Steady-State Errors

- Three performance criteria in analysis and design of control systems:
  - Transient response
  - Stability
  - Steady-state errors
- Transient response of 1st and 2nd order systems have been discussed in previous lectures.
- This section focuses on steady-state errors of the time response of a particular system.

Definition

- Steady-state error is the difference between the input and the output for a prescribed test input as $t \to \infty$.
- Test inputs used for steady-state error analysis and design are summarized in Table 7.1.
- Use for stable systems only.

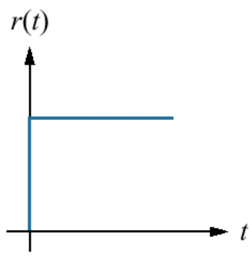
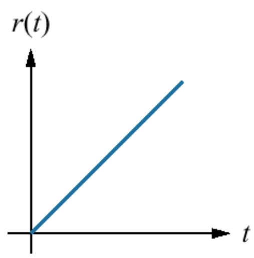
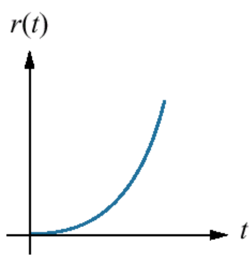
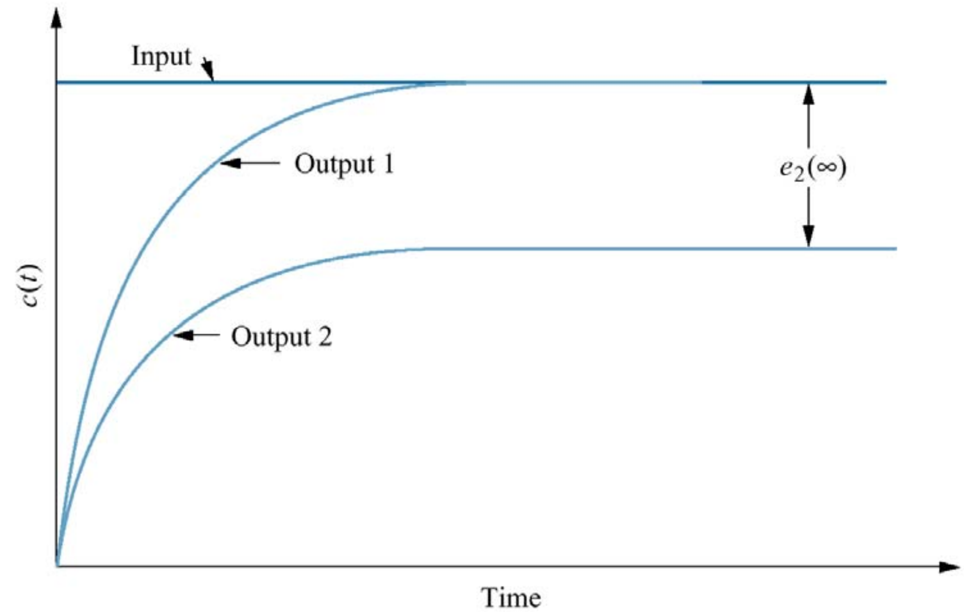
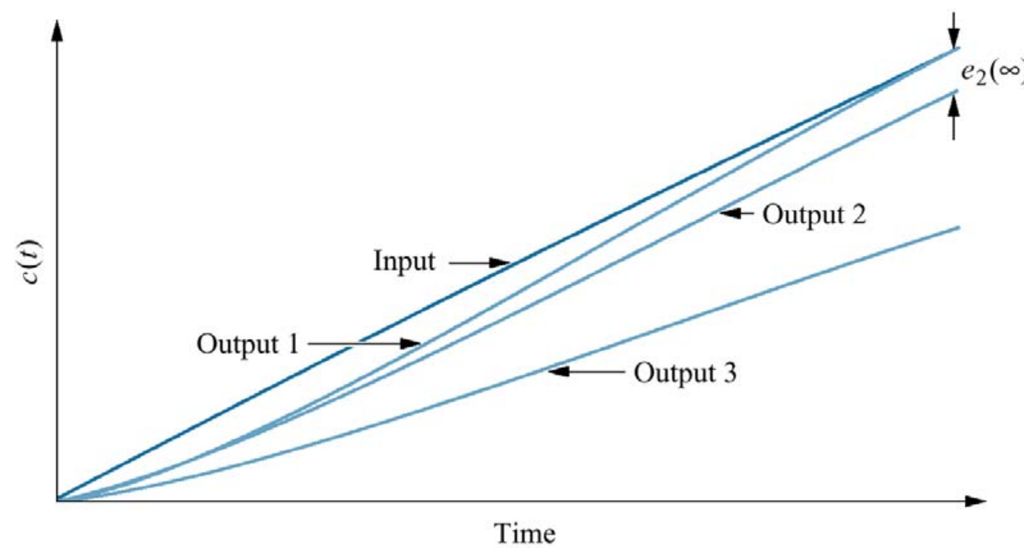
Waveform	Name	Physical interpretation	Time function	Laplace transform
	Step	Constant position	1	$\frac{1}{s}$
	Ramp	Constant velocity	t	$\frac{1}{s^2}$
	Parabola	Constant acceleration	$\frac{1}{2}t^2$	$\frac{1}{s^3}$

Table 7.1

Steady-state errors:



- Output 1: zero e_{ss}; Output2: $e_{ss} = e_2(\infty)$



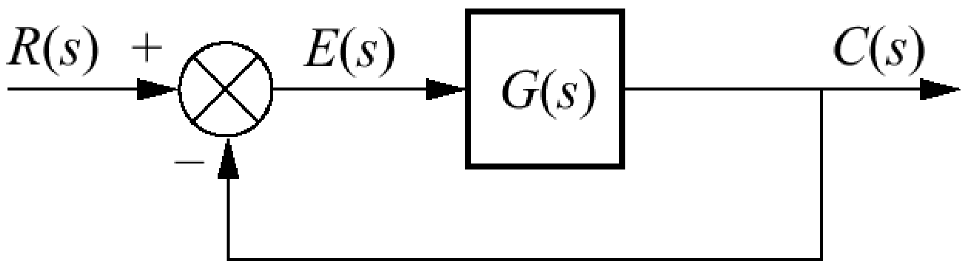
- Output 1: zero e_{ss}; Output2: $e_{ss} = e_2(\infty)$

- Steady-state error, ess can be obtained using the final value theorem:

- **Example**: Find the steady-state error for an open loop system with $G(s) = \dfrac{5}{s^2 + 7s + 10}$ and the input is unit step input.

Steady-state errors for Unity Feedback Systems

- Consider a unity feedback system



- Assume system is stable, we can apply the final value theorem:

- The steady-state error depends on the input signal.

Step input

- Consider the case when $R(s) = \frac{1}{s}$,

- Hence in order to have zero steady state error, the term, $\lim_{s \to 0} G(s)$, must be big. i.e.:

- This can only happen if

- If $n = 0$,

Ramp Input

- Consider the case when $r(t) = t, R(s) = \frac{1}{s^2}$.

- Hence, to have zero steady state error for a ramp input,

- This can only happen if

Parabolic input

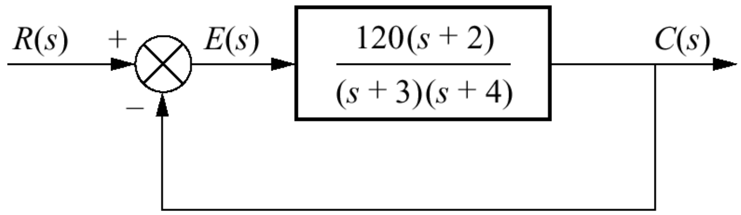
- Consider the case when $r(t) = \frac{1}{2}t^2, R(s) = \frac{1}{s^3}$.

- Hence, to have zero steady state error for a ramp input,

- This can only happen if

- If *n=2*,

- **Example**: Find the steady-state error for input $u(t), tu(t), t^2u(t)$ to the system shown below. The function $u(t)$ is the unit step.



- **Example**: Find the steady-state error for input $5u(t), 5tu(t), 5t^2u(t)$ to the system shown below. The function $u(t)$ is the unit step.