**Steady-State Errors**

- Three performance criteria in analysis and design of control systems:
  - Transient response
  - Stability
  - Steady-state errors

- Transient response of 1\(^{st}\) and 2\(^{nd}\) order systems have been discussed in previous lectures.

- This section focuses on steady-state errors of the time response of a particular system.

**Definition**

- Steady-state error is the difference between the input and the output for a prescribed test input as \( t \to \infty \).

- Test inputs used for steady-state error analysis and design are summarized in Table 7.1.

- Use for stable systems only.
<table>
<thead>
<tr>
<th>Waveform</th>
<th>Name</th>
<th>Physical interpretation</th>
<th>Time function</th>
<th>Laplace transform</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Step</td>
<td>Constant position</td>
<td>1</td>
<td>$\frac{1}{s}$</td>
</tr>
<tr>
<td></td>
<td>Ramp</td>
<td>Constant velocity</td>
<td>$t$</td>
<td>$\frac{1}{s^2}$</td>
</tr>
<tr>
<td></td>
<td>Parabola</td>
<td>Constant acceleration</td>
<td>$\frac{1}{2}t^2$</td>
<td>$\frac{1}{s^3}$</td>
</tr>
</tbody>
</table>

Table 7.1
Steady-state errors:

- Output 1: zero $e_{ss}$; Output 2: $e_{ss} = e_2(\infty)$

- Output 1: zero $e_{ss}$; Output 2: $e_{ss} = e_2(\infty)$
• Steady-state error, $\text{ess}$ can be obtained using the final value theorem:

\[
\text{Example: Find the steady-state error for an open loop system with } G(s) = \frac{5}{s^2 + 7s + 10} \text{ and the input is unit step input.}
\]
Steady-state errors for Unity Feedback Systems

- Consider a unity feedback system

\[ R(s) + E(s) \rightarrow \frac{1}{1+G(s)} \rightarrow C(s) \]

- Assume system is stable, we can apply the final value theorem:

- The steady-state error depends on the input signal.
Step input

- Consider the case when $R(s) = \frac{1}{s}$,

- Hence in order to have zero steady state error, the term, $\lim_{s \to 0} G(s)$, must be big. i.e.:

- This can only happen if

- If $n = 0$, 
Ramp Input

- Consider the case when \( r(t) = t, R(s) = \frac{1}{s^2}. \)

- Hence, to have zero steady state error for a ramp input,

- This can only happen if
Parabolic input

- Consider the case when \( r(t) = \frac{1}{2} t^2, R(s) = \frac{1}{s^3} \).

- Hence, to have zero steady state error for a ramp input,

- This can only happen if

- If \( n=2 \),
**Example:** Find the steady-state error for input $u(t)$, $tu(t)$, $t^2u(t)$ to the system shown below. The function $u(t)$ is the unit step.
**Example:** Find the steady-state error for input $5u(t), 5tu(t), 5t^2u(t)$ to the system shown below. The function $u(t)$ is the unit step.