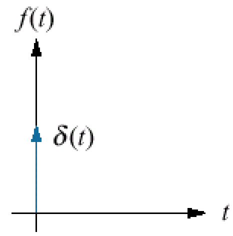
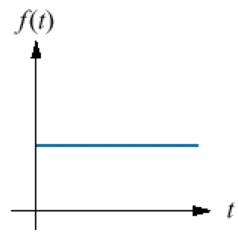
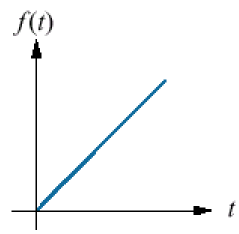
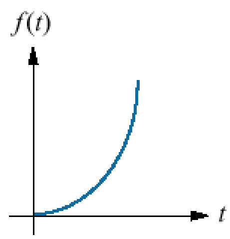
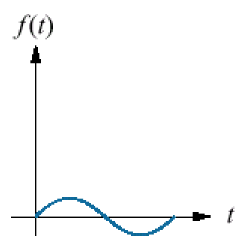
TIME RESPONSE

- This topic involves analysis of a control system in terms of time response and steady-state error.
- At the end of this topic, the students should be able to:
  1. Recognize standard input test signal.
  2. Find and sketch the response of first, second and third order systems.
  3. Identify the poles and zeros of a transfer function.
  4. State the important specifications of a time domain response.
  5. Identify and state the order, type and steady state error coefficient given a transfer function.

Standard Input Test Signal

- Test input signals are used, both analytically and during testing, to verify the design.
- Engineer usually select standard test inputs, as shown in Table 4.1: impulse, step, ramp, parabola and sinusoid.
- An *impulse* is infinite at $t=0$ and zero elsewhere. The area under the unit impulse is *1*.
- A *step* input represents a *constant command*, such as position, velocity, or acceleration.
- Typically, the step input command is of the same form as the output. For example, if the system's output is position, the step input represents a desired position, and the output represents the actual position.
- The designer uses step inputs because both the transient response and the steady-state response are clearly visible and can be evaluated.
- The *ramp* input represents a *linearly increasing command*. For example, if the system's output is position, the input ramp represents a linearly increasing position.

Table 4.1

Input	Function	Description	Sketch	Use
Impulse	$\delta(t)$	$\delta(t) = \infty$ for $0- < t < 0+$ $= 0$ elsewhere $\int_{0-}^{0+} \delta(t)\,dt = 1$		Transient response Modeling
Step	$u(t)$	$u(t) = 1$ for $t > 0$ $= 0$ for $t < 0$		Transient response Steady-state error
Ramp	$tu(t)$	$tu(t) = t$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error
Parabola	$\frac{1}{2}t^2u(t)$	$\frac{1}{2}t^2u(t) = \frac{1}{2}t^2$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error
Sinusoid	$\sin \omega t$			Transient response Modeling Steady-state error

- The response to an input ramp test signal yields additional information about the steady-state error.
- The previous discussion can be extended to *parabolic* inputs, which are also used to evaluate a system's steady-state error.
- *Sinusoidal* inputs can also be used to test a physical system to arrive at a mathematical model (Frequency Response).

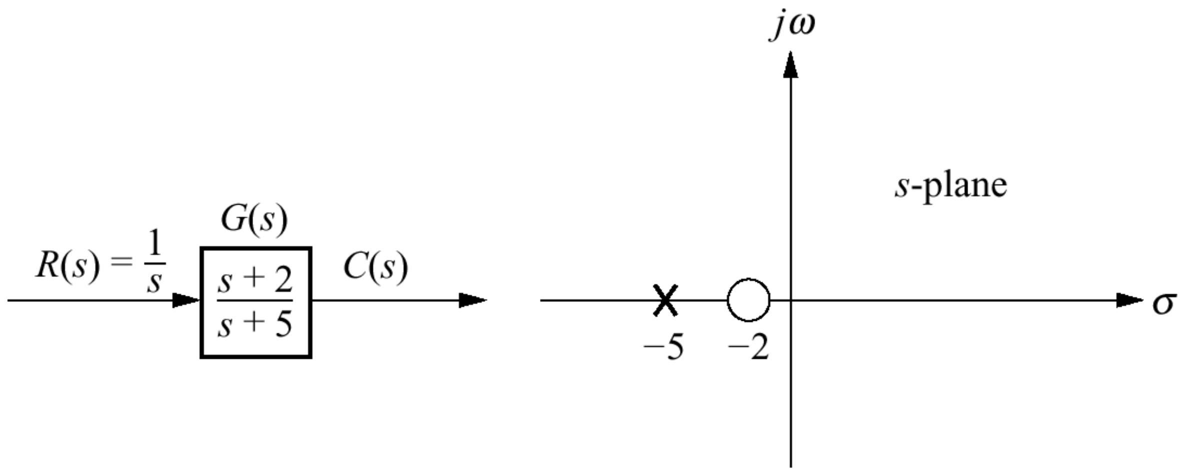
Poles, Zeros and System Response

- The output response of a system is the sum of two responses: the *forced response (steady-state response)* and *the natural response*.
- It is possible to relate the poles and zeros of a system to its output response.
- The *poles* of a transfer function are:
    - The values of the Laplace transform variable, *s*, that cause the transfer function to become infinite.
    - Any roots of the denominator of the transfer function that are common to roots of the numerator.

- The *zeros* of a transfer function are:
    - The values of the Laplace transform variable, *s*, that cause the transfer function to become zero.
    - Any roots of the numerator of the transfer function that are common to roots of the denominator.

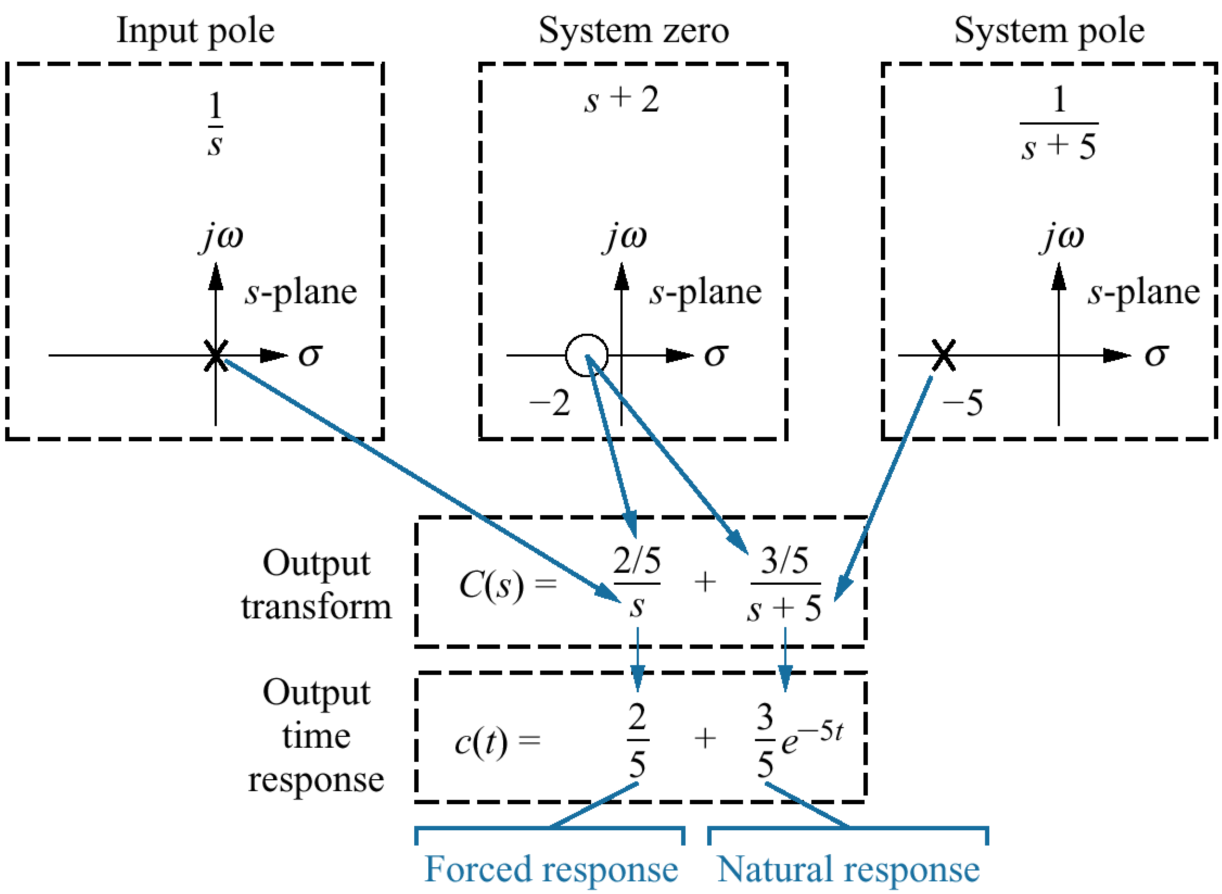
Poles and Zeros of a First-Order System

- Example: Given $\frac{C(s)}{R(s)} = \frac{(s+2)}{(s+5)}$. Find the pole and zero of the system and plot on the s plane. Find the unit step response of the system.



R(s) = 1/s
G(s)
(s + 2)/(s + 5)
C(s)
jω
s-plane
σ
−5
−2

(*a*) (*b*)



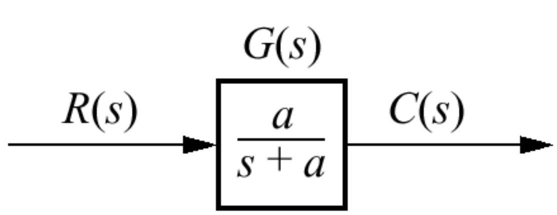
Input pole
1/s
jω
s-plane
σ
System zero
s + 2
jω
s-plane
σ
−2
System pole
1/(s + 5)
jω
s-plane
σ
−5
Output transform
C(s) = 2/5 / s + 3/5 / (s + 5)
Output time response
c(t) = 2/5 + 3/5 e^−5t
Forced response
Natural response

(*c*)

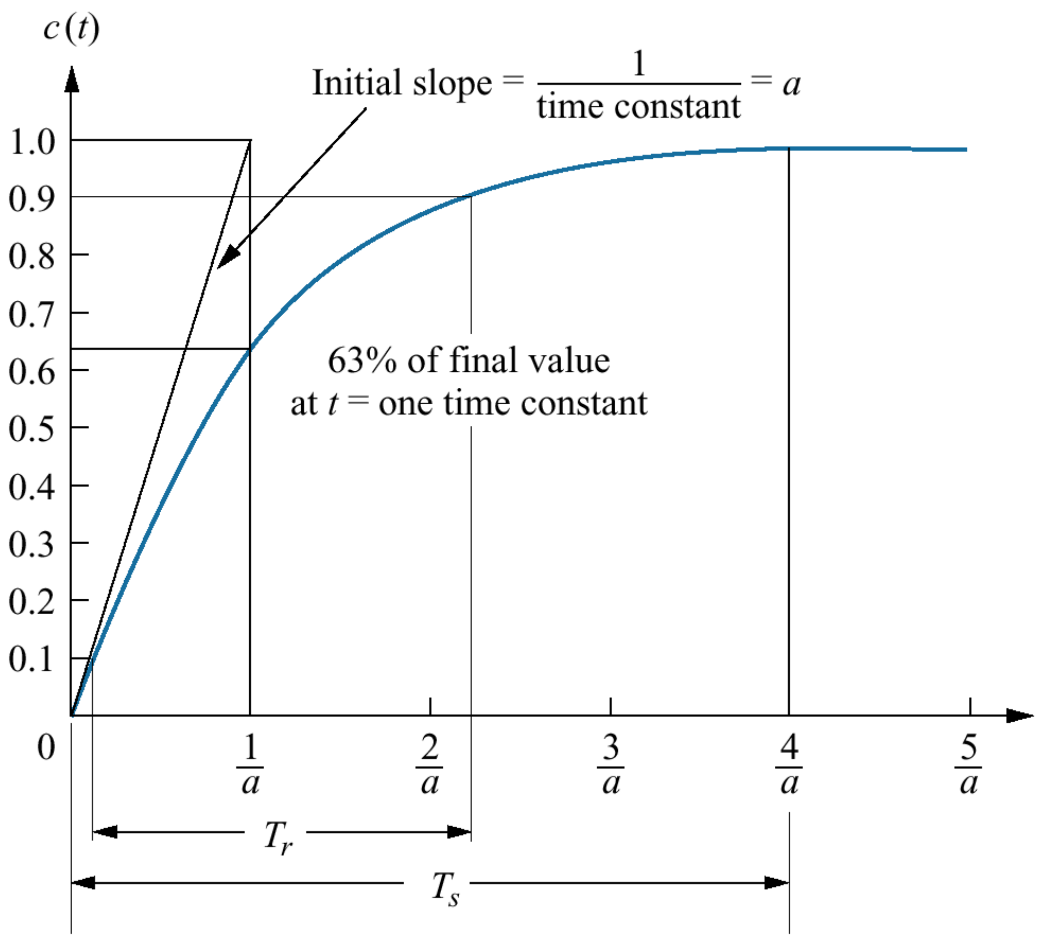
- Example conclusions:
  - A pole of the input function generates the form of the *forced response*.
  - A pole of the transfer function generates the form of the *natural response* (the pole at –5 generated e^{-5t}).
  - A pole on the real axis generates an *exponential* response of the form $e^{-\alpha t}$, where $-\alpha$ is the pole location on the real axis.
  - The farther to the left a pole is on the negative real axis, the faster the exponential transient response will decay to zero.
  - The zeros and poles generate the *amplitudes* for both the forced and natural responses.

First-Order Systems

- Consider a first order system, $\dfrac{C(s)}{R(s)} = \dfrac{a}{(s+a)}$ with a unit step input



- Time response:



- At t=1/a,

Specification: Time Constant

- The Time constant, T_C is the time for the step response to rise to 63% of its final value.
- From the response, $T_C = 1/a$.
- The bigger *a* is, the farther it will be from the imaginary axis and the faster the transient response will be.

Specification: Rise Time

- Rise time, T_r is the time it takes for the response to go from 0.1 to 0.9 of its final value.

Specifications: Settling Time

- Settling time, T_S is the time for the response to reach and stay within, 2% of its final value.

First-Order Transfer Functions via Testing

- Often it is not possible or practical to obtain a system's transfer function analytically (i.e. using techniques from last topic).
- We can have an approximation of a system's transfer function by looking at its step response.
- The actual system is fed with a step input, and its response recorded experimentally.
- Given that the step response is known, the transfer function can be found.
- Consider a simple first order system:

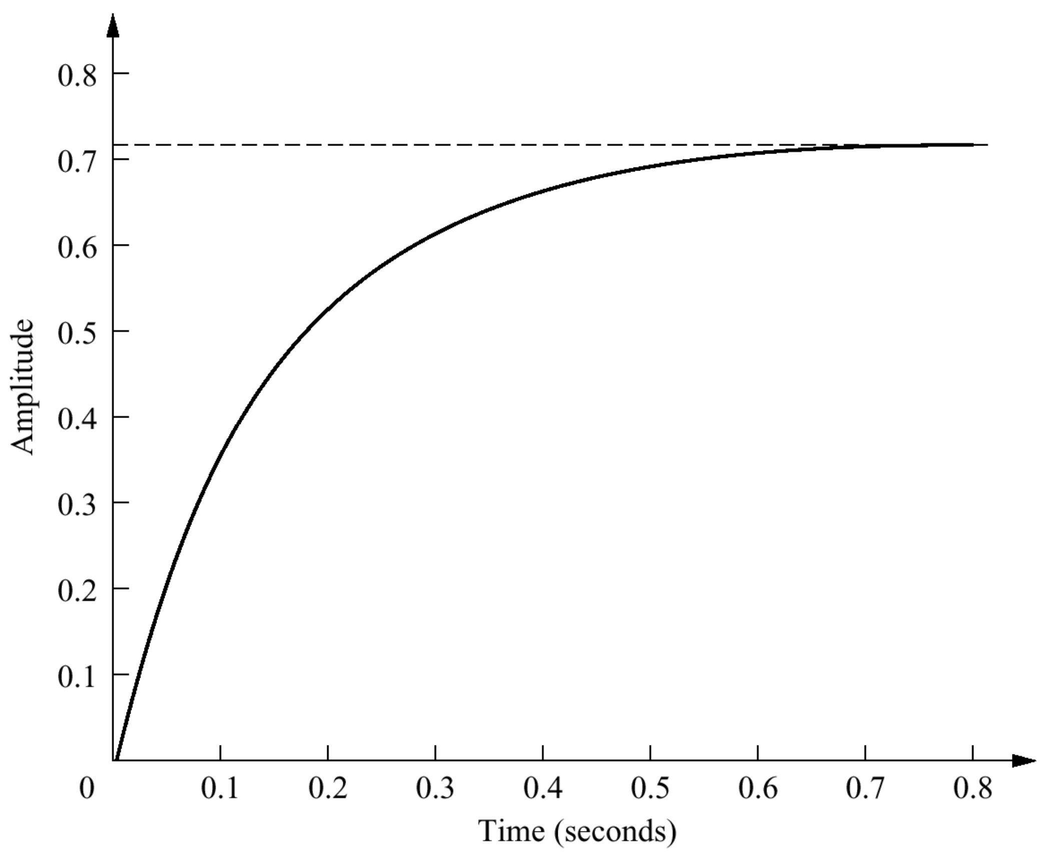
$$G(s) = \frac{K}{(s+a)}$$

For step response, $C(s) = \dfrac{K}{s(s+a)} = \dfrac{K/a}{s} - \dfrac{K/a}{s+a}$

$$\Rightarrow c(t) = \frac{K}{a} - \frac{K}{a}e^{-at}$$

- Hence, if we can identify K and a from laboratory testing, we can obtain the transfer function of the system.

- **Example**: Find the transfer function for the step response below:



- **Example**: A system has a transfer function, $G(s) = \dfrac{50}{s+50}$. Find the time constant, settling time and rise time. Sketch the unit step response of the system.

- **Example**: Sketch the unit step response for a system with transfer function, $G(s) = \dfrac{100}{s+50}$.