Sistem Cerdas : PTK – Pasca Sarjana - UNY

Fuzzy Rules & Fuzzy Reasoning

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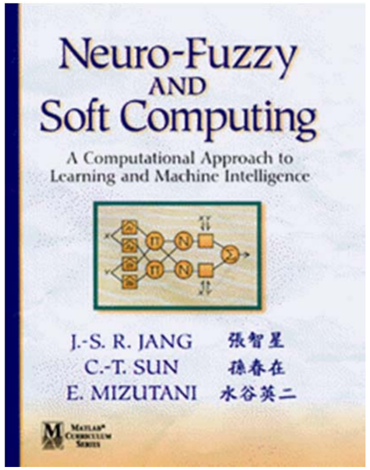
Referensi:



Jyh-Shing Roger Jang et al., *Neuro-Fuzzy and Soft Computing: A Computational Approach to Learning and Machine Intelligence*, First Edition, Prentice Hall, 1997

Fuzzy if-then rules (3.3) (cont.)

– Orthogonality

A term set T = $t_1, \ldots, t_n$ of a linguistic variable x on the universe X is orthogonal if:

$$\sum_{i=1}^{n} \mu_{t_i}(x) = 1, \quad \forall x \in X$$

Where the t_i's are convex & normal fuzzy sets defined on X.

Fuzzy if-then rules (3.3) (cont.)

General format:

- If x is A then y is B (where A & B are linguistic values defined by fuzzy sets on universes of discourse X & Y).
  - "x is A" is called the antecedent or premise
  - "y is B" is called the consequence or conclusion
- Examples:
  - If pressure is high, then volume is small.
  - If the road is slippery, then driving is dangerous.
  - If a tomato is red, then it is ripe.
  - If the speed is high, then apply the brake a little.

Fuzzy if-then rules (3.3) (cont.)

– Meaning of fuzzy if-then-rules (A ⇒ B)

- It is a relation between two variables x & y; therefore it is a binary fuzzy relation R defined on X * Y

- There are two ways to interpret A ⇒ B:

  – A coupled with B
  – A entails B

  if A is coupled with B then:

$$\mathbf{R = A \Rightarrow B = A * B = \int_{X*Y} \mu_A(x) \tilde{*} \mu_B(y)/(x, y)}$$

$$\textbf{where } \tilde{*} \textbf{ is a T-normoperator.}$$

Fuzzy if-then rules (3.3) (cont.)

If A entails B then:

$R = A \Rightarrow B = \neg A \cup B$ (material implication)

$R = A \Rightarrow B = \neg A \cup (A \cap B)$ (propositional calculus)

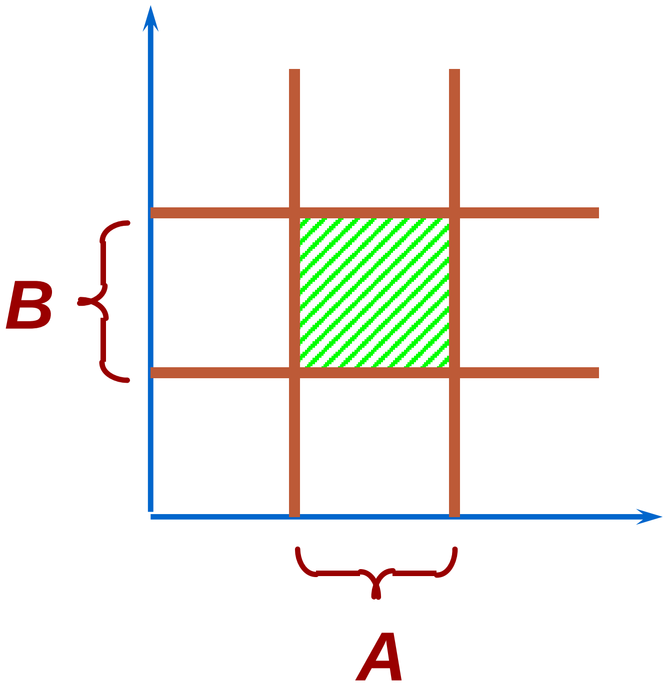
$R = A \Rightarrow B = (\neg A \cap \neg B) \cup B$ (extended propositional calculus)

$$\mu_R(x, y) = \sup\left\{c; \mu_A(x) \tilde{*} c \le \mu_B(y), 0 \le c \le 1\right\}$$

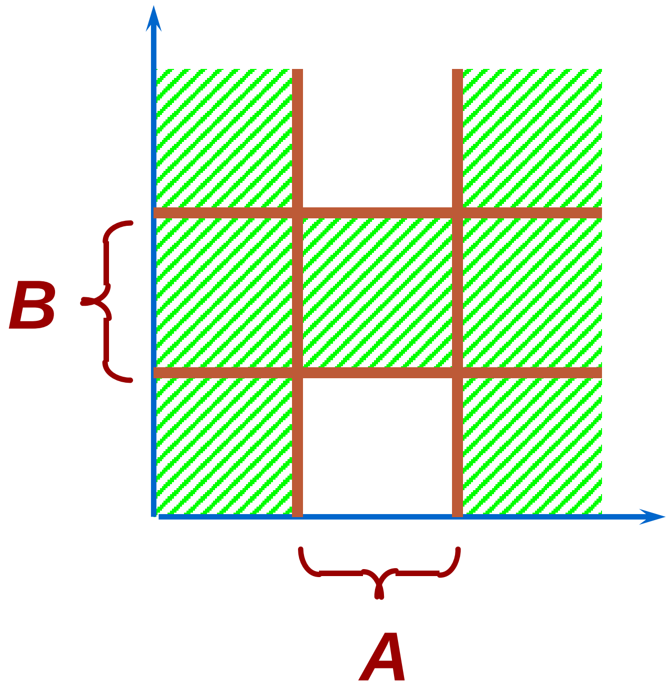
Fuzzy if-then rules (3.3) (cont.)

Two ways to interpret "If x is A then y is B":

A coupled with B



A entails B



Fuzzy if-then rules (3.3) (cont.)

- Note that R can be viewed as a fuzzy set with a two-dimensional MF

$$\mu_R(x, y) = f(\mu_A(x), \mu_B(y)) = f(a, b)$$

With $a = \mu_A(x)$, $b = \mu_B(y)$ and f called the fuzzy implication function provides the membership value of (x, y)

Fuzzy if-then rules (3.3) (cont.)

– Case of "A coupled with B"

$$R_m = A * B = \int_{X*Y} \mu_A(x) \wedge \mu_B(y)/(x,y)$$

(minimum operator proposed by Mamdani, 1975)

$$\mathbf{R_p = A * B = \int_{X*Y} \mu_A(x)\, \mu_B(y)/(x,y)}$$

(product proposed by Larsen, 1980)

$$\mathbf{R_{bp} = A * B = \int_{X*Y} \mu_A(x) \otimes \mu_B(y)/(x,y)}$$

$$\mathbf{= \int_{X*Y} 0 \vee (\mu_A(x) + \mu_B(y) - 1)/(x,y)}$$

(bounded product operator)

Fuzzy if-then rules (3.3) (cont.)

– Case of "A coupled with B" (cont.)

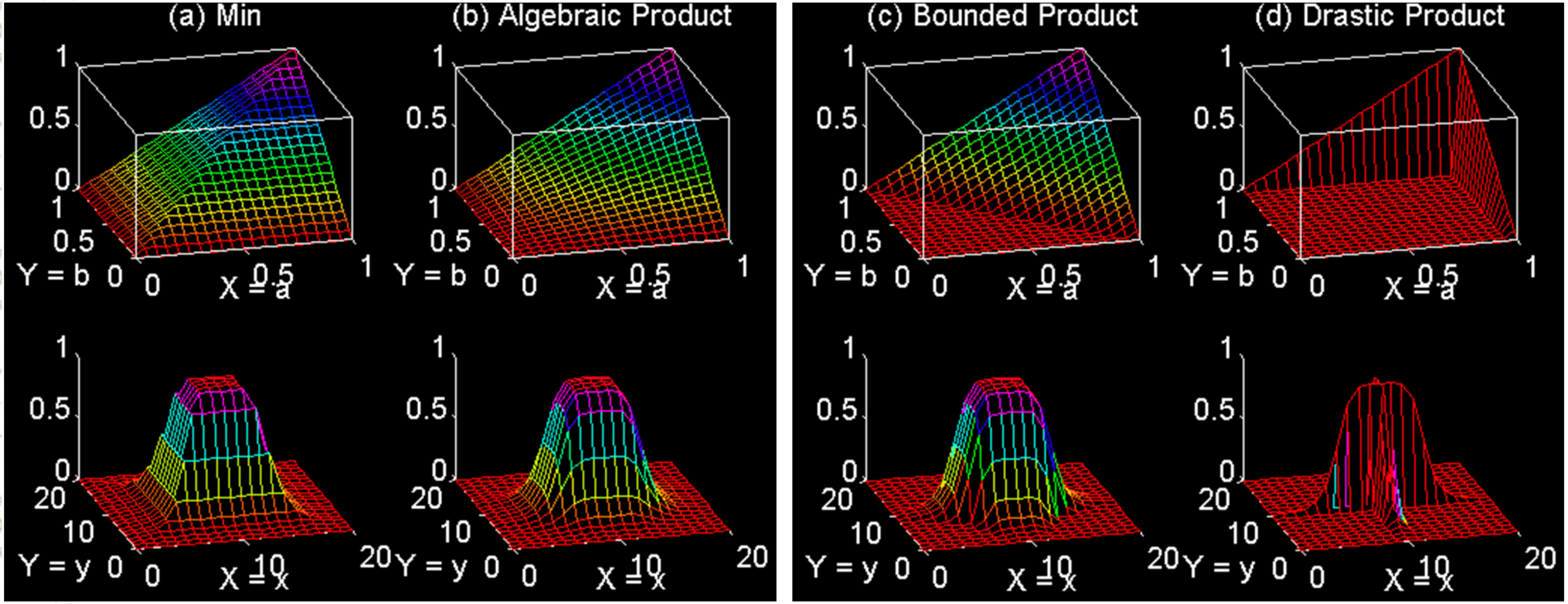
$$R_{dp} = A * B = \int_{X*Y} \mu_A(x) \hat{\cdot} \mu_B(y) / (x, y)$$

$$where: \text{f(a, b)} = \text{a} \hat{\cdot} b = \begin{cases} a & \text{if b} = 1 \\ b & \text{if a} = 1 \\ 0 & \text{if otherwise} \end{cases}$$

$$Example \text{ for } \mu_{\text{A}}(x) = bell(x; 4,3,10) \text{ and } \mu_{\text{B}}(y) = bell(y; 4,3,10)$$

(Drastic operator)

Fuzzy if-then rules (3.3) (cont.)



A coupled with B

Fuzzy if-then rules (3.3) (cont.)

– Case of "A entails B"

$$R_a = \neg A \cup B = \int_{X*Y} 1 \wedge (1 - \mu_A(x) + \mu_B(y) / (x, y)$$

$$\text{where}: f_a(a, b) = 1 \wedge (1 - a + b)$$

(Zadeh's arithmetic rule by using bounded sum operator for union)

$$R_{mm} = \neg A \cup (A \cap B) = \int_{X*Y} (1 - \mu_A(x)) \vee (\mu_A(x) \wedge \mu_B(y)) / (x, y)$$

$$\text{where}: f_m(a, b) = (1 - a) \vee (a \wedge b)$$

(Zadeh's max-min rule)

Fuzzy if-then rules (3.3) (cont.)

– Case of "A entails B" (cont.)

$$R_s = \neg A \cup B = \int_{X*Y} (1-\mu_A(x)) \vee \mu_B(y) / (x,y)$$

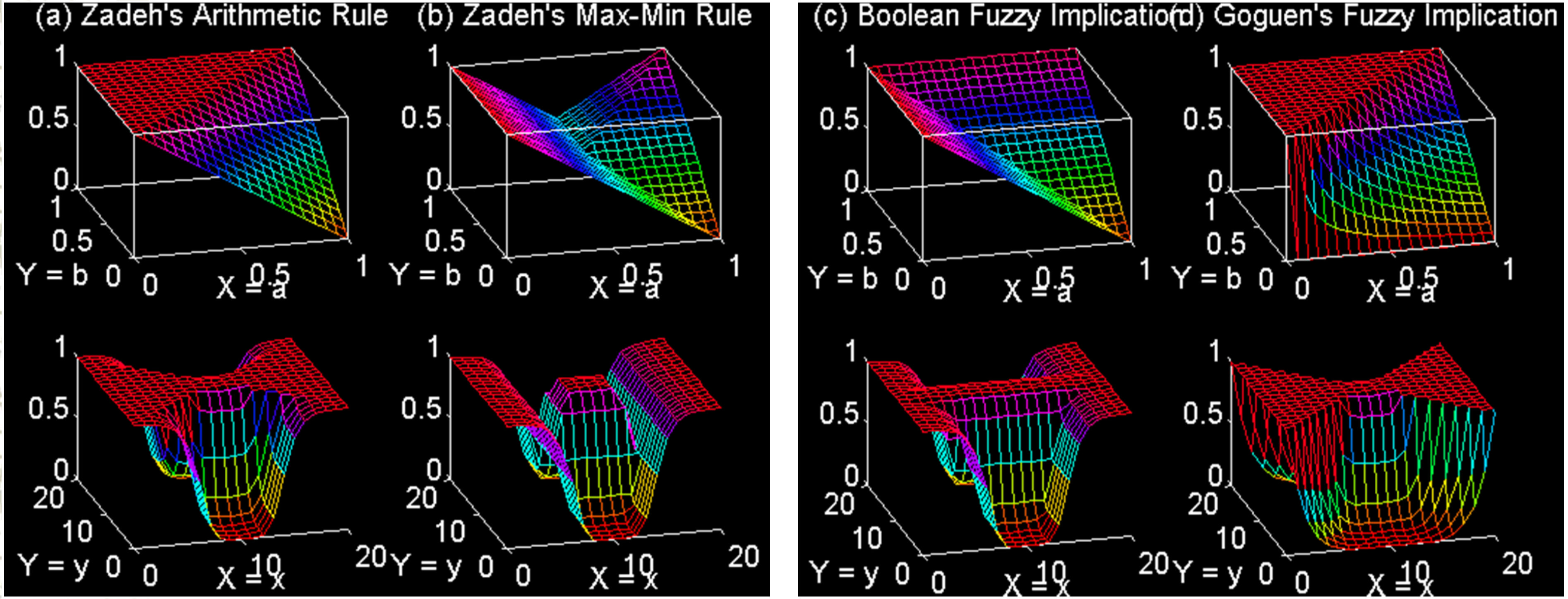
$$\text{where}: \; f_s(a,b) = (1-a) \vee b$$

(Boolean fuzzy implication with max for union)

$$R_\Delta = \int_{X*Y} (\mu_A(x) \tilde{<} \mu_B(y)) / (x,y)$$

$$\text{where}: \; a \tilde{<} b = \begin{cases} 1 & \text{if } a \le b \\ b/a & \text{otherwise} \end{cases}$$

(Goguen's fuzzy implication with algebraic product for T-norm)


(a) Zadeh's Arithmetic Rule
(b) Zadeh's Max-Min Rule
(c) Boolean Fuzzy Implication
(d) Goguen's Fuzzy Implication
Y = b
X = a
Y = y
X = x

A entails B

Fuzzy Reasoning (3.4)

- Definition

  – Known also as approximate reasoning

  – It is an inference procedure that derives conclusions from a set of fuzzy if-then-rules & known facts

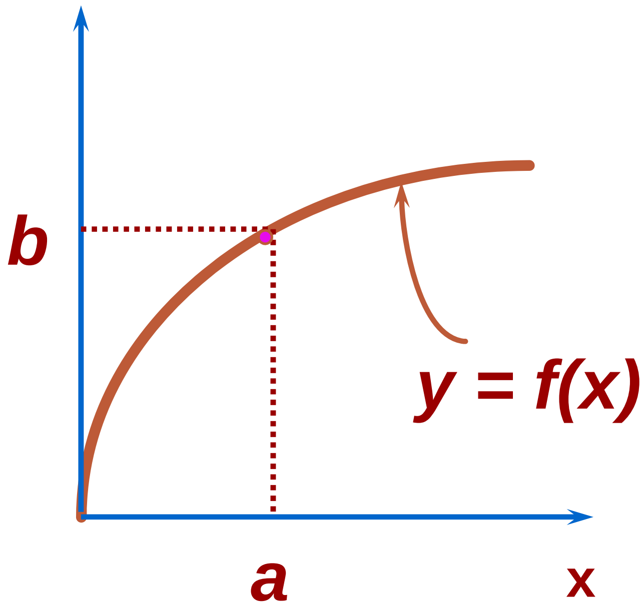
Fuzzy Reasoning (3.4) (cont.)

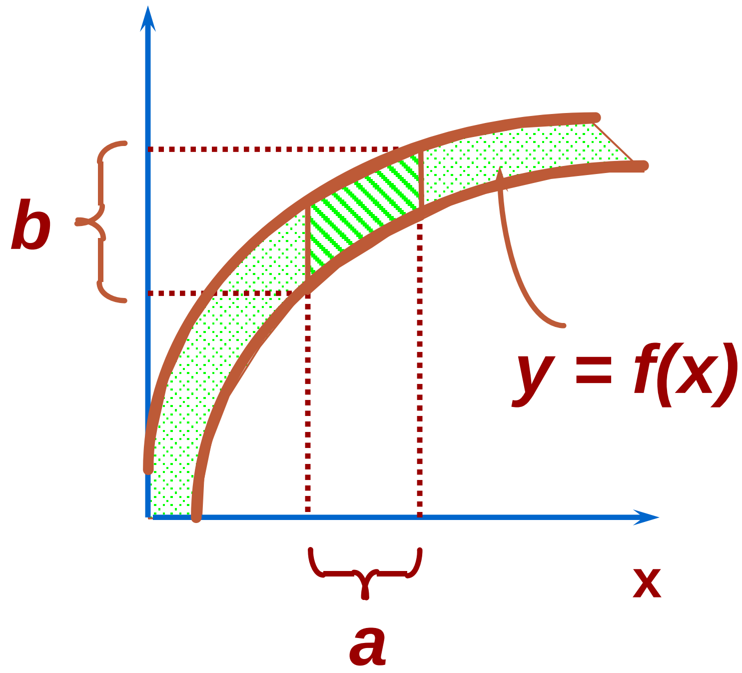
Compositional rule of inference

- Idea of composition (cylindrical extension & projection)
  - Computation of b given a & f is the goal of the composition
    - Image of a point is a point
    - Image of an interval is an interval

Fuzzy Reasoning (3.4) (cont.)

Derivation of *y* = *b* from *x* = *a* and *y* = *f(x)*:





a **and** ***b*****: points**

y = f(x) **: a curve**

a **and** ***b*****: intervals**

y = f(x) **: an interval-valued function**

Fuzzy Reasoning (3.4) (cont.)

– The extension principle is a special case of the compositional rule of inference

- F is a fuzzy relation on X*Y, A is a fuzzy set of X & the goal is to determine the resulting fuzzy set B

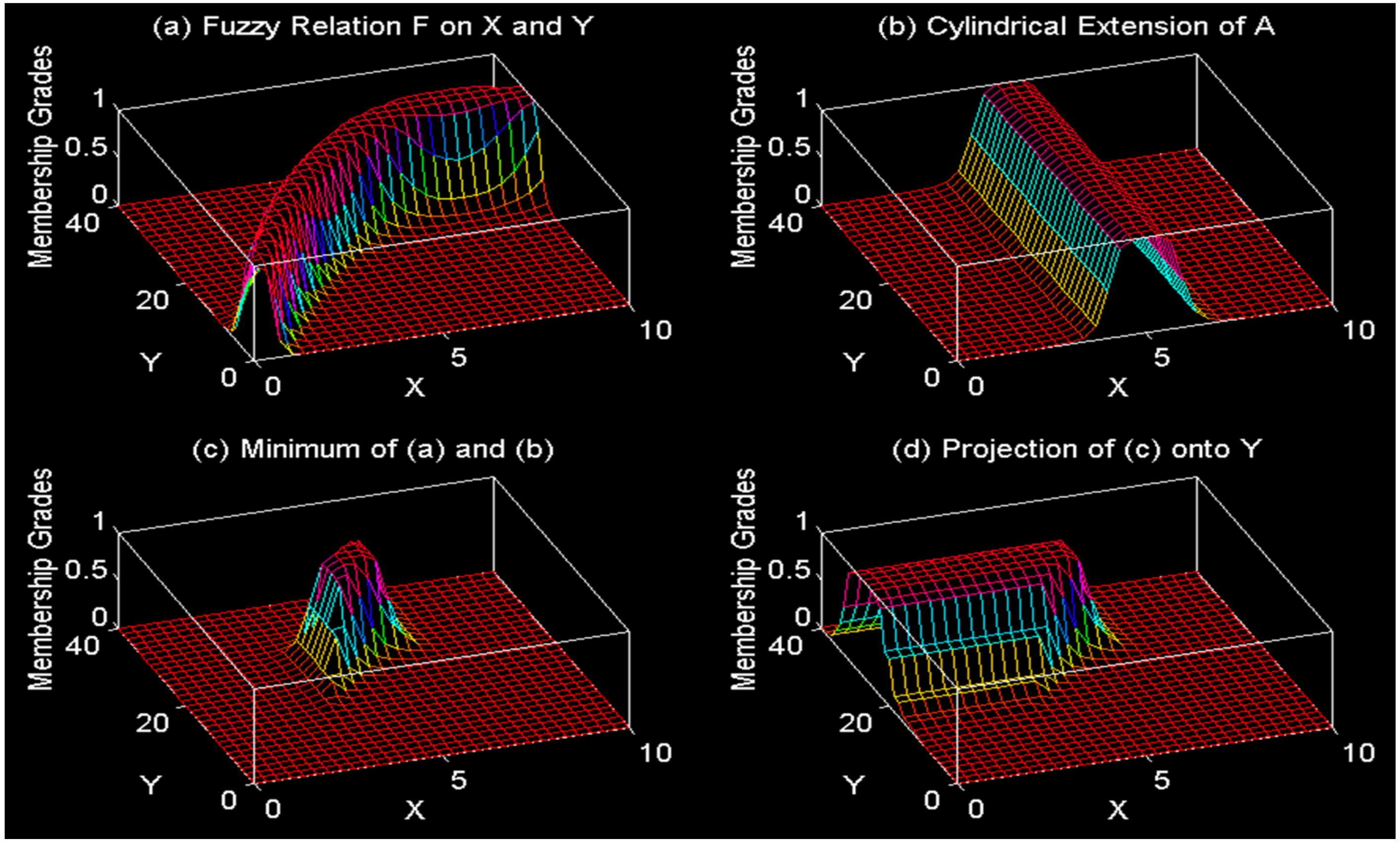
  – Construct a cylindrical extension c(A) with base A

  – Determine c(A) $\wedge$ F (using minimum operator)

  – Project c(A) $\wedge$ F onto the y-axis which provides B

Fuzzy Reasoning (3.4) (cont.)

a is a fuzzy set and *y* = *f(x)* is a fuzzy relation:



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Fuzzy Reasoning (3.4) (cont.)

- Given A, A $\Rightarrow$ B, infer B

A = "today is sunny"

A $\Rightarrow$ B: day = sunny then sky = blue

infer: "sky is blue"

- illustration

Premise 1 (fact):	x is A
Premise 2 (rule):	if x is A then y is B
Consequence:	y is B

Fuzzy Reasoning (3.4) (cont.)

Approximation

A’ = “ today is more or less sunny”

B’ = “ sky is more or less blue”

- illustration

Premise 1 (fact): x is A’

Premise 2 (rule): if x is A then y is B

Consequence: y is B’

(approximate reasoning or fuzzy reasoning!)

Fuzzy Reasoning (3.4) (cont.)

- Definition of fuzzy reasoning

  Let A, A' and B be fuzzy sets of X, X, and Y, respectively. Assume that the fuzzy implication

  $A \Rightarrow B$ is expressed as a fuzzy relation R on X*Y. Then the fuzzy set B induced by "x is A' " and the fuzzy rule "if x is A then y is B' is defined by:

$$\mu_{B'}(y) = \max_x \min\left[\mu_{A'}(x), \mu_R(x, y)\right]$$

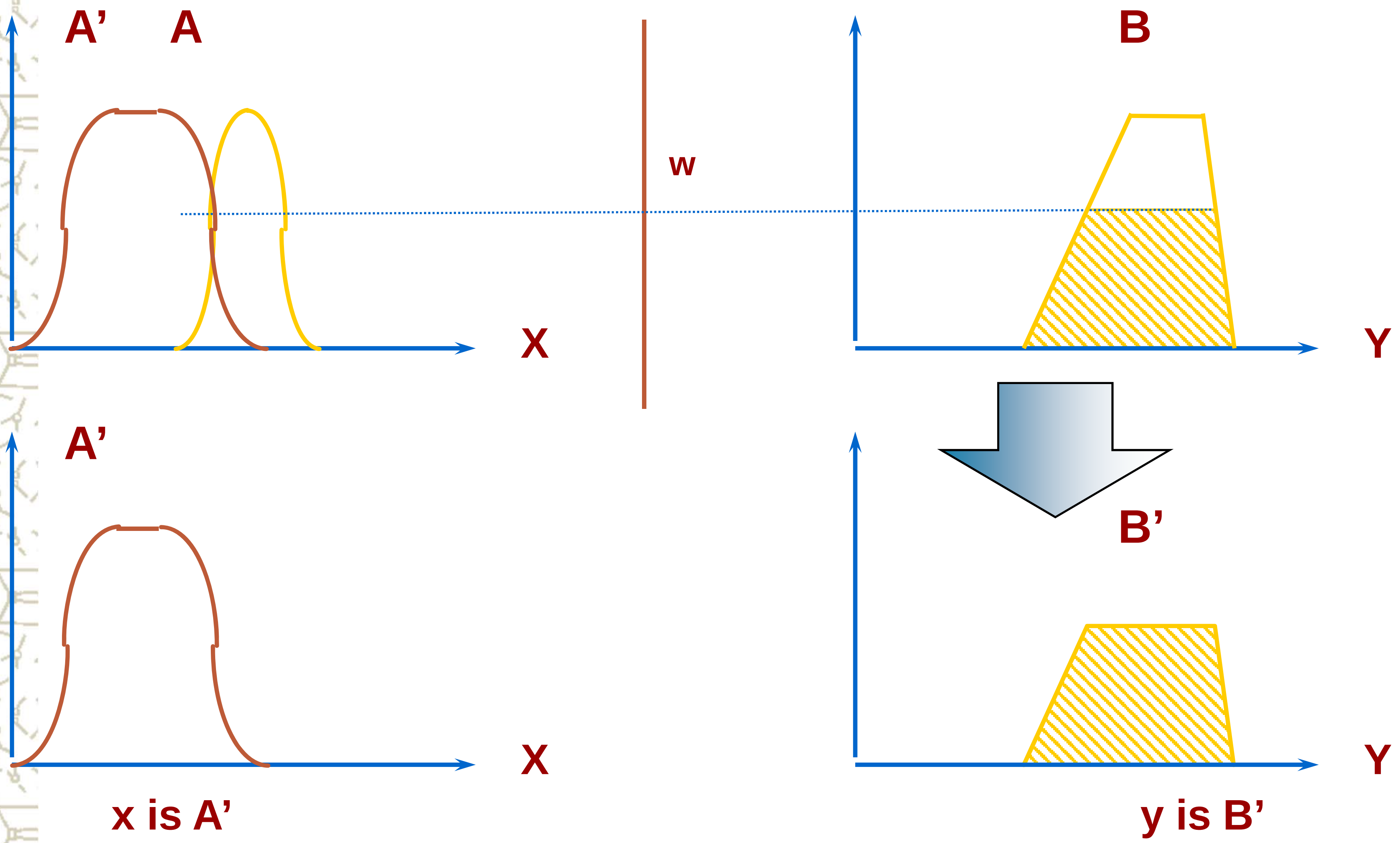
- Single rule with single antecedent

  Rule : if x is A then y is B

  Fact: x is A'

  Conclusion: y is B' ($\mu_{B'}(y) = [\vee_x (\mu_{A'}(x) \wedge \mu_A(x)] \wedge \mu_B(y)$)

Fuzzy Reasoning (3.4) (cont.)



– Single rule with multiple antecedents

Premise 1 (fact): x is A’ and y is B’

Premise 2 (rule): if x is A and y is B then z is C

Conclusion: z is C’

Premise 2: A*B → C

$$R_{mamdani}(A,B,C) = (A*B)*C = \int_{X*Y*Z} \mu_A(x) \wedge \mu_B(y) \wedge \mu_C(z)/(x,y,z)$$

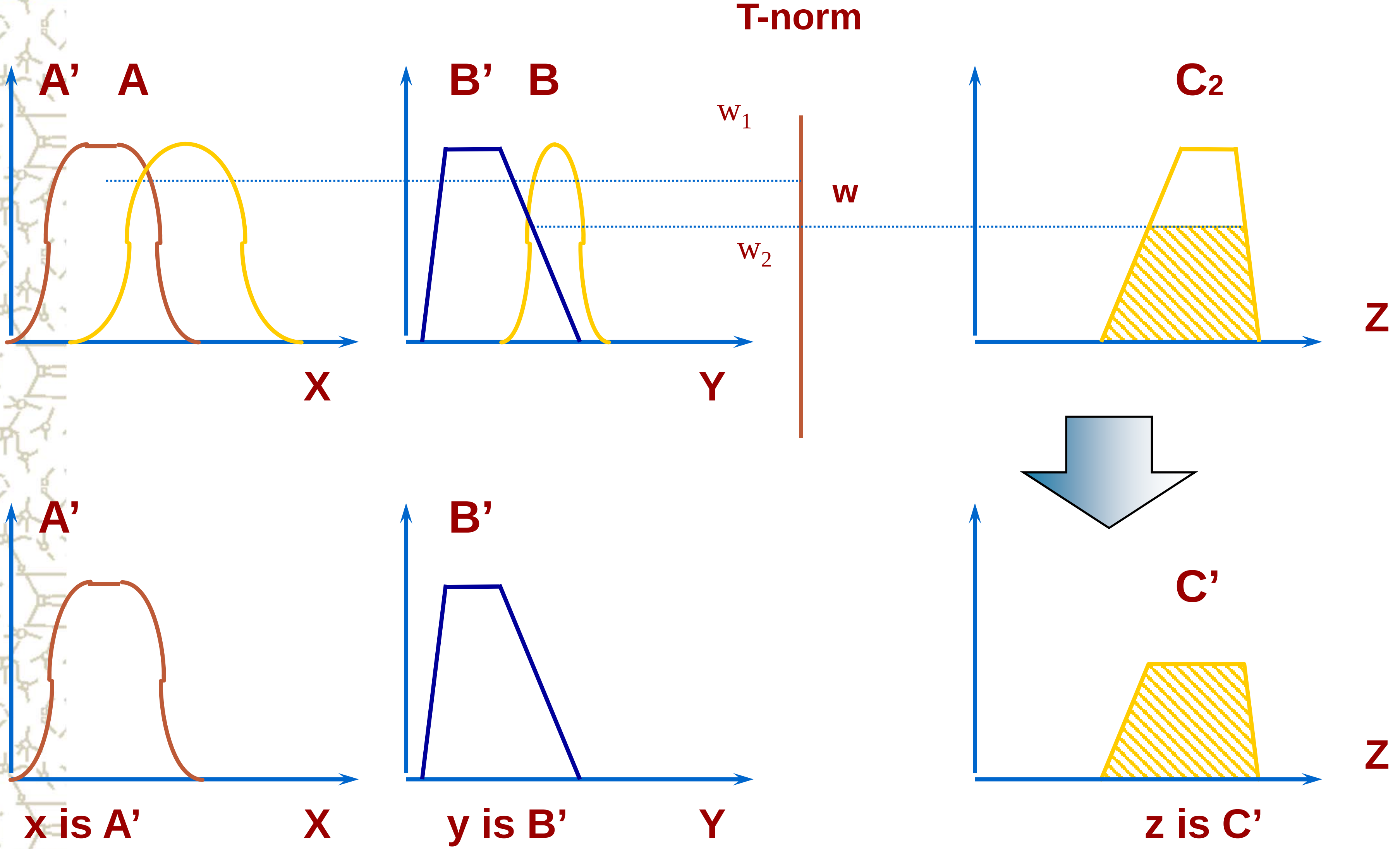
$$C' = \underbrace{(A'*B')}_{\text{premise 1}} \circ \underbrace{(A*B \to C)}_{\text{premise 2}}$$

$$\mu_{C'}(z) = \vee_{x,y} [\mu_{A'}(x) \wedge \mu_{B'}(y)] \wedge [\mu_A(x) \wedge \mu_B(y) \wedge \mu_C(z)]$$

$$= \vee_{x,y} \{[\mu_{A'}(x) \wedge \mu_{B'}(y) \wedge \mu_A(x) \wedge \mu_B(y)]\} \wedge \mu_C(z)$$

$$= \underbrace{\left\{\vee_x [\mu_{A'}(x) \wedge \mu_A(x)]\right\}}_{w_1} \wedge \underbrace{\left\{\vee_y [\mu_{B'}(y) \wedge \mu_B(y)]\right\}}_{w2} \wedge \mu_C(z)$$

$$= (w_1 \wedge w_2) \wedge \mu_C(z)$$


T-norm
A'
A
X
B'
B
Y
w_1
w
w_2
C_2
Z
A'
x is A'
X
B'
y is B'
Y
C'
Z
z is C'

Fuzzy Reasoning (3.4) (cont.)

– Multiple rules with multiple antecedents

Premise 1 (fact): x is A' and y is B'

Premise 2 (rule 1): if x is A_1 and y is B_1 then z is C_1

Premise 3 (rule 2): If x is A_2 and y is B_2 then z is C_2

Consequence (conclusion): z is C'

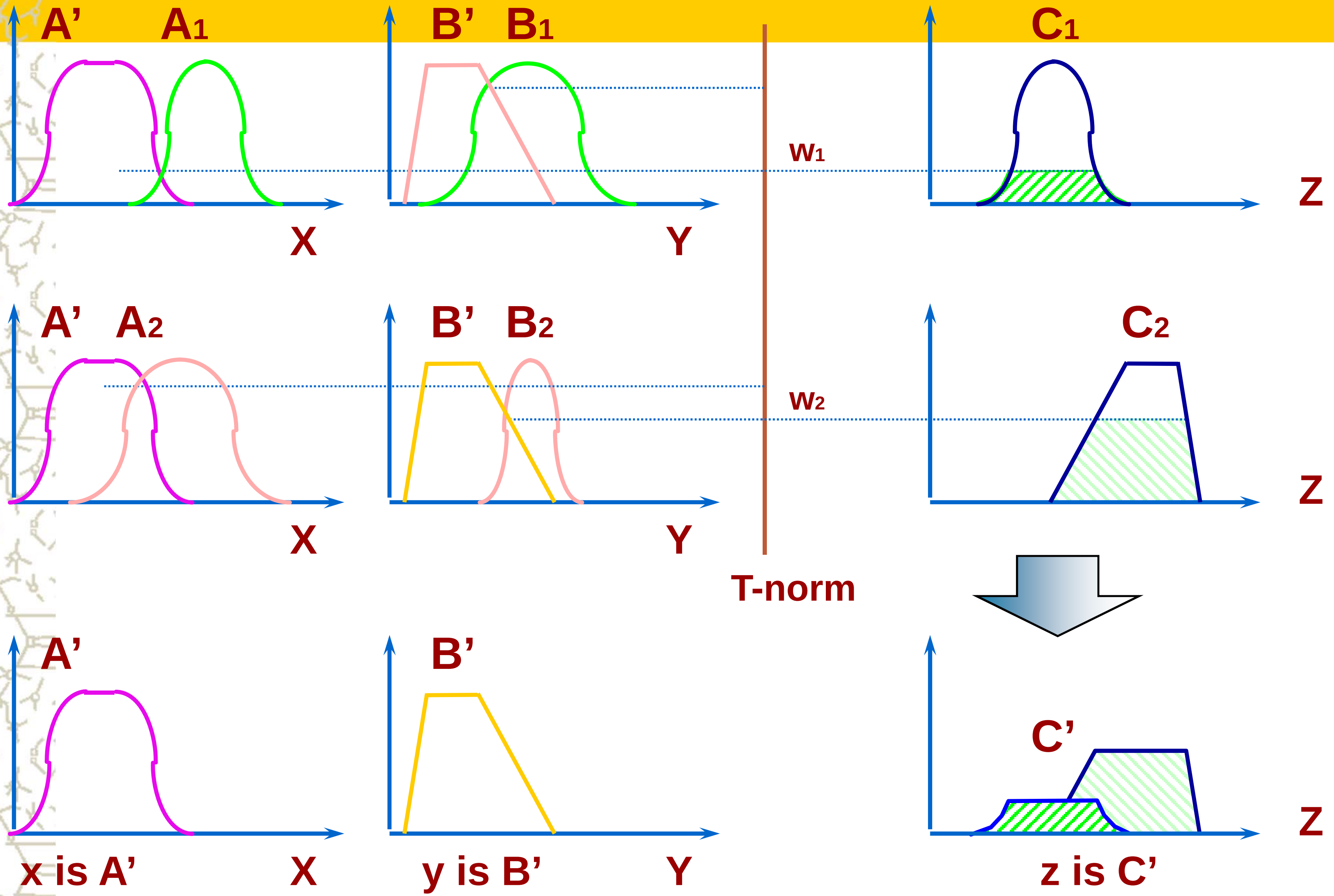
$R_1 = A_1 * B_1 \rightarrow C_1$

$R_2 = A_2 * B_2 \rightarrow C_2$

Since the max-min composition operator o is distributive over the union operator, it follows:

$C' = (A' * B') \circ (R_1 \cup R_2) = [(A' * B') \circ R_1] \cup [(A' * B') \circ R_2] = C'_1 \cup C'_2$

Where C'_1 & C'_2 are the inferred fuzzy set for rules 1 & 2 respectively


A'
A1
B'
B1
C1
w1
Z
X
Y
A'
A2
B'
B2
C2
w2
Z
X
Y
T-norm
A'
B'
C'
Z
x is A'
X
y is B'
Y
z is C'