Kinetic Theory of Gases I:

- Gas Pressure
- Translational Kinetic Energy
- Root Mean Square Speed
GASES

- Gases are one of the most pervasive aspects of our environment on the Earth. We continually exist with constant exposure to gases of all forms.
- The steam formed in the air during a hot shower is a gas.
- The Helium used to fill a birthday balloon is a gas.
- The oxygen in the air is an essential gas for life.
A windy day or a still day is a result of the difference in pressure of gases in two different locations. A fresh breeze on a mountain peak is a study in basic gas laws.
Mixture of gases
The Kinetic Molecular Model for Gases

- Gas consists of large number of small individual particles with negligible size
- Particles in constant random motion and collisions
- No forces exerted among each other
- Kinetic energy directly proportional to temperature in Kelvin

\[ KE = \frac{3}{2} \cdot R \cdot T \]
The Ideal Gas Law

\[ PV = nRT \]

\( n \): the number of moles in the ideal gas

\[ n = \frac{N}{N_A} \]

total number of molecules

Avogadro’s number: the number of atoms, molecules, etc, in a mole of a substance: \( N_A = 6.02 \times 10^{23}/\text{mol} \).

\( R \): the Gas Constant: \( R = 8.31 \ \text{J/mol} \cdot \text{K} \)
Pressure and Temperature

**Pressure**: Results from collisions of molecules on the surface

\[ P = \frac{F}{A} \]

**Force**: Rate of momentum given to the surface

\[ F = \frac{dp}{dt} \]

**Momentum**: momentum given by each collision times the number of collisions in time \( dt \)
Only molecules moving toward the surface hit the surface. Assuming the surface is normal to the $x$ axis, half the molecules of speed $v_x$ move toward the surface.

Only those close enough to the surface hit it in time $dt$, those within the distance $v_x dt$.
The number of collisions hitting an area $A$ in time $dt$ is

$$\frac{1}{2} \left( \frac{N}{V} \right) \cdot A \cdot v_x \cdot dt$$

The momentum given by each collision to the surface is $2mv_x$.

Average density

Cylinder; volume $A \cdot v_x \cdot dt$
Momentum in time $dt$:

$$dp = (2mv_x) \cdot \frac{1}{2} \cdot \left( \frac{N}{V} \right) \cdot A \cdot v_x dt$$

Force:

$$F = \frac{dp}{dt} = (2mv_x) \cdot \frac{1}{2} \cdot \left( \frac{N}{V} \right) \cdot A \cdot v_x$$

Pressure:

$$P = \frac{F}{A} = \frac{N}{V} mv_x^2$$

Not all molecules have the same $v_x$, $\Rightarrow$ average $\overline{v_x^2}$
\[ v_x^2 = \frac{1}{3} v^2 = \frac{1}{3} \left( v_x^2 + v_y^2 + v_z^2 \right) \]

\[ \bar{v}_x = \frac{1}{3} \bar{v}^2 = \frac{1}{3} v_{rms}^2 \]  

\( v_{rms} \) is the root-mean-square speed

\[ v_{rms} = \sqrt{\bar{v}^2} = \sqrt{\frac{v_x^2 + v_y^2 + v_z^2}{3}} \]

Pressure:

\[ P = \frac{1}{3} \frac{N}{V} \bar{m} \bar{v}^2 = \frac{2}{3} \left( \frac{N}{V} \right) \left( \frac{1}{2} \bar{m} \bar{v}^2 \right) \]

Average Translational Kinetic Energy:

\[ \bar{K} = \frac{1}{2} \bar{m} \bar{v}^2 = \frac{1}{2} \bar{m} v_{rms}^2 \]
Pressure:

\[ P = \frac{2}{3} \cdot \frac{N}{V} \cdot \bar{K} \]

From

\[ PV = \frac{2}{3} \cdot N \cdot \bar{K} \]

and

\[ PV = nRT \]

Temperature:

\[ \bar{K} = \frac{3}{2} \cdot \frac{nRT}{N} = \frac{3}{2} \cdot k_BT \]

Boltzmann constant:

\[ k_B = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ J/K} \]
From

\[ PV = \frac{1}{3} \cdot N \cdot m v_{rms}^2 \]

and

\[ PV = nRT = \frac{N}{N_A} \cdot RT \]

Avogadro’s number

\[ N = nN_A \]

Molar mass

\[ M = mN_A \]

\[ v_{rms} = \sqrt{\frac{3RT}{M}} \]
Pressure ✓ Density x Kinetic Energy

Temperature ✓ Kinetic Energy
(a) Compute the root-mean-square speed of a nitrogen molecule at 20.0 °C. At what temperatures will the root-mean-square speed be (b) half that value and (c) twice that value?

\[ \sqrt{v_{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.31 \text{ J/mol} \cdot \text{K})(293 \text{ K})}{28.0 \times 10^{-3} \text{ kg/mol}}} = 511 \text{ m/s} \]

(b) Since \( v_{rms} \propto \sqrt{T} \)

for 0.5 \( v_{rms} \quad T' = 0.5^2 T = 73.3 \text{ K} = -200 \text{ °C} \)

for 2 \( v_{rms} \quad T'' = 2^2 T = 1.17 \times 10^3 \text{ K} = 899 \text{ °C} \)
Please estimate the root mean square mean velocity of Hydrogen gas.

A. 2000  B. 1000  C. 500
D. 100 m/s
E. 250
What is the average translational kinetic energy of nitrogen molecules at 1600K, (a) in joules and (b) in electron-volts?

(a) 
\[ \overline{K} = \frac{3}{2} k_B T = \frac{3}{2} \left( 1.38 \times 10^{-23} \text{ J/K} \right) \left( 1600 \text{ K} \right) \]
\[ = 3.31 \times 10^{-20} \text{ J} \]

(b) 1 eV = 1.60 \times 10^{-19} \text{ J}

\[ \overline{K} = \frac{3.31 \times 10^{-20} \text{ J}}{1.60 \times 10^{-19} \text{ J/eV}} = 0.21 \text{ eV} \]

\[ \overline{K} = \frac{3}{2} \cdot k_B T \]