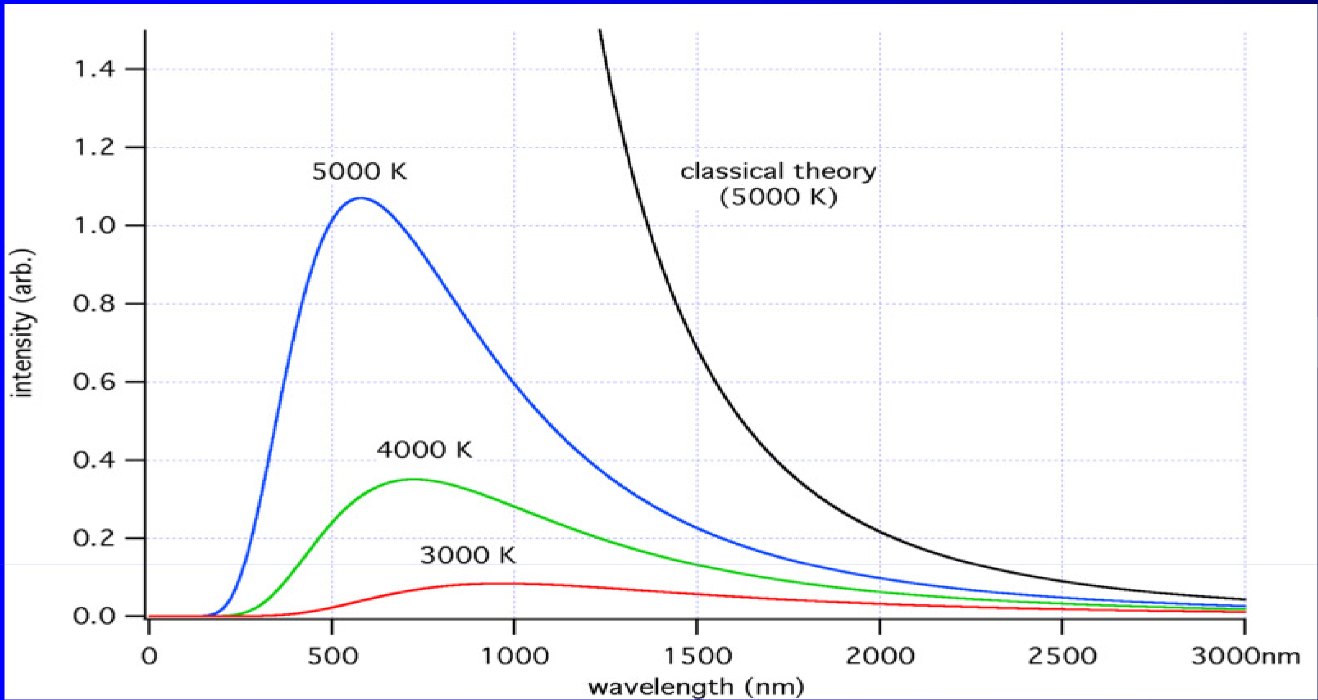
Lecture 3

- Need for new theory
- Stern-Gerlach Experiments
  - Some doubts
- Analogy with mathematics of light
- Feynman's double slit thought experiment



Three empirical laws
To explain one physical phenomenon

⇓

Radiation is emitted in Quantas of energy

⇓

Radiation is absorbed in Quantas of energy

⇓

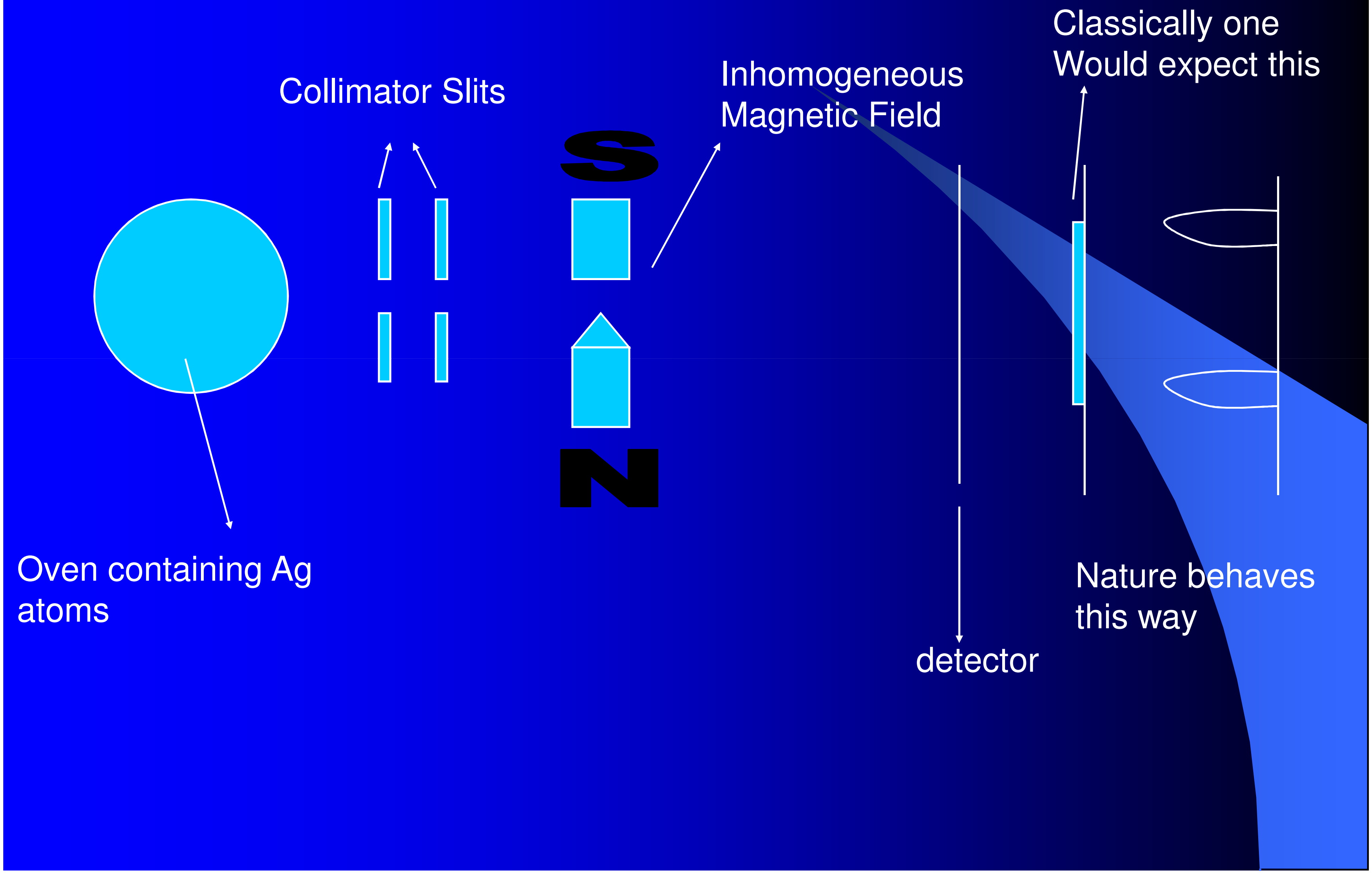
Radiation is quantum of energy

- We are formulating a new theory!
- Radiation sometimes behaves as
  - Particles
  - Waves
- Same is true for Matter

Thought Experiments

- Feyman's logical tightrope?
- We have given up asking whether the electron is a particle or a wave
- What we demand from our theory is that given an experiment we must be able to tell whether it will behave as a particle or a wave.
- We need to develop the Mathematics which the language of TRUTH which we all seek
- What kind of Language we seek is the motivation right now.

Stern-Gerlach Experiment



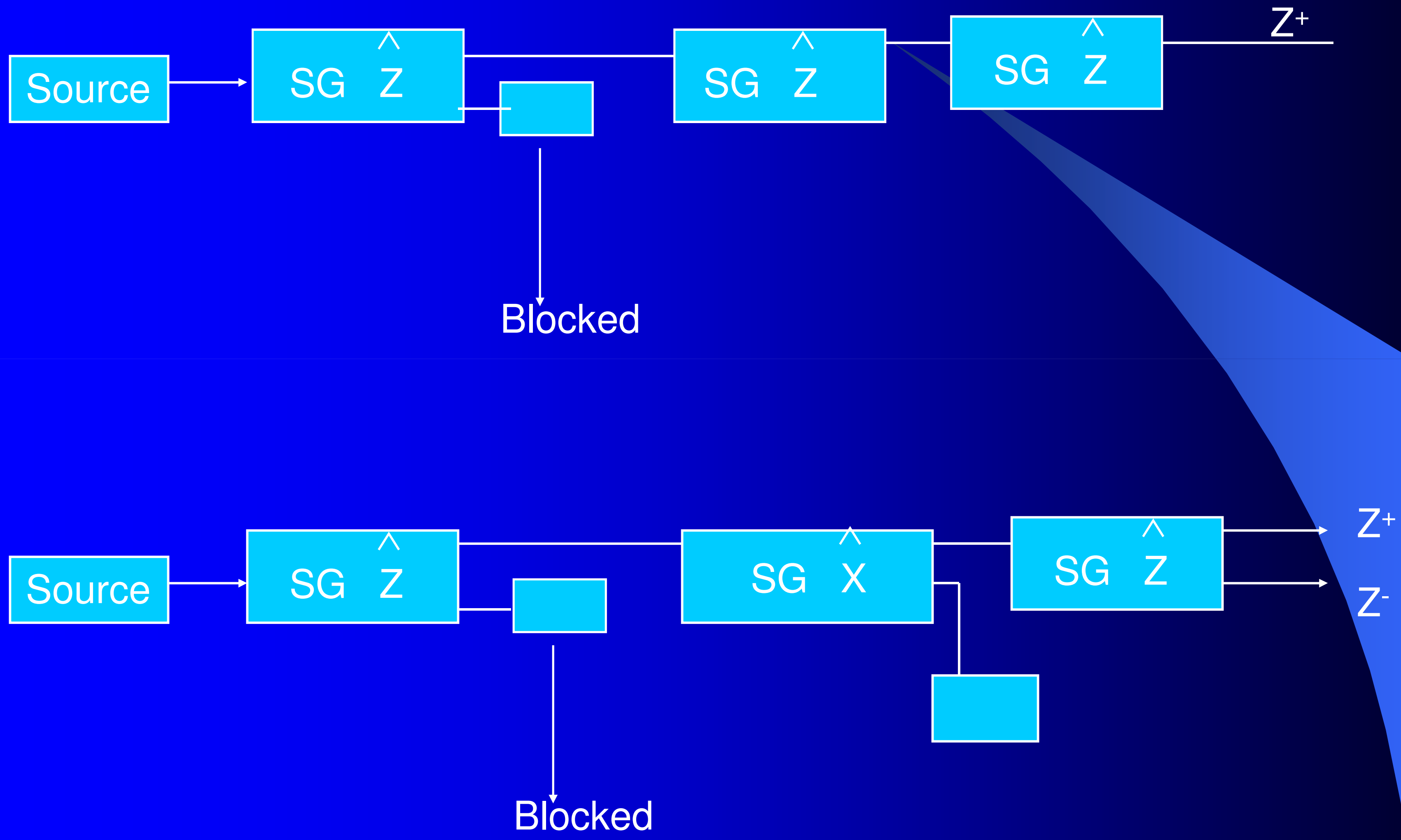
Stern-Gerlach Experiment (contd)

- One can say it is an apparatus which measures the z component of $\mu \Rightarrow S_z$
- If atoms randomly oriented
  - No preferred direction for the orientation of μ
  - Classically spinning object $\Rightarrow$ μ_z will take all possible values between μ & $-\mu$
- Experimentally we observe two distinct blobs
- Original silver beam into 2 distinct component

What have we learnt from the experiment

- Two possible values of the Z component of S observed S_Z^{UP} & S_Z^{down}
- Refer to them as S_Z^+ & S_Z^- $\Rightarrow$ Multiples of some fundamental constants, turns out to be $+\frac{\hbar}{2}$ & $-\frac{\hbar}{2}$
- Spin is quantised
- Nothing is sacred about the z direction, if our apparatus was in x direction we would have observed S_x^+ & S_x^- instead

Thought Experiments start



Thought Experiment continues

- Silver atoms were oriented in all possible directions
- The Stern-Gerlach Apparatus which is a measuring device puts those atoms which were in all possible states in either one of the two states specific to the Apparatus
- No matter how many measurements we make to measure S_z in z direction we put, there is only one beam coming out
- Once the SG App. put it into one of the states repeated measurements OF THE SAME KIND did not disturb the system

Conclusions from Coupled experiment

- Measurements disturb a quantum system in an essential way
- Measurements put the QM System in one of the special states associated with that measurement
- Any further measurement of the same variable does not change the state of the system
- Measurement of another variable may disturb the system and put it in one of its special states.

Complete Departure from Classical Physics

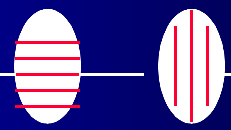
- Measurement of S_x destroys the information about S_z
    - We can never measure S_x & S_z together
        - Incompatible measurements
- How do you measure angular momentum of a spinning top, $L = I\omega$
    - Measure ω_x , ω_y , ω_z
    - No difficulty in specifying L_x L_y L_z

Analogy

- Consider a monochromatic light wave propagating in Z direction & it is polarised in x direction $E = E_0\hat{x}Cos(kz - \varpi t)$
- Similarly linearly polarised light in y direction is represented by $E = E_0\hat{y}Cos(kz - \varpi t)$
- A filter which polarises light in the x direction is called an X filter and one which polarises light in y direction is called a y filter
- An X filter becomes a Y filter when rotated by 90°

An Experiment with Light

Source — X Filter — Y Filter → No LIGHT

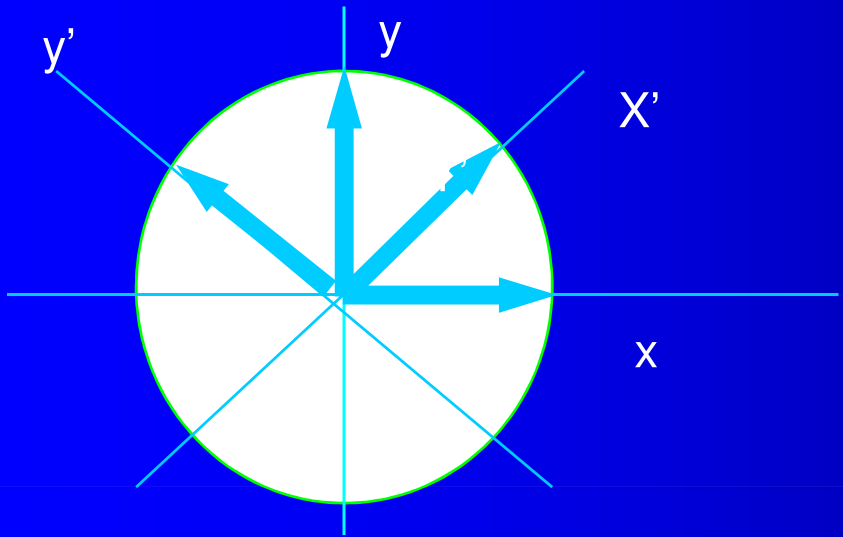


No LIGHT

Source — X Filter — X' Filter — Y Filter — LIGHT

- The selection of x` filter destroyed the information about the previous state of polarisation of light
- Quite analogous to situation earlier
- Carry the analogy further
  - $S_z \pm$ x & y polarised light
  - $S_x \pm$ x` & y` polarised light

Mathematics of Polarisation



$$E_0 \hat{x}' Cos(kz - \omega t) = E_0 \left[\frac{1}{\sqrt{2}} \hat{x} Cos(kz - \omega t) + \frac{1}{\sqrt{2}} \hat{y} Sin(kz - \omega t) \right]$$

$$E_0 \hat{y}' Cos(kz - \omega t) = E_0 \left[-\frac{1}{\sqrt{2}} \hat{x} Cos(kz - \omega t) + \frac{1}{\sqrt{2}} \hat{y} Sin(kz - \omega t) \right]$$

Mathematics of Polarisation

- In the triple filter arrangement
  - First Filter An x polarised beam – linear combination of x ` & y` polarised beam
    - An x polarised beam – linear combination of x ` & y` polarised beam
  - Second Filter– Selects x ` polarised beam
    - An x` polarised beam – linear combination of x & y polarised beam
  - Third Filter– Selects y polarised beam
- This is quite similar to the sequential Stern-Gerlach Experiment
  - We represent the spin state of silver atom by some kind of vector in some abstract space. NOT THE USUAL VECTOR SPACE

The Analogy

- In case of light x and y was my basis
  - I could expand x` in terms of x and y…
- Suppose now I want to describe the SG apparatus
  - I could use two vectors $|S_z^+\rangle$ and $|S_z^-\rangle$
  - Notice I am using the hat on the side
  - Then $|S_x^+\rangle = 1/\sqrt{2}\ [\ |S_z^+\rangle + |S_z^-\rangle\ ]$
  - $|S_y^-\rangle = 1/\sqrt{2}\ [\ |S_z^+\rangle - |S_z^-\rangle\ ]$
- Nothing sacred about z or x direction
  - What about y Direction?
  - S_y^+ & S_y^-
  - They have to be independent of $|S_x^+\rangle$ and $|S_y^-\rangle$
  - Basis is of two vectors

Analogy further

- Circularly polarised light Now
  - When we pass it thru a x filter only x component goes thru
  - When we pass it thru a y filter only y component goes thru
- Circularly polarised light different from linearly polarised light along x` and y`
- Mathematically --circularly polarised light

  y polarised component is 90° out of phase with x component

$$\vec{E} = E_0\left[\frac{1}{\sqrt{2}}\hat{x}\cos(kz-\omega t)+\frac{1}{\sqrt{2}}\hat{y}\cos\left(kz-\omega t+\frac{\pi}{2}\right)\right]$$

More elegant to use complex notation by introducing ε

$$\mathrm{Re}(\varepsilon) = \frac{E}{E_0}$$

$$\vec{E} = E_0\left[\frac{1}{\sqrt{2}}\hat{x}e^{i(kz-\omega t)}+\frac{i}{\sqrt{2}}\hat{y}e^{i(kz-\omega t)}\right]$$

Analogy with circularly polarised light

- Now we can represent S_y^+ and S_y^-
- Thus $|S_y^+\rangle = 1/\sqrt{2}\,[\,|S_z^+\rangle + \iota\,|S_z^-\rangle\,]$
    - $|S_y^-\rangle = 1/\sqrt{2}\,[\,|S_z^+\rangle - \iota\,|S_z^-\rangle\,]$

- We can describe the SG experiment using the language of vectors
- However no connection with ordinary vectors having magnitude and direction
- That the vector space must be complex

Feynman's thought experiments