Lecture 3

- Need for new theory
- Stern-Gerlach Experiments
  - Some doubts
- Analogy with mathematics of light
- Feynman’s double slit thought experiment
Three empirical laws
To explain one physical phenomenon

Radiation is emitted in Quantas of energy

Radiation sometimes behaves as

- Particles
- Waves

Radiation is absorbed in Quantas of energy

Radiation is quantum of energy

- We are formulating a new theory!
- Radiation sometimes behaves as
  - Particles
  - Waves
- Same is true for Matter
Thought Experiments

- Feyman’s logical tightrope?
- We have given up asking whether the electron is a particle or a wave
- What we demand from our theory is that given an experiment we must be able to tell whether it will behave as a particle or a wave.
- We need to develop the Mathematics which the language of TRUTH which we all seek
- What kind of Language we seek is the motivation right now.
Stern-Gerlach Experiment

Collimator Slits

Inhomogeneous Magnetic Field

Oven containing Ag atoms

Nature behaves this way

Classically one would expect this

detector
Stern-Gerlach Experiment (contd)

- One can say it is an apparatus which measures the z component of $\mu \Rightarrow S_z$

- If atoms randomly oriented
  - No preferred direction for the orientation of $\mu$
  - Classically spinning object $\Rightarrow \mu_z$ will take all possible values between $\mu$ & -$\mu$

- Experimentally we observe two distinct blobs

- Original silver beam into 2 distinct component
What have we learnt from the experiment

- Two possible values of the Z component of S observed $S_Z^\text{UP}$ & $S_Z^\text{down}$

- Refer to them as $S_Z^+$ & $S_Z^-$ ⇒ Multiples of some fundamental constants, turns out to be $+\frac{\hbar}{2}$ & $-\frac{\hbar}{2}$

- Spin is quantised

- Nothing is sacred about the z direction, if our apparatus was in x direction we would have observed $S_x^+$ & $S_x^-$ instead
Thought Experiments start

Source → SG $\hat{Z}$ → SG $\hat{Z}$ → SG $\hat{Z}$ → Z$^+$

Source → SG $\hat{Z}$ → SG $\hat{X}$ → SG $\hat{Z}$ → Z$^+$, Z$^-$
Thought Experiment continues

- Silver atoms were oriented in all possible directions.
- The Stern-Gerlach Apparatus which is a measuring device puts those atoms which were in all possible states in either one of the two states specific to the Apparatus.
- No matter how many measurements we make to measure $S_z$ in $z$ direction we put, there is only one beam coming out.
- Once the SG App. put it into one of the states repeated measurements OF THE SAME KIND did not disturb the system.
Conclusions from Coupled experiment

- Measurements disturb a quantum system in an essential way.
- Measurements put the QM System in one of the special states associated with that measurement.
- Any further measurement of the same variable does not change the state of the system.
- Measurement of another variable may disturb the system and put it in one of its special states.
Complete Departure from Classical Physics

- Measurement of $S_x$ destroys the information about $S_z$
  - We can never measure $S_x$ & $S_z$ together
    - Incompatible measurements

- How do you measure angular momentum of a spinning top, $L = I\omega$
  - Measure $\omega_x$, $\omega_y$, $\omega_z$
  - No difficulty in specifying $L_x$, $L_y$, $L_z$
Consider a monochromatic light wave propagating in $Z$ direction & it is polarised in $x$ direction

\[ E = E_0 \hat{x} \cos(\omega t) \]

Similarly linearly polarised light in $y$ direction is represented by

\[ E = E_0 \hat{y} \cos(\omega t) \]

A filter which polarises light in the $x$ direction is called an $X$ filter and one which polarises light in $y$ direction is called a $Y$ filter

An $X$ filter becomes a $Y$ filter when rotated by $90^\circ$
An Experiment with Light

- The selection of \( x' \) filter destroyed the information about the previous state of polarisation of light
- Quite analogous to situation earlier
- Carry the analogy further
  - \( S_z \pm x \) & \( y \) polarised light
  - \( S_x \pm x' \) & \( y' \) polarised light
Mathematics of Polarisation

\[ E_0 \hat{x}' \cos(kz - \omega t) = E_0 \left[ \frac{1}{\sqrt{2}} \hat{x} \cos(kz - \omega t) + \frac{1}{\sqrt{2}} \hat{y} \sin(kz - \omega t) \right] \]

\[ E_0 \hat{y}' \cos(kz - \omega t) = E_0 \left[ -\frac{1}{\sqrt{2}} \hat{x} \cos(kz - \omega t) + \frac{1}{\sqrt{2}} \hat{y} \sin(kz - \omega t) \right] \]
Mathematics of Polarisation

- In the triple filter arrangement
  - **First Filter** An x polarised beam – linear combination of x` & y` polarised beam
  - An x polarised beam – linear combination of x` & y` polarised beam
  - **Second Filter**– Selects x` polarised beam
  - An x` polarised beam – linear combination of x & y polarised beam
  - **Third Filter**– Selects y polarised beam

- This is quite similar to the sequential Stern-Gerlach Experiment
  - We represent the spin state of silver atom by some kind of vector in some abstract space. **NOT THE USUAL VECTOR SPACE**
The Analogy

- In case of light x and y was my basis
  - I could expand x` in terms of x and y…
- Suppose now I want to describe the SG apparatus
  - I could use two vectors \(|S_z^+\rangle\) and \(|S_z^-\rangle\)
  - Notice I am using the hat on the side
  - Then \(|S_x^+\rangle = \frac{1}{\sqrt{2}} \left[ |S_z^+\rangle + |S_z^-\rangle \right]\)
  - \(|S_y^-\rangle = \frac{1}{\sqrt{2}} \left[ |S_z^+\rangle - |S_z^-\rangle \right]\)
- Nothing sacred about z or x direction
  - What about y Direction?
  - \(S_y^+\) & \(S_y^-\)
  - They have to be independent of \(|S_x^+\rangle\) and \(|S_y^-\rangle\)
  - Basis is of two vectors
Analogy further

- Circularly polarised light Now
  - When we pass it thru a x filter only x component goes thru
  - When we pass it thru a y filter only y component goes thru
- Circularly polarised light different from linearly polarised light along x` and y`
- Mathematically -- circularly polarised light
  
  \[ \text{y polarised component is } 90^\circ \text{ out of phase with x component} \]

More elegant to use complex notation by introducing \( \varepsilon \)

\[
\bar{E} = E_0 \left[ \frac{1}{\sqrt{2}} \hat{x} \cos(kz - \omega t) + \frac{1}{\sqrt{2}} \hat{y} \cos(kz - \omega t + \frac{\pi}{2}) \right]
\]

\[ \text{Re}(\varepsilon) = \frac{E}{E_0} \]

\[
\bar{E} = E_0 \left[ \frac{1}{\sqrt{2}} \hat{x} e^{i(kz - \omega t)} + \frac{\text{i}}{\sqrt{2}} \hat{y} e^{i(kz - \omega t)} \right]
\]
Analogy with circularly polarised light

- Now we can represent $S_y^+$ and $S_y^-$
- Thus $|S_y^+\rangle = 1/\sqrt{2} \left[ |S_z^+\rangle + i |S_z^-\rangle \right]$
  \[ |S_y^-\rangle = 1/\sqrt{2} \left[ |S_z^+\rangle - i |S_z^-\rangle \right] \]

- We can describe the SG experiment using the language of vectors
- However no connection with ordinary vectors having magnitude and direction
- That the vector space must be complex
Feynman’s thought experiments