}{dx} \rightarrow U = f(x)$$

$$\frac{d}{dx} a^u = a^u \ln a \frac{du}{dx}$$

$$\frac{d}{dx} \log U = \frac{1}{U \ln a} \frac{du}{dx}$$

$$\frac{d}{dx} e^{-x} = -e^{-x}$$

Catatan : Gunakanlah selalu cara diferensial logaritmik bila ada lebih dari dua fungsi dalam suatu perkalian atau pembagian maupun dua-duanya.

Contoh:

Carilah harga $\frac{dy}{dx}$ dari persamaan $y = \frac{x^2 \cdot \sin x}{\cos 2x}$

Penyelesaian :

$$\ln y = \ln (x^2) + \ln (\sin x) - \ln (\cos 2x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x^2} \cdot 2x + \frac{1}{\sin x} \cdot \cos x - \frac{1}{\cos 2x} (-2 \sin 2x)$$

$$\text{ingat : } \frac{\cos x}{\sin x} = \text{ctgx} ; \frac{\sin x}{\cos x} = \text{tgx}$$

jadi

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{2x}{x^2} + \frac{\cos x}{\sin x} + \frac{2 \sin 2x}{\cos 2x}$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{2}{x} + \text{ctgx} + 2 \text{tg} 2x$$

$$\text{Karena } y = \frac{x^2 \cdot \sin x}{\cos 2x} \text{ maka } \frac{dy}{dx} = \frac{x^2 \sin x}{\cos 2x} (2/x + \text{ctgx} + 2 \text{tg} 2x)$$