**Abstract Algebra**

4th meeting

Materials: subring and its examples

Motivation:
1. \( \mathbb{Z}, \mathbb{R}, \mathbb{Q}, \mathbb{C} \) with ordinary operations additive (+) and multiplicative (\( \cdot \)) are ring and \( \mathbb{Z} \subset \mathbb{Q}, \mathbb{Z} \subset \mathbb{R}, \mathbb{Z} \subset \mathbb{C}, \mathbb{Q} \subset \mathbb{R}, \mathbb{Q} \subset \mathbb{C}, \mathbb{R} \subset \mathbb{C} \).

2. \( M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \) with additive (+) and multiplicative (\( \times \)) operations on matrix is ring.

3. \( N = \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \) with additive (+) and multiplicative (\( \times \)) operations on matrix is ring.

4. \( K = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \) with additive (+) and multiplicative (\( \times \)) operations on matrix is ring.

We know that \( K \subset N \subset M \).

From this fact, the concept of subring of ring is defined as follows:

**Definition 1:** Let \((R, +, \cdot)\) be a ring, \( S \neq \emptyset \) and \( S \subset R \). \( S \) is called subring of \( R \) if \((S, +, \cdot)\) is also ring.

Examples:
1. \( \mathbb{Z}, \mathbb{R}, \mathbb{Q}, \mathbb{C} \) with ordinary operations additive (+) and multiplicative (\( \cdot \)) are ring and \( \mathbb{Z} \subset \mathbb{Q}, \mathbb{Z} \subset \mathbb{R}, \mathbb{Z} \subset \mathbb{C}, \mathbb{Q} \subset \mathbb{R}, \mathbb{Q} \subset \mathbb{C}, \mathbb{R} \subset \mathbb{C} \). So we conclude that \( \mathbb{Z} \) is subring of \( \mathbb{R}, \mathbb{Q}, \) and \( \mathbb{C} \).

   Then, \( \mathbb{Q} \) is subring of \( \mathbb{R} \) and \( \mathbb{C} \). And \( \mathbb{R} \) is subring of \( \mathbb{C} \).

2. \( K \) is subring of \( N \) and \( M \). Then, \( N \) is subring of \( M \).

3. Let \( \mathbb{Z}_{15} \) be set of integer classes of modulo 15. Find all subring of \( \mathbb{Z}_{15} \)!

4. Let \( \mathbb{Z}_{7} \) be set of integer classes of modulo 7. Find all subrings of \( \mathbb{Z}_{7} \)!

**Theorem 1:** Let \((R, +, \cdot)\) be a ring, \( S \neq \emptyset \) and \( S \subset R \). \( S \) is called subring of \( R \) if and only if for every \( a, b \in S \) (i). \( a - b \in S \) (ii). \( ab \in S \)

Proof: (see sukirman, 2006, page: 36)

**Theorem 2:** If \( S \) and \( T \) are subrings of ring \( R \), then \( S \cap T \) is subring of \( R \).

Proof: (see sukirman, 2006, page: 42)

Let \( S \) and \( T \) be subrings of ring \( R \). Is \( S \cup T \) subring of \( R \)? explain.