Hypothesis Testing

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Sources:
Anderson, Sweeney,wiliams, ***Statistics for Business and Economics*** , 6e Pearson Education, 2007
http://business.clayton.edu/arjomand/

What is a Hypothesis?

- A hypothesis is a claim (assumption) about a population parameter:

  - population mean

    Example: The mean monthly cell phone bill of this city is μ = $42

  - population proportion

    Example: The proportion of adults in this city with cell phones is p = .68

The Null Hypothesis, H_0

- States the assumption (numerical) to be tested

  Example: The average number of TV sets in U.S. Homes is equal to three ($H_0 : \mu = 3$)

- Is always about a population parameter, not about a sample statistic

$H_0 : \mu = 3$ ~~$H_0 : \bar{X} = 3$~~

The Null Hypothesis, H_0

(continued)

- Begin with the assumption that the null hypothesis is true
  - Similar to the notion of innocent until proven guilty

- Refers to the status quo
- Always contains "=" , "≤" or "≥" sign
- May or may not be rejected

The Alternative Hypothesis, H_1

- Is the opposite of the null hypothesis
  - e.g., The average number of TV sets in U.S. homes is not equal to 3 (H_1: $\mu \neq 3$)
- Challenges the status quo
- Never contains the "=" , "≤" or "≥" sign
- May or may not be supported
- Is generally the hypothesis that the researcher is trying to support

Hypothesis Testing Process

Claim: the population mean age is 50. (Null Hypothesis: H_0: μ = 50)

Population

Now select a random sample

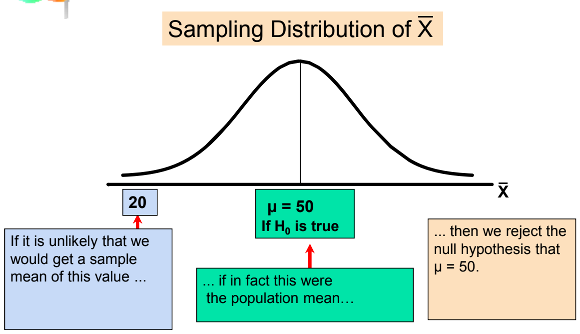
Is $\bar{X}$=20 likely if μ = 50?

Suppose the sample mean age is 20: $\bar{X}$ = 20

Sample

If not likely, REJECT Null Hypothesis

Reason for Rejecting H_0



Level of Significance, α

- **Defines the unlikely values of the sample statistic if the null hypothesis is true**
  - Defines rejection region of the sampling distribution
- Is designated by α , (level of significance)
  - Typical values are .01, .05, or .10
- Is selected by the researcher at the beginning
- Provides the critical value(s) of the test

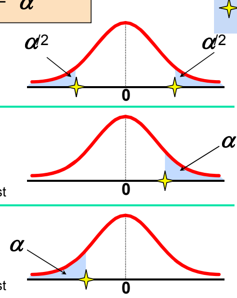
Level of Significance and the Rejection Region

Level of significance = α

H_0: μ = 3
H_1: μ ≠ 3
Two-tail test

H_0: μ ≤ 3
H_1: μ > 3
Upper-tail test

H_0: μ ≥ 3
H_1: μ < 3
Lower-tail test



Represents critical value

Rejection region is shaded

Errors in Making Decisions

- **Type I Error**
  - Reject a true null hypothesis
  - Considered a serious type of error

The probability of Type I Error is α

- Called level of significance of the test
- Set by researcher in advance

Errors in Making Decisions

(continued)

- **Type II Error**
  - Fail to reject a false null hypothesis

The probability of Type II Error is β

Outcomes and Probabilities

Possible Hypothesis Test Outcomes

	Actual Situation	
Decision	H_0 True	H_0 False
Do Not Reject H_0	**No error** ($1 - \alpha$)	**Type II Error** (β)
Reject H_0	**Type I Error** (α)	**No Error** ($1 - \beta$)

Key:
Outcome
(Probability)

Type I & II Error Relationship

- Type I and Type II errors can not happen at the same time
  - Type I error can only occur if H_0 is true
  - Type II error can only occur if H_0 is false

If Type I error probability (α) ↑, then Type II error probability (β) ↓

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Factors Affecting Type II Error

- All else equal,
  - β ↑ when the difference between hypothesized parameter and its true value ↓
  - β ↑ when α ↓
  - β ↑ when σ ↑
  - β ↑ when n ↓

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Power of the Test

- The power of a test is the probability of rejecting a null hypothesis that is false
- i.e., Power = P(Reject H_0 | H_1 is true)
  - Power of the test increases as the sample size increases

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Two Basic Approaches to Hypothesis Testing

- There are **two** basic approaches to conducting a hypothesis test:
- **1- p-Value Approach, and**
- **2- Critical Value Approach**

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1- *p*-Value Approach to One-Tailed Hypothesis Testing

- In order to accept or reject the null hypothesis the *p-value* is *computed using the test statistic* --Actual Z value.
- Reject H_0 if the *p*-value $\leq \alpha$
- Do not reject (accept) H_0 if the *p*-value $> \alpha$

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2- Critical Value Approach One-Tailed Hypothesis Testing

- Use the *Z table* to find the ***critical Z value**, and*
- Use the equation to find the ***actual Z***--Z statistics.
- *The rejection rule is:*
  - Lower tail: Reject H_0 if Actual $z \leq$ Critical $-z_\alpha$
  - Upper tail: Reject H_0 if Actual $z \geq$ Critical z_α

In other words, if the actual Z (Z statistics) is in the rejection region, then reject the null hypothesis.

Equation for finding the actual Z value: $z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

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Hypothesis Tests for the Mean

- Hypothesis Tests for μ
  - σ Known
  - σ Unknown

Test of Hypothesis for the Mean (σ Known)

- Convert sample result ($\bar{x}$) to a z value

- Hypothesis Tests for μ
  - σ Known
  - σ Unknown

Consider the test

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu > \mu_0$$

(Assume the population is normal)

The decision rule is:

$$\text{Reject } H_0 \text{ if } z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} > z_\alpha$$

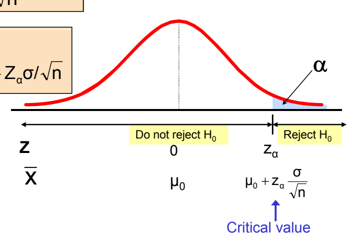
Decision Rule

$$\text{Reject } H_0 \text{ if } z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} > z_\alpha$$

H_0: $\mu = \mu_0$
H_1: $\mu > \mu_0$

Alternate rule:

$$\text{Reject } H_0 \text{ if } \bar{X} > \mu_0 + Z_\alpha \sigma/\sqrt{n}$$



p-Value Approach to Testing

- p-value: Probability of obtaining a test statistic more extreme (≤ or ≥) than the observed sample value given H_0 is true
  - Also called observed level of significance
  - Smallest value of α for which H_0 can be rejected

p-Value Approach to Testing

(continued)

- Convert sample result (e.g., $\bar{x}$) to test statistic (e.g., z statistic)
- Obtain the p-value
  - For an upper tail test:

$$\text{p-value} = P\left(Z > \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}, \text{ given that } H_0 \text{ is true}\right)$$
$$= P\left(Z > \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \mid \mu = \mu_0\right)$$

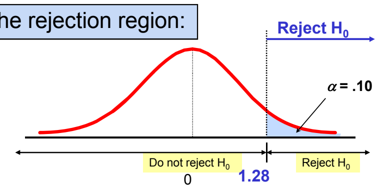
- Decision rule: compare the p-value to α
  - If p-value < α , reject H_0
  - If p-value ≥ α , do not reject H_0

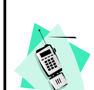
Example: Find Rejection Region

(continued)

- Suppose that α = .10 is chosen for this test

Find the rejection region:





$$\text{Reject } H_0 \text{ if } z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} > 1.28$$

Example: Upper-Tail Z Test for Mean (σ Known)

A phone industry manager thinks that customer monthly cell phone bill have increased, and now average over \$52 per month. The company wishes to test this claim. (Assume $\sigma = 10$ is known)

Form hypothesis test:

$H_0: \mu \le 52$ the average is not over \$52 per month

$H_1: \mu > 52$ the average **is** greater than \$52 per month (i.e., sufficient evidence exists to support the manager's claim)

Example: Sample Results

(continued)

Obtain sample and compute the test statistic

Suppose a sample is taken with the following results: $n = 64$, $\bar{x} = 53.1$ (σ=10 was assumed known)

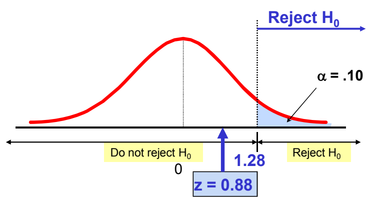
- Using the sample results,

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{53.1 - 52}{\frac{10}{\sqrt{64}}} = 0.88$$

Example: Decision

(continued)

Reach a decision and interpret the result:



Do not reject H_0 since $z = 0.88 < 1.28$

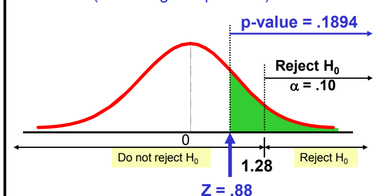
i.e.: there is not sufficient evidence that the mean bill is over \$52

Example: p-Value Solution

(continued)

Calculate the p-value and compare to α

(assuming that μ = 52.0)



$$P(\bar{x} \ge 53.1 \mid \mu = 52.0)$$

$$= P\left(z \ge \frac{53.1 - 52.0}{10/\sqrt{64}}\right)$$

$$= P(z \ge 0.88) = 1 - .8106$$

$$= .1894$$

Do not reject H_0 since p-value = .1894 > α = .10

One-Tail Tests

- In many cases, the alternative hypothesis focuses on one particular direction

$H_0: \mu \le 3$
$H_1: \mu > 3$

This is an upper-tail test since the alternative hypothesis is focused on the upper tail above the mean of 3

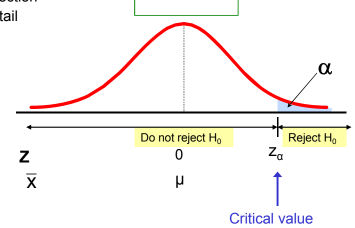
$H_0: \mu \ge 3$
$H_1: \mu < 3$

This is a lower-tail test since the alternative hypothesis is focused on the lower tail below the mean of 3

Upper-Tail Tests

- There is only one critical value, since the rejection area is in only one tail

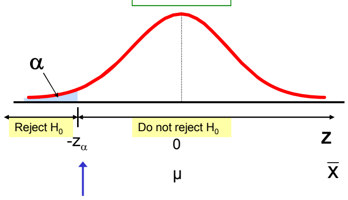
$H_0: \mu \le 3$
$H_1: \mu > 3$



Lower-Tail Tests

- There is only one critical value, since the rejection area is in only one tail

$H_0: \mu \geq 3$
$H_1: \mu < 3$

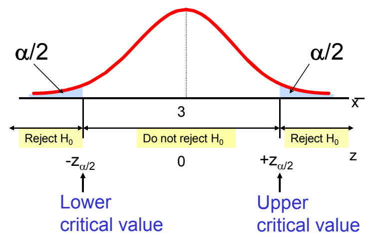


Two-Tail Tests

- In some settings, the alternative hypothesis does not specify a unique direction

$H_0: \mu = 3$
$H_1: \mu \neq 3$

- There are two critical values, defining the two regions of rejection



Hypothesis Testing Example

Test the claim that the true mean # of TV sets in US homes is equal to 3. (Assume σ = 0.8)

- State the appropriate null and alternative hypotheses
  - $H_0: \mu = 3$, $H_1: \mu \neq 3$ (This is a two tailed test)
- Specify the desired level of significance
  - Suppose that $\alpha = .05$ is chosen for this test
- Choose a sample size
  - Suppose a sample of size n = 100 is selected

Hypothesis Testing Example

(continued)

- Determine the appropriate technique
  - σ is known so this is a z test
- Set up the critical values
  - For $\alpha = .05$ the critical z values are ±1.96
- Collect the data and compute the test statistic
  - Suppose the sample results are
    n = 100, $\bar{x} = 2.84$ (σ = 0.8 is assumed known)

So the test statistic is:

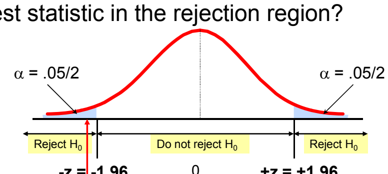
$$z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{.08} = -2.0$$

Hypothesis Testing Example

(continued)

- Is the test statistic in the rejection region?

Reject H_0 if z < -1.96 or z > 1.96; otherwise do not reject H_0

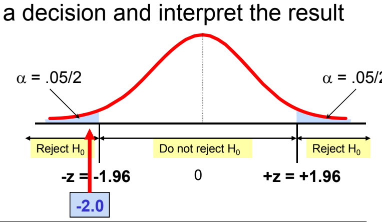


Here, z = -2.0 < -1.96, so the test statistic is in the rejection region

Hypothesis Testing Example

(continued)

- Reach a decision and interpret the result



Since z = -2.0 < -1.96, we reject the null hypothesis and conclude that there is sufficient evidence that the mean number of TVs in US homes is not equal to 3

Example: p-Value

- Example: How likely is it to see a sample mean of 2.84 (or something further from the mean, in either direction) if the true mean is $\mu = 3.0$?

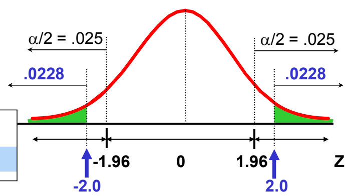
$\bar{x} = 2.84$ is translated to a z score of z = -2.0

$P(z < -2.0) = .0228$

$P(z > 2.0) = .0228$

p-value

= .0228 + .0228 = .0456



t Test of Hypothesis for the Mean (σ Unknown)

- Convert sample result ($\bar{x}$) to a t test statistic

Hypothesis Tests for μ

σ Known | σ Unknown

Consider the test

$H_0: \mu = \mu_0$

$H_1: \mu > \mu_0$

(Assume the population is normal)

The decision rule is:

Reject H_0 if $t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} > t_{n-1, \alpha}$

t Test of Hypothesis for the Mean (σ Unknown)

(continued)

- For a two-tailed test:

Consider the test

$H_0: \mu = \mu_0$

$H_1: \mu \neq \mu_0$

(Assume the population is normal, and the population variance is unknown)

The decision rule is:

Reject H_0 if $t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} < -t_{n-1, \alpha/2}$ or if $t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} > t_{n-1, \alpha/2}$

Example: Two-Tail Test (σ Unknown)

The average cost of a hotel room in New York is said to be $168 per night. A random sample of 25 hotels resulted in $\bar{x}$ = $172.50 and s = $15.40. Test at the $\alpha = 0.05$ level.

(Assume the population distribution is normal)

H_0: μ = 168

H_1: μ ≠ 168

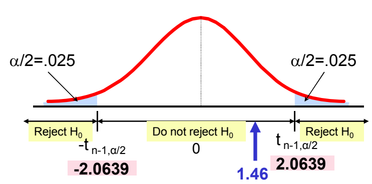
Example Solution: Two-Tail Test

H_0: μ = 168

H_1: μ ≠ 168

- **α = 0.05**
- **n = 25**
- **σ is unknown, so use a t statistic**
- **Critical Value:**

$t_{24, .025} = \pm 2.0639$



$$t_{n-1} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

Do not reject H_0: not sufficient evidence that true mean cost is different than $168

Tests of the Population Proportion

- Involves categorical variables
- Two possible outcomes
  - "Success" (a certain characteristic is present)
  - "Failure" (the characteristic is not present)
- Fraction or proportion of the population in the "success" category is denoted by P
- Assume sample size is large

Proportions

(continued)

- Sample proportion in the success category is denoted by $\hat{p}$
  - $$\hat{p} = \frac{x}{n} = \frac{\text{number of successes in sample}}{\text{sample size}}$$
- When $nP(1 – P) > 9$, $\hat{p}$ can be approximated by a normal distribution with mean and standard deviation
  - $\mu_{\hat{p}} = P$ $\qquad$ $\sigma_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}}$

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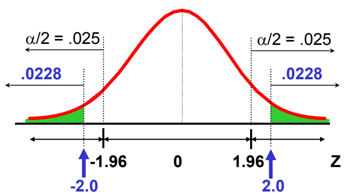
Example: p-Value

(continued)

- Compare the p-value with α
  - If p-value $<$ α, reject H_0
  - If p-value $\geq$ α, do not reject H_0

Here: p-value = .0456
α = .05

Since .0456 < .05, we reject the null hypothesis



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Power of the Test

- Recall the possible hypothesis test outcomes:

Key:
Outcome
(Probability)

	Actual Situation	
Decision	H_0 True	H_0 False
Do Not Reject H_0	**No error** $(1 - \alpha)$	**Type II Error** (β)
Reject H_0	**Type I Error** (α)	**No Error** $(1 - \beta)$

- β denotes the probability of Type II Error
- $1 – \beta$ is defined as the power of the test

Power = $1 – \beta$ = the probability that a false null hypothesis is rejected

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Type II Error

Assume the population is normal and the population variance is known. Consider the test

$$H_0: \mu = \mu_0$$
$$H_1: \mu > \mu_0$$

The decision rule is:

Reject H_0 if $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} > z_\alpha$ or Reject H_0 if $\bar{x} = \bar{x}_c > \mu_0 + Z_\alpha \sigma / \sqrt{n}$

If the null hypothesis is false and the true mean is μ^*, then the probability of type II error is

$$\beta = P(\bar{x} < \bar{x}_c \mid \mu = \mu^*) = P\left(z < \frac{\bar{x}_c - \mu^*}{\sigma / \sqrt{n}}\right)$$

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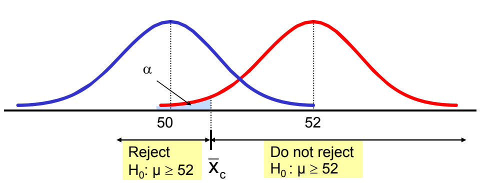
Type II Error Example

- Type II error is the probability of failing to reject a false H_0

Suppose we fail to reject **H_0: $\mu \geq 52$** when in fact the true mean is **$\mu^* = 50$**



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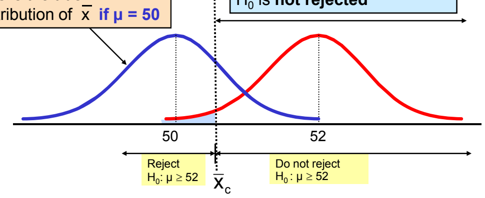
Type II Error Example

(continued)

- Suppose we do not reject H_0: $\mu \geq 52$ when in fact the true mean is $\mu^* = 50$

This is the true distribution of $\bar{x}$ **if $\mu = 50$**

This is the range of $\bar{x}$ where H_0 is **not rejected**



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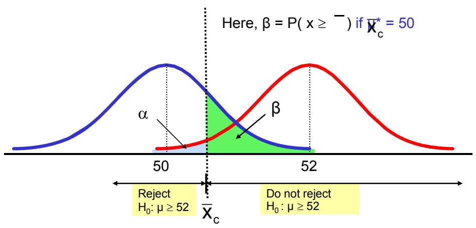
Type II Error Example

(continued)

- Suppose we do not reject $H_0: \mu \geq 52$ when in fact the true mean is $\mu^* = 50$

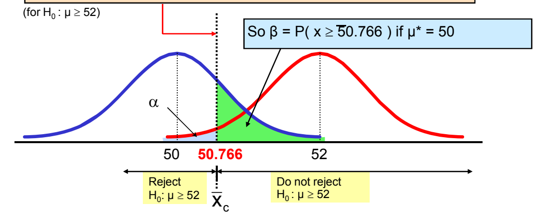


Calculating β

- Suppose n = 64 , σ = 6 , and α = .05

$$\bar{x}_c = \mu_0 - z_\alpha \frac{\sigma}{\sqrt{n}} = 52 - 1.645 \frac{6}{\sqrt{64}} = 50.766$$

(for $H_0: \mu \geq 52$)

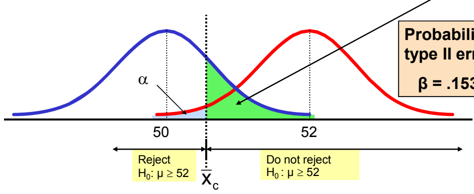


Calculating β

(continued)

- Suppose n = 64 , σ = 6 , and α = .05

$$P(\bar{x} \geq 50.766 \mid \mu^* = 50) = P\left(z \geq \frac{50.766 - 50}{6/\sqrt{64}}\right) = P(z \geq 1.02) = .5 - .3461 = .1539$$



Power of the Test Example

If the true mean is $\mu^* = 50$,

- The probability of Type II Error = β = 0.1539
- The power of the test = $1 - \beta = 1 - 0.1539 = 0.8461$

Key: **Outcome** **(Probability)**

	Actual Situation	
Decision	H_0 True	H_0 False
Do Not Reject H_0	**No error** **1 - α = 0.95**	**Type II Error** **β = 0.1539**
Reject H_0	**Type I Error** **α = 0.05**	**No Error** **1 - β = 0.8461**

(The value of β and the power will be different for each μ*)

Chapter Summary

- Addressed hypothesis testing methodology
- Performed Z Test for the mean (σ known)
- Discussed critical value and p-value approaches to hypothesis testing
- Performed one-tail and two-tail tests
- Performed t test for the mean (σ unknown)
- Performed Z test for the proportion
- Discussed type II error and power of the test