The Second International Conference on Control, Instrumentation and Mechatronic Engineering (CIM09)
Malacca, Malaysia, June 2-3, 2009

Modelling of a Two-Link Flexible Manipulator

M. Khairudin, Z. Mohamed, Z. M. Zain and A.W.I. Mohd Hashim
Faculty of Electrical Engineering, Universiti Teknologi Malaysia, 81310, Skudai, Malaysia
zahar@fke.utm.my

Abstract

This paper presents development of a dynamic model of a two-link flexible manipulator. An explicit, complete, and accurate nonlinear dynamic model of the system is developed using assumed mode method. The Lagrangian approach is used to derive the dynamic model of the system. Several responses including angular positions and displacements of both links are obtained and analysed.

1. Introduction

Robot manipulators are constructed very massively to make them precise and stiff. The arms of these can be considered rigid, which allows a simple control joints. The drawback of their heavy construction is that the robots need very powerful actuators and their operating speed is strongly limited by their own inertia. Lightweight robots are developed to overcome these drawbacks and allow high speed movements with the same or even better precision. These are commonly used in space applications, because the take-off weight of space shuttles is strongly limited. In order to improve higher productivity, it is required to reduce the weight of the arms and/or to increase their speed of operation. Moreover, flexible manipulators are lighter, faster and less expensive than rigid ones. However, control of a flexible manipulator is challenging and the difficulty increases dramatically for a two-link flexible manipulator.

Each flexible link can be modeled as distributed parameter system where the motion is described by a coupled system of ordinary and partial differential equations. Various approaches have been developed for modelling of the system. These can mainly be divided into two categories: the numerical analysis approach and the Assumed Mode Method (AMM). The numerical analysis methods that are utilised include finite difference and finite element methods. Both approaches have been used in obtaining the dynamic characterisation of a single-link flexible manipulator system incorporating damping, hub inertia and payload [1,2].

Subudhi and Morris [3] have used a combined Euler-Lagrange formulation and AMM approach to model the planar motion of a manipulator consisting of two flexible links and joints. The conventional Lagrangian modeling of flexible link robots does not fully incorporate the bending mechanism of flexible link as it allows free link elongation in addition to link deflection. De Luca and Siciliano [4] have utilised the AMM to derive a dynamic model of multilink flexible robot arms limiting to the case of planar manipulators with no torsional effects. The equations of motion which can be arranged in a computationally efficient closed form that is also linear with respect to a suitable set of constant mechanical parameters have been obtained [5].

The controller strategies for flexible manipulator systems can be classified as feedforward and feedback control schemes. A number of techniques have been proposed as feedforward control schemes for control of vibration [6].

This paper presents modeling of a two-link flexible manipulator using Lagrangian technique in conjunction with the AMM. The links are modeled as Euler-Bernoulli beams satisfying proper mass boundary conditions. A payload is added to the tip of the outer link, while hub inertias are included at the actuator joints. Several system responses including angular positions and displacements of both links are obtained and analysed.

2. A Two-Link Flexible Manipulator

Figure 1 shows the schematic diagram of a two-link flexible manipulator system considered in this study. The links are cascaded in serial fashion and are actuated by rotors and hubs with individual motors. An inertial payload of mass M_P and inertia I_P is connected to the distal link. The proximal link is clamped and connected to the rotor with a hub. The *i*th link has length l_i and uniform mass density per unit length ρ_i. E and I represent Young modulus and area moment of inertia of both links respectively. A payload is attached at the end-point of link-2.

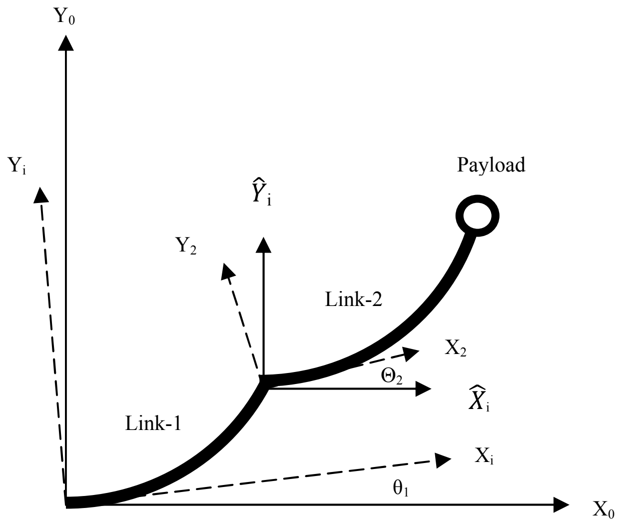

Figure 1. Structure of a two-link flexible manipulator

The first link is clamped on the rotor of the first motor. The second motor is attached to the tip of the first link. X_0 and Y_0 are the inertial coordinate frame. θ_1 and θ_2 are the angular position and $v_i(x_i,t)$ are the transverse component of the displacement vector. M_p is an inertial payload mass with inertia J_p at the end-point of second link.

3. Modelling

The description of kinematics is developed for a chain of *n* serially connected flexible links, revolute joints and motion of the manipulator on a two-dimensional plane, the rigid transformation matrix, A_i, from $X_{i-1}Y_{i-1}$ to $X_i\,Y_i$ is written as

$$A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix} \tag{1}$$

The global transformation matrix T_i transforming co-ordinates from X_0Y_0 to $X_i\,Y_i$ follow a recursion as

$$T_i = T_{i-1}E_{i-1}A_i \tag{2}$$

To derive the dynamic equations of motion of the system, the total energy associated with the manipulator system needs to be computed using the kinematics formulations explained previously. The total kinetic energy of the manipulator (T) can be written as

$$T = T_R + T_L + T_{PL} \tag{3}$$

where T_R, T_L and T_{PL} are the kinetic energies associated with the rotors, links and the hubs, respectively. As shown in Figure 1 and the kinematics formulation described previously, the kinetic energy associated with the payload can be written as

$$T_{PL} = \frac{1}{2}M_p\dot{p}_{n+1}^T\dot{p}_{n+1} + \frac{1}{2}I_P(\dot{\Omega}_n + \dot{\upsilon}'_n(l_n))^2 \tag{4}$$

where $\dot{\Omega}_n = \sum_{j=1}^{n}\theta_j + \sum_{k=1}^{n-1}\dot{\upsilon}'_k(l_k)$; n being the link number, prime and dot represent the first derivatives with respect to spatial variable x and time, respectively. The potential energy of the system due to the deformation of the link i by neglecting the effects of the gravity can be written as

$$U = \sum_i^n \frac{1}{2}\int_0^{l_i}(EI)_i\left(\frac{d^2\upsilon_i(x_i)}{dx_i^2}\right)^2 dx_i \tag{5}$$

The effectiveness masses at the end of the individual links can be obtained as

$$M_{L1} = m_2 + m_{h2} + M_p$$

$$J_{L1} = J_{o2} + J_{h2} + J_p + M_p l_2^2 \tag{6}$$

$$MD_1 = 0\,,\quad M_{L2} = M_p\,,\quad J_{L2} = J_p\,,\quad MD_2 = 0$$

The co-ordinate vector consists of link positions, (θ_1, θ_2) and modal displacements (q_{11}, q_{12}, q_{21}, q_{22}). The input vector, F = $\{\tau_1, \tau_2, 0, 0, 0, 0\}^T$, where τ_1 and τ_2 are the torques applied by rotor-1 and rotor-2, respectively. Therefore, the Euler–Lagrange's equations, with i = 1 and 2 and j = 1 and 2 can be obtained as:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial\dot{\theta}_i}\right) - \frac{\partial L}{\partial\theta_i} = \tau_i \tag{7}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_{ij}}\right)-\frac{\partial L}{\partial q_{ij}}=0 \quad (8)$$

Finally, a set of dynamic equations can be written in compact form as

$$M(\theta,q)\begin{Bmatrix}\ddot{\theta}\\ \ddot{q}\end{Bmatrix}+\begin{Bmatrix}f_1(\theta,\dot{\theta})\\ f_2(\theta,\dot{\theta})\end{Bmatrix}+\begin{Bmatrix}g_1(\theta,\dot{\theta},q,\dot{q})\\ g_2(\theta,\dot{\theta},q,\dot{q})\end{Bmatrix}$$
$$+\begin{Bmatrix}0\\ D\dot{q}\end{Bmatrix}+\begin{Bmatrix}0\\ Kq\end{Bmatrix}=\begin{Bmatrix}\tau\\ 0\end{Bmatrix} \quad (9)$$

where f_1 and f_2 are the vectors containing terms due to coriolis and centrifugal forces, M is the mass matrix, and g_1 and g_2 are the vectors containing terms due to the interactions of the link angles and their rates with the modal displacements. K is the diagonal stiffness matrix which takes on the values $\omega_{ij}{}^2 m_i$ and D is the passive structural damping.

4. Simulation Results

Table 1 shows the physical parameter of the two-link flexible manipulator used in this simulation. M_{Li}, EI and G represent the mass of link, flexural rigidity and gear ratio respectively. J_{hi} is the inertia of the motor and hub and M_{h2} is the mass of centre rotor at the second motor.

Table 1. Parameters of The System

Symbol	Parameter	Value	Unit
M_{L1}=M_{L2}	Mass of link	0.1	kg
ρ	Mass density	0.2	kgm^{-1}
G	Gear ratio	1	-
EI	Flexural rigidity	1.0	Nm^2
J_h	Motor and hub inertia	0.02	kgm^2
M_p	Payload mass	0.5	kg
l	Length	0.5	m
I_p	Payload inertia	0.0025	kgm^2
M_{h2}	Mass of the centre rotor	1	kg

A two-link flexible manipulator was excited with symmetric bang-bang torque inputs with amplitude of 0.2 Nm and 1 s width as shown in Figure. 2. For validation of the dynamic models, simulations were conducted on the manipulators with the same input trajectories. A bang-bang torque has a positive (acceleration) and negative (deceleration) period allowing the manipulator to, initially, accelerate and then decelerate and eventually stop at target location.

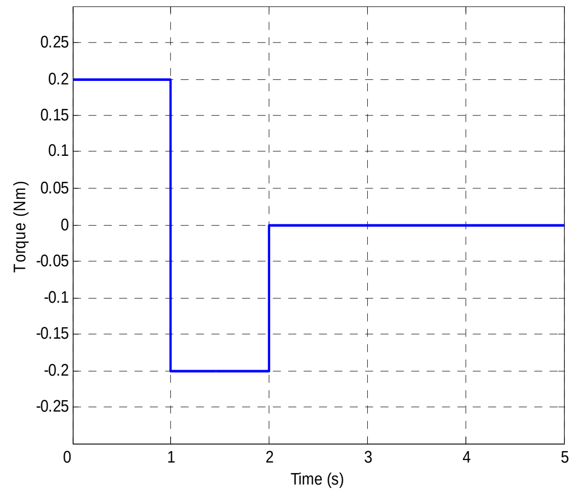

Figure 2. The Bang–bang Input Torque

Figure 3 shows the response of the angular positions of link-1 and link-2 of the system with payload of 0.5 kg. It is noted that the angular positions for Link-1 of 16.5 degrees and Link-2 is -34 degrees, respectively, were achieved within 2 s.

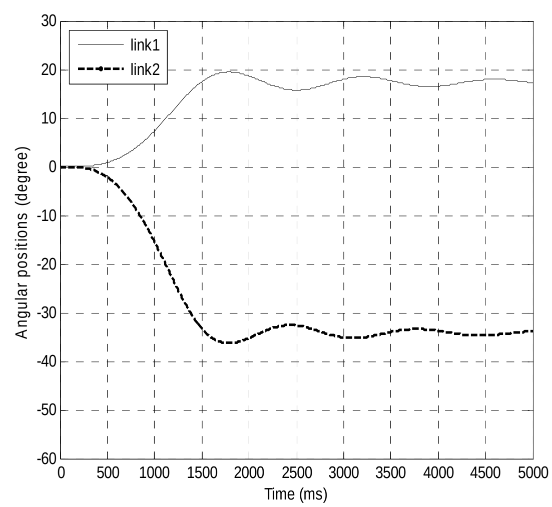

Figure 3. Angular Position of Link-1 and Link-2

Figures 4 and 5 show modal displacements of link-1 and link-2 of the system with a payload of 0.5 kg. For link-1, it is noted that the maximum displacements are 8.8×10^{-3} m and 5.5×10^{-3} m for mode 1 and mode 2 respectively. For link-2, the maximum displacements are 0.08 m and 2.5×10^{-3} m.

Figure 6 shows response of the end-point acceleration of link-1 and link-2. The maximum end-point accelerations for link-1 and link-2 of the system were found to be 0.23 m/sec^2 and 1.30 m/sec^2 respectively.

.

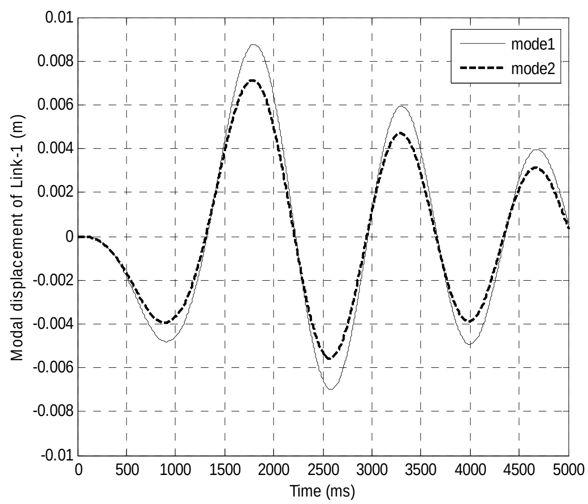

Figure 4. Modal Displacement of Link-1.

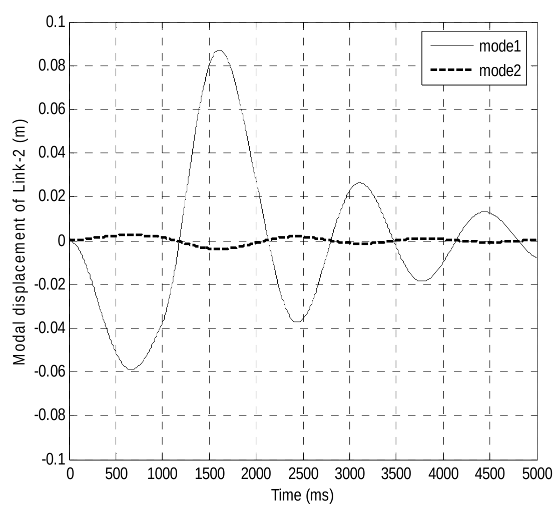

Figure 5. Modal Displacement of Link-2.

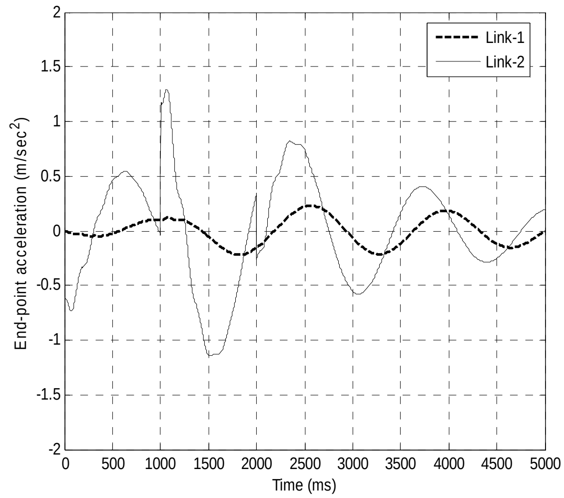

Figure 6. Response of end-point acceleration.

Figure 7 shows frequency response of end-point acceleration of link-1 and link-2. In this work, power spectral density (PSD) of the response is analysed. For payload 0.5 kg, the resonance frequencies for the first two modes of vibration respectively were obtained as 1 Hz and 7 Hz.

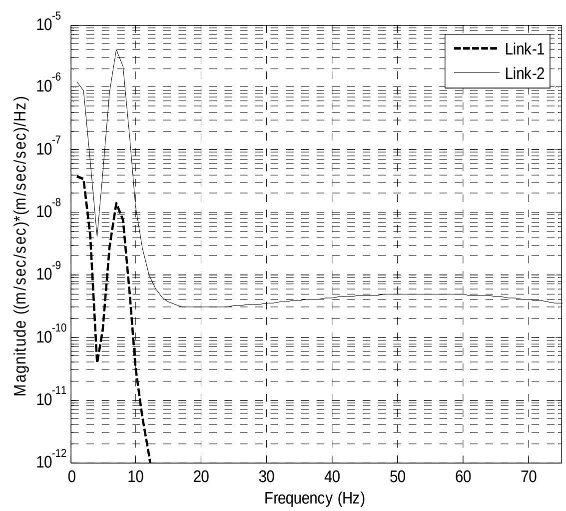

Figure 7. PSD of end-point acceleration response.

5. Conclusion

The development of dynamic model of a two-link flexible manipulator incorporating payload has been presented. The model has been derived using Eular-Lagrange and AMM. Simulation exercises have been conducted to study and verify the performance of the system. Several responses including angular positions and displacements of both links are obtained and analysed.

Acknowledgement

The authors gratefully acknowledge Universiti Teknologi Malaysia (UTM) and the Ministry of Science, Technology and Innovation (MOSTI) for the research funding under ScienceFund No. 03-01-06-SF0026.

References

[1] Tokhi MO, Mohamed Z, Azad AKM, "Finite difference and finite element approaches to dynamic modelling of a flexible manipulator," Proceeding of IMechE-I: Journal of Systems and Control Engineering, 1997; 211:145-156.

[2] Tokhi MO, Mohamed Z, Shaheed MH, "Dynamic characterization of a flexible manipulator system," Robotica, 2001;19:571-580.

[3] Subudhi B, Morris AS., "Dynamic Modelling, Simulation and Control of a manipulator with flexible links and joints," Robotics and Autonomous System, 2002; 41:257-270.

[4] De Luca, B. Siciliano, "Closed-form dynamic model of planar multi-link lightweight robots," IEEE Transactions on Systems, Man, and Cybernetics **21** (4) .1991. 826-839.

[5] S. Morris, A. Madani, "Inclusion of Shear Deformation Term to Improve Accuracy in Flexible-Link Robot Modelling", Mechatronics, Vol. 6, No. 6. 1996, pp. 631-647.

[6] Z. Mohammed, J.M. Martins, M.O. Tokhi, J. Sa'da Costa, M.A. Botto, "Vibration control of very flexible manipulator system," Control Engineering Practise 13 (2005), 267-277.