1. Mathematical Induction

Theorem: (Principle of Finite Induction)

Let S be a set of positive integer with the following properties:
(a) The integer 1 belongs to S.
(b) Whenever the integer k belongs to S, then the next integer (k+1) must also be in S.
Then S is the set of all positive integers.

Prove a proposition using mathematical induction:

Let p(n) be proposition that true for all positive integer n. The proposition can be proved by mathematical induction using the following steps:
Step 1. P(1) is true
Step 2. Assume that p(k) is true for positive integer k, then we must show that p(k+1) is true.

If Step 1 and Step 2 are true, then we can conclude that p(n) is true for all positive integer n.

Example 1: Prove that 4 + 10 + 16 + … + (6n-2) = n (3n + 1) for all positive integer n.

2. Binomial Theorem

If n and k are any positive integer with $0 \le k \le n$, then the combination of k objects from n objects, denoted by $\binom{n}{k}$, is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

The Binomial Theorem: Let x and y be variables and n a positive integer, then

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$$

Example 2: Compute $\sum_{k=0}^{n} (-1)^k \binom{n}{k}$.

Discussion:

1. Conjecture a formula for A^n where $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and prove your conjecture using mathematical induction.
2. Conjecture a formula for $\prod_{j=1}^{n} 2^j$ and prove your conjecture using mathematical induction.
3. Compute $3 + 3.5^2 + 3.5^4 + ... + 3.5^{1000}$
4. Find $\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + ...$
5. Find $\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + ...$
6. Show that $\binom{k}{k} + \binom{k+1}{k} + \binom{k+2}{k} + ... + \binom{k+r}{k} = \binom{k+r+1}{k+1}$

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4. Find $\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\ldots$

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