The Kinetic Theory of Gas II:

- Maxwell-Boltzmann Distribution
- Root Mean Square Speed
- Average Speed
- Most probable Speed

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Maxwell-Boltzmann Velocity Distribution

✓ *M-B Equation gives distribution of molecules in terms of:*

- *Speed/Velocity, and*
- *Energy*

✓ *One-dimensional Velocity Distribution in the x-direction: [$1D_{v\text{-}x}$]*

Ratio Constant

$$\frac{dN}{N} = A \cdot e^{-\frac{1}{2}\cdot m \cdot v_x^{\,2} / k \cdot T} \cdot dv_x$$

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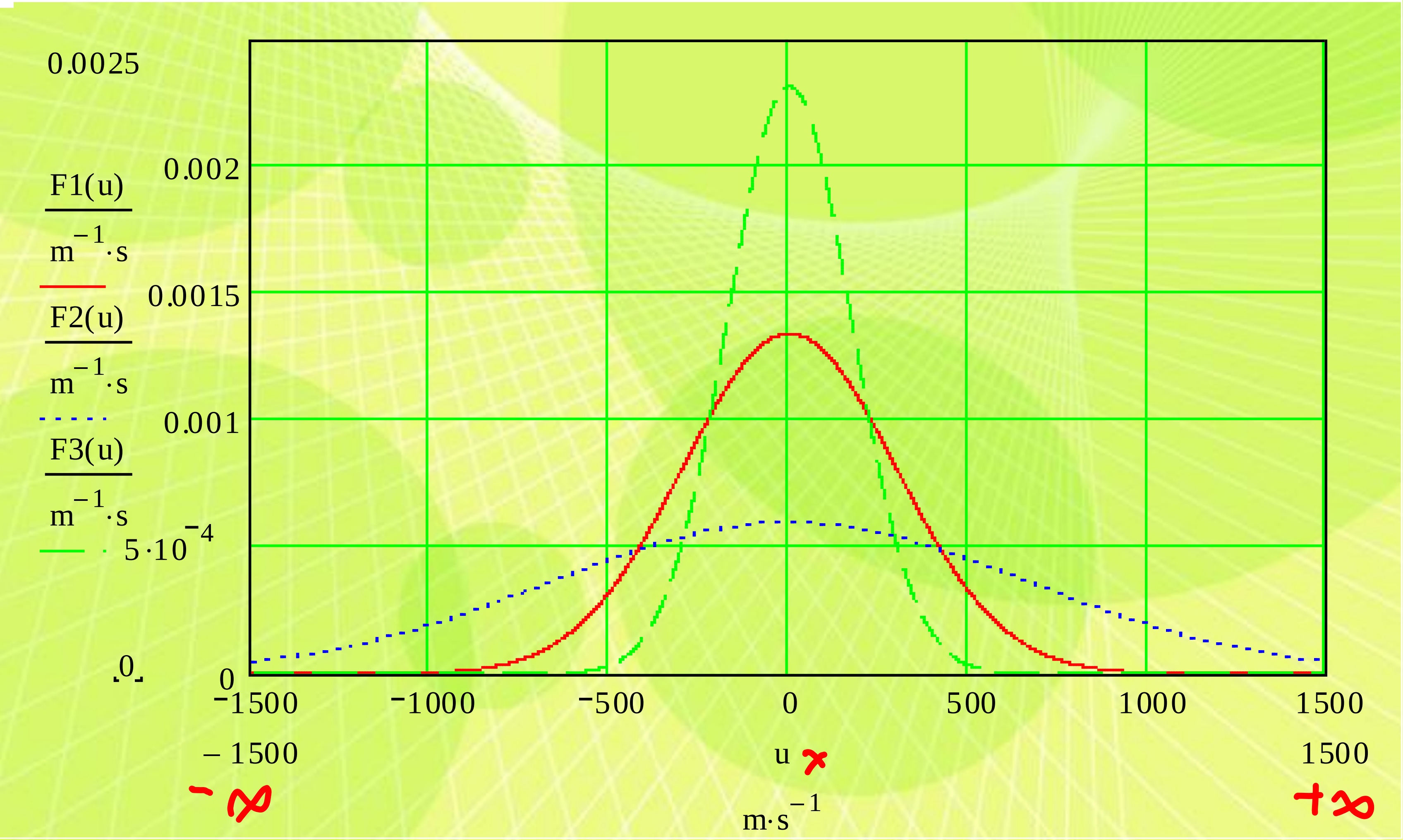

$$\int_{-\infty}^{\infty} A\cdot e^{-\frac{1}{2}\cdot m\cdot v_x{}^2/k\cdot T}\cdot dv_x = 1$$

$$A = \frac{1}{\int_{-\infty}^{\infty} e^{-\frac{1}{2}\cdot m\cdot v_x{}^2/k\cdot T}\cdot dv_x}$$

$$\int_{-\infty}^{\infty} e^{-\frac{1}{2}\cdot m\cdot v_x{}^2/k\cdot T}\cdot dv_x = \sqrt{\frac{2\pi kT}{m}}$$

$$A = \sqrt{\frac{m}{2\pi kT}}$$

Mathematics Solutions:

$$\int_{-\infty}^{\infty} v^n\cdot e^{-a\cdot v^2}\cdot dv =$$

$$\int_{-\infty}^{\infty} e^{-a\cdot v^2}\cdot dv = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} v\cdot e^{-a\cdot v^2}\cdot dv = 0$$

$$\int_{-\infty}^{\infty} v^2\cdot e^{-a\cdot v^2}\cdot dv = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}$$

$$\int_{-\infty}^{\infty} v^3\cdot e^{-a\cdot v^2}\cdot dv = 0$$

Maxwell-Boltzmann Distribution

3D Velocity Distribution: [$3D_v$],

So: $A = \sqrt{\dfrac{m}{2\pi kT}}$

Let: $a = \dfrac{m}{2kT}$

$$\left(\frac{dN}{N}\right)_{1D-v_x} = \sqrt{\frac{a}{\pi}} \cdot e^{-a v_x^{\,2}} \cdot dv_x$$

$$\left(\frac{dN}{N}\right)_{1D-v_y} = \sqrt{\frac{a}{\pi}} \cdot e^{-a v_y^{\,2}} \cdot dv_y$$

$$\left(\frac{dN}{N}\right)_{1D-v_z} = \sqrt{\frac{a}{\pi}} \cdot e^{-a v_z^{\,2}} \cdot dv_z$$

Cartesian Coordinates:

$$\left(\frac{dN}{N}\right)_{3D} = \left(\frac{a}{\pi}\right)^{3/2} \cdot e^{-a[v_x^{\,2} + v_y^{\,2} + v_z^{\,2}]} \cdot dv_x \cdot dv_y \cdot dv_z$$

Maxwell-Boltzmann Speed Distribution

Re-shape box into sphere of same volume with radius v.

$V = (4/3)\,\pi\, v^3$ with $v^2 = v_x^2 + v_y^2 + v_z^2$

$dv = dv_x\, dv_y\, dv_z = 4\,\pi\, v^2\, dv$

$$\left(\frac{dN/N}{dv}\right)_{3D} = \frac{4}{\sqrt{\pi}} \cdot a^{3/2} \cdot v^2 \cdot e^{-a\cdot v^2}$$

$$f(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$$

3D Maxwell-Boltzmann Speed Distribution

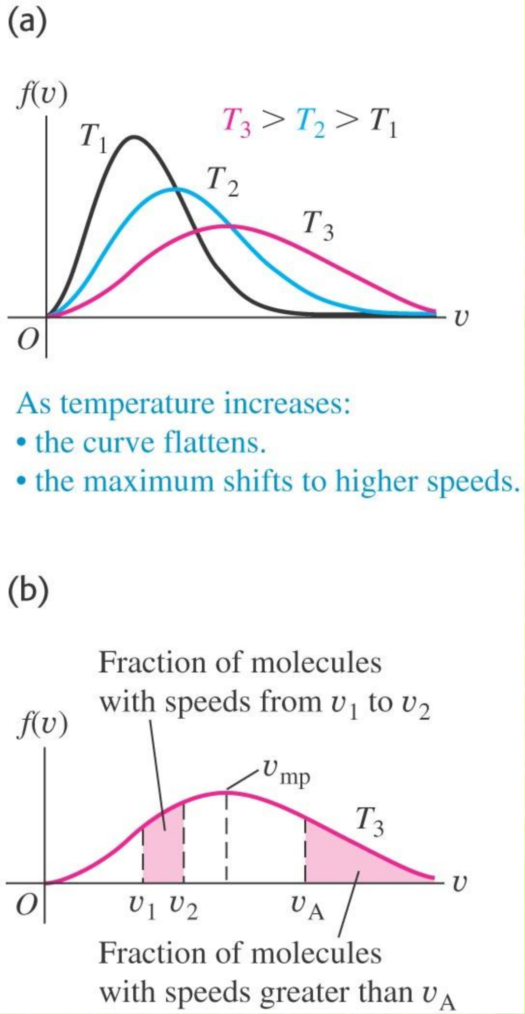

Table 18.2 Fractions of Molecules in an Ideal Gas with Speeds Less than Various Multiples of v/v_{rms}

v/v_{rms}	Fraction
0.20	0.011
0.40	0.077
0.60	0.218
0.80	0.411
1.00	0.608
1.20	0.771
1.40	0.882
1.60	0.947
1.80	0.979
2.00	0.993

Velocity Values from M-B Distribution

- v_{rms} = root mean square velocity
- v_{avg} = average velocity
- v_{mp} = most probable velocity

$$(x^n)_{average} = \int x^n \cdot \left(\frac{dN}{N} \right)_x$$

Example: Evaluate the values of v_{rms} by use of M-B Distribution!

Mathematics Solutions:

$$\int_0^\infty v^n \cdot e^{-a\cdot v^2} \cdot dv = I$$

$$\text{if } n=0,\ I = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\text{if } n=1,\ I = \frac{1}{2a}$$

$$\text{if } n=2,\ I = \frac{1}{4}\left(\frac{\pi}{a^3}\right)^{\frac{1}{2}}$$

$$\text{if } n=3,\ I = \frac{1}{2a^2}$$

$$\text{if } n=4,\ I = \frac{3}{8}\left(\frac{\pi}{a^5}\right)^{\frac{1}{2}}$$

$$\sqrt{\overline{v^2}} = \int_0^\infty v^2.f(v).dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}\int_0^\infty v^4 e^{-mv^2/2kT}.dv$$

$$v^2 = 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} x\,\frac{3}{8}\left[\pi\left(\frac{2kT}{m}\right)^5\right]^{\frac{1}{2}}$$

$$\overline{v^4} = \pi^2\left(\frac{m}{2\pi kT}\right)^3 x\,\frac{9}{4}\left[\pi\left(\frac{2kT}{m}\right)^5\right]$$

$$\overline{v^2} = \left(\frac{3kT}{m}\right) \quad so \quad v_{rms} = \sqrt{\frac{3kT}{m}}$$

Evaluate the v_{avg} by use of M-B Distribution!

– **The root-mean square speed :**

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

DONE

– **The average speed:**

$$v_{avg} = \sqrt{\frac{8kT}{\pi m}}$$

– **The most possible speed:**

$$v_{mp} = \sqrt{\frac{2kT}{m}}$$

Remember:

- You can use R, instead of k, and $R=kN_A$
- When you use R, you have to use M (mass of mole) instead of m, and $M=mN_A$.

Comparison of Velocity Values

Ratio in terms of : $\sqrt{\frac{kT}{m}}$		
V_{rms}	V_{avg}	V_{mp}
$\sqrt{3}\cdot\sqrt{\frac{kT}{m}}$	$\sqrt{\frac{8}{\pi}}\cdot\sqrt{\frac{kT}{m}}$	$\sqrt{2}\cdot\sqrt{\frac{kT}{m}}$
1.73	1.60	1.41

Thank You

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