Static Error Constants and System Type

- Here we will define parameters that can be used as steady-state error performance specifications for unit negative feedback systems.

- These parameters are called *static error constants*.

- Recall previously that we have found the expressions for $e_{ss}$ for the case of unity feedback systems, namely:
  - Step input:
  - Ramp input:
  - Parabolic input:

- The three terms in the denominator that are taken to the limit determine $e_{ss}$. These are the static error constants, i.e.:
• Note that the values of $e_{ss}$ decreases as the static error constants increases.

**Example:**

For each system shown, evaluate the static error constants and find the expected error for the standard step, ramp, and parabolic inputs.

(a)

\[
\frac{500(s + 2)(s + 5)}{(s + 8)(s + 10)(s + 12)}
\]

(b)

\[
\frac{500(s + 2)(s + 5)(s + 6)}{s(s + 8)(s + 10)(s + 12)}
\]

(c)

\[
\frac{500(s + 2)(s + 4)(s + 5)(s + 6)(s + 7)}{s^2(s + 8)(s + 10)(s + 12)}
\]
**System type**

- As seen in the previous example, the values of static error constants depend on the form of $G(s)$ i.e. they depend on the number of integrations in existing in $G(s)$.

- Hence, define system type to be the value of $n$ in the denominator of $G(s)$.

![Block diagram](image_url)

- The relationship between input, system type, static error constants, and steady state error is summarized in Table 7.2.
<table>
<thead>
<tr>
<th>Input</th>
<th>Steady-state error formula</th>
<th>Type 0</th>
<th></th>
<th>Type 1</th>
<th></th>
<th>Type 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Static error constant</td>
<td>Error</td>
<td>Static error constant</td>
<td>Error</td>
<td>Static error constant</td>
<td>Error</td>
</tr>
<tr>
<td>Step, $u(t)$</td>
<td>$\frac{1}{1 + K_p}$</td>
<td>$K_p = \frac{1}{1 + K_p}$</td>
<td>Constant</td>
<td>$K_p = \infty$</td>
<td>0</td>
<td>$K_p = \infty$</td>
<td>0</td>
</tr>
<tr>
<td>Ramp, $tu(t)$</td>
<td>$\frac{1}{K_v}$</td>
<td>$K_v = 0$</td>
<td>$\infty$</td>
<td>$K_v = \frac{1}{K_v}$</td>
<td></td>
<td>$K_v = \infty$</td>
<td>0</td>
</tr>
<tr>
<td>Parabola, $\frac{1}{2}t^2u(t)$</td>
<td>$\frac{1}{K_a}$</td>
<td>$K_a = 0$</td>
<td>$\infty$</td>
<td>$K_a = 0$</td>
<td>$\infty$</td>
<td>$K_a = \frac{1}{K_a}$</td>
<td></td>
</tr>
</tbody>
</table>
**Steady state error specifications**

- A lot of information is contained within the specification of a static error constant.
- For example, if a control system has the specification $K_v=1000$, we can draw several conclusions:
  - The system is stable
  - The system is of Type 1, since only Type 1 systems have $K_v$’s that are finite constants.
  - A ramp input is used as the test signal.
  - $e_{ss}$ between input ramp and output ramp is $\frac{1}{K_v}$.

**Example:**
Find the value of $K$ so that there is 10% error in the steady state.
Steady-state error for Non-unity Feedback Systems

- Example: Find the steady-state errors for the closed-loop system for the unit step, ramp and parabolic inputs where

\[ G(s) = \frac{100}{s(s + 10)}; \quad H(s) = \frac{1}{s + 5} \]