The Kinetic Theory of Gas II:

- Maxwell-Boltzmann Distribution
- Root Mean Square Speed
- Average Speed
- Most probable Speed

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Maxwell-Boltzmann Velocity Distribution

✓ M-B Equation gives distribution of molecules in terms of:
  • Speed/Velocity, and
  • Energy

✓ One-dimensional Velocity Distribution in the x-direction: [ 1D \( v_x \) ]

\[
\frac{dN}{N} = A \cdot e^{-\frac{1}{2} \cdot \frac{m \cdot v_x^2}{k \cdot T}} \cdot dv_x
\]
\[
\frac{dN}{N} = A \cdot e^{-\frac{1}{2} \cdot m \cdot v_x^2 / k \cdot T} \cdot dv_x
\]
\[ \int_{-\infty}^{\infty} A \cdot e^{-\frac{1}{2} \cdot m \cdot v_x^2 / k \cdot T} \cdot dv_x = 1 \]

\[ A = \frac{1}{\int_{-\infty}^{\infty} e^{-\frac{1}{2} \cdot m \cdot v_x^2 / k \cdot T} \cdot dv_x} \]

\[ \int_{-\infty}^{\infty} e^{-\frac{1}{2} \cdot m \cdot v_x^2 / k \cdot T} \cdot dv_x = \sqrt{\frac{2\pi kT}{m}} \]

\[ A = \sqrt{\frac{m}{2\pi kT}} \]

Mathematics Solutions:

\[ \int_{-\infty}^{\infty} v^n \cdot e^{-a \cdot v^2} \cdot dv = \]

\[ \int_{-\infty}^{\infty} e^{-a \cdot v^2} \cdot dv = \sqrt{\frac{\pi}{a}} \]

\[ \int_{-\infty}^{\infty} v \cdot e^{-a \cdot v^2} \cdot dv = 0 \]

\[ \int_{-\infty}^{\infty} v^2 \cdot e^{-a \cdot v^2} \cdot dv = \frac{1}{2} \sqrt{\frac{\pi}{a^3}} \]

\[ \int_{-\infty}^{\infty} v^3 \cdot e^{-a \cdot v^2} \cdot dv = 0 \]
Maxwell-Boltzmann Distribution

3D Velocity Distribution: \( [3D_v] \),

So:

Let: \( a = \frac{m}{2kT} \)

Cartesian Coordinates:

\[
\left( \frac{dN}{N} \right)_{3D} = \left( \frac{a}{\pi} \right)^{3/2} \cdot e^{-a[v_x^2 + v_y^2 + v_z^2]} \cdot dv_x \cdot dv_y \cdot dv_z
\]

\[
\left( \frac{dN}{N} \right)_{1D-v_x} = \sqrt{\frac{a}{\pi}} \cdot e^{-av_x^2} \cdot dv_x
\]

\[
\left( \frac{dN}{N} \right)_{1D-v_y} = \sqrt{\frac{a}{\pi}} \cdot e^{-av_y^2} \cdot dv_y
\]

\[
\left( \frac{dN}{N} \right)_{1D-v_z} = \sqrt{\frac{a}{\pi}} \cdot e^{-av_z^2} \cdot dv_z
\]
Re-shape box into sphere of same volume with radius \( v \).

\[ V = \frac{4}{3} \pi v^3 \quad \text{with} \quad v^2 = v_x^2 + v_y^2 + v_z^2 \]

\[ dv = dv_x \, dv_y \, dv_z = 4 \pi v^2 \, dv \]

\[
\left( \frac{dN / N}{dv} \right)_{3D} = \frac{4}{\sqrt{\pi}} \cdot a^{3/2} \cdot v^2 \cdot e^{-a \cdot v^2}
\]

\[
f(v) = 4\pi \left( \frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/2kT}
\]
3D Maxwell-Boltzmann Speed Distribution

As temperature increases:
- the curve flattens.
- the maximum shifts to higher speeds.

**Table 18.2** Fractions of Molecules in an Ideal Gas with Speeds Less than Various Multiples of $v/v_{\text{rms}}$

<table>
<thead>
<tr>
<th>$v/v_{\text{rms}}$</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>0.011</td>
</tr>
<tr>
<td>0.40</td>
<td>0.077</td>
</tr>
<tr>
<td>0.60</td>
<td>0.218</td>
</tr>
<tr>
<td>0.80</td>
<td>0.411</td>
</tr>
<tr>
<td>1.00</td>
<td>0.608</td>
</tr>
<tr>
<td>1.20</td>
<td>0.771</td>
</tr>
<tr>
<td>1.40</td>
<td>0.882</td>
</tr>
<tr>
<td>1.60</td>
<td>0.947</td>
</tr>
<tr>
<td>1.80</td>
<td>0.979</td>
</tr>
<tr>
<td>2.00</td>
<td>0.993</td>
</tr>
</tbody>
</table>
Velocity Values from M-B Distribution

- $v_{\text{rms}} = \text{root mean square velocity}$
- $v_{\text{avg}} = \text{average velocity}$
- $v_{\text{mp}} = \text{most probable velocity}$
Example: Evaluate the values of $v_{\text{rms}}$ by use of M-B Distribution!

Mathematics Solutions:

\[
\int_0^\infty v^n e^{-a v^2} \cdot dv = I
\]

if $n = 0$, $I = \frac{1}{2} \sqrt{\frac{\pi}{a}}$

if $n = 1$, $I = \frac{1}{2a}$

if $n = 2$, $I = \frac{1}{4} \left( \frac{\pi}{a^3} \right)^{\frac{1}{2}}$

if $n = 3$, $I = \frac{1}{2a^2}$

if $n = 4$, $I = \frac{3}{8} \left( \frac{\pi}{a^5} \right)^{\frac{1}{2}}$

\[
\sqrt{v^2} = \int_0^\infty v^2 f(v) dv = 4\pi \left( \frac{m}{2\pi kT} \right)^{\frac{3}{2}} \int_0^\infty v^4 e^{-mv^2/2kT} dv
\]

\[
v^2 = 4\pi \left( \frac{m}{2\pi kT} \right)^{\frac{3}{2}} \times \frac{3}{8} \left[ \pi \left( \frac{2kT}{m} \right)^5 \right]^{\frac{1}{2}}
\]

\[
v^4 = \pi^2 \left( \frac{m}{2\pi kT} \right)^3 \times \frac{9}{4} \left[ \pi \left( \frac{2kT}{m} \right)^5 \right]
\]

\[
\sqrt{v^2} = \left( \frac{3kT}{m} \right) \quad \text{so} \quad v_{\text{rms}} = \sqrt{\frac{3kT}{m}}
\]
Evaluate the $v_{avg}$ by use of M-B Distribution!

- The root-mean square speed:

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

- The average speed:

$$v_{avg} = \sqrt{\frac{8kT}{\pi m}}$$

- The most possible speed:

$$v_{mp} = \sqrt{\frac{2kT}{m}}$$

Remember:
- You can use $R$, instead of $k$, and $R=kN_A$
- When you use $R$, you have to use $M$ (mass of mole) instead of $m$, and $M=mN_A$. 

DONE
## Comparison of Velocity Values

Ratio in terms of: \( \sqrt{\frac{kT}{m}} \)

<table>
<thead>
<tr>
<th></th>
<th>( V_{\text{rms}} )</th>
<th>( V_{\text{avg}} )</th>
<th>( V_{\text{mp}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{3} \cdot \sqrt{\frac{kT}{m}} )</td>
<td>( \sqrt{\frac{8}{\pi}} \cdot \sqrt{\frac{kT}{m}} )</td>
<td>( \sqrt{2} \cdot \sqrt{\frac{kT}{m}} )</td>
<td></td>
</tr>
<tr>
<td>1.73</td>
<td>1.60</td>
<td>1.41</td>
<td></td>
</tr>
</tbody>
</table>
Thank You

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