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Fuzzy Rules & Fuzzy Reasoning

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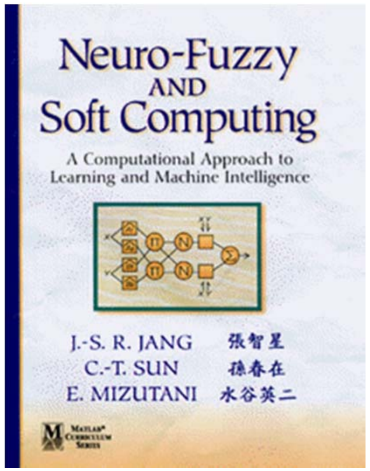
Referensi:



Jyh-Shing Roger Jang et al., *Neuro-Fuzzy and Soft Computing: A Computational Approach to Learning and Machine Intelligence*, First Edition, Prentice Hall, 1997

Extension Principle & Fuzzy Relations (3.2)

- Extension principle

  A is a fuzzy set on *X* :

$$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \cdots + \mu_A(x_n)/x_n$$

  The image of *A* under *f(.)* is a fuzzy set *B:*

$$B = \mu_B(x_1)/y_1 + \mu_B(x_2)/y_2 + \cdots + \mu_B(x_n)/y_n$$

  where $y_i = f(x_i)$, $i = 1$ to n

  If *f(.)* is a many-to-one mapping, then

$$\mu_B(y) = \max_{x = f^{-1}(y)} \mu_A(x)$$

Extension Principle & Fuzzy Relations (3.2) (cont.)

– Example:

Application of the extension principle to fuzzy sets with discrete universes

Let A = 0.1 / -2+0.4 / -1+0.8 / 0+0.9 / 1+0.3 / 2

and f(x) = x2 – 3

Applying the extension principle, we obtain:

B = 0.1 / 1+0.4 / -2+0.8 / -3+0.9 / -2+0.3 /1

= 0.8 / -3+(0.4V0.9) / -2+(0.1V0.3) / 1

= 0.8 / -3+0.9 / -2+0.3 / 1

where "V" represents the "max" operator

Same reasoning for continuous universes

Extension Principle & Fuzzy Relations (3.2) (cont.)

Fuzzy relations

- A fuzzy relation R is a 2D MF:

$$R = \{((x, y), \mu_R(x, y)) | (x, y) \in X \times Y\}$$

- Examples:

Let X = Y = IR+

and R(x,y) = “y is much greater than x”

The MF of this fuzzy relation can be subjectively defined as:

$$\mu_R(x,y) = \begin{cases} \frac{y-x}{x+y+2}, & \text{if } y > x \\ 0, & \text{if } y \leq x \end{cases}$$

if X = {3,4,5} & Y = {3,4,5,6,7}

Extension Principle & Fuzzy Relations (3.2) (cont.)

- Then R can be Written as a matrix:

$$\mathbf{R} = \begin{bmatrix} \mathbf{0} & \mathbf{0.111} & \mathbf{0.200} & \mathbf{0.273} & \mathbf{0.333} \\ \mathbf{0} & \mathbf{0} & \mathbf{0.091} & \mathbf{0.167} & \mathbf{0.231} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0.077} & \mathbf{0.143} \end{bmatrix}$$

where R{i,j} = μ[xi, yj]

  - x is close to y (x and y are numbers)
  - x depends on y (x and y are events)
  - x and y look alike (x and y are persons or objects)
  - If x is large, then y is small (x is an observed reading and Y is a corresponding action)

Extension Principle & Fuzzy Relations (3.2) (cont.)

– Max-Min Composition

- The max-min composition of two fuzzy relations R_1 (defined on X and Y) and R_2 (defined on Y and Z) is

$$\mu_{R_1 \circ R_2}(x, z) = \vee_y [\mu_{R_1}(x, y) \wedge \mu_{R_2}(y, z)]$$

- Properties:
  - Associativity: $R \circ (S \circ T) = (R \circ S) \circ T$
  - Distributivity over union: $R \circ (S \cup T) = (R \circ S) \cup (R \circ T)$
  - Week distributivity over intersection:

$$R \circ (S \cap T) \subseteq (R \circ S) \cap (R \circ T)$$

  - Monotonicity: $S \subseteq T \Rightarrow (R \circ S) \subseteq (R \circ T)$

Extension Principle & Fuzzy Relations (3.2) (cont.)

- Max-min composition is not mathematically tractable, therefore other compositions such as max-product composition have been suggested

– Max-product composition

$$\mu_{R_1 \circ R_2}(x, z) = \vee_y [\mu_{R_1}(x, y)\mu_{R_2}(y, z)]$$

Extension Principle & Fuzzy Relations (3.2) (cont.)

– Example of max-min & max-product composition

- Let R_1 = "x is relevant to y"
  R_2 = "y is relevant to z"
  be two fuzzy relations defined on X*Y and Y*Z respectively
  X = {1,2,3}, Y = $\{\alpha,\beta,\chi,\delta\}$ and Z = {a,b}.

  Assume that:

$$\mathbf{R_1} = \begin{bmatrix} 0.1 & 0.3 & 0.5 & 0.7 \\ 0.4 & 0.2 & 0.8 & 0.9 \\ 0.6 & 0.8 & 0.3 & 0.2 \end{bmatrix} \qquad \mathbf{R_2} = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.3 \\ 0.5 & 0.6 \\ 0.7 & 0.2 \end{bmatrix}$$

Extension Principle & Fuzzy Relations (3.2) (cont.)

The derived fuzzy relation "x is relevant to z" based on R_1 & R_2

Let's assume that we want to compute the degree of relevance between $2 \in X$ & $a \in Z$

Using max-min, we obtain:

$$\mu_{R_1 \circ R_2}(2, a) = \max\{0.4 \wedge 0.9, 0.2 \wedge 0.2, 0.8 \wedge 0.5, 0.9 \wedge 0.7\}$$
$$= \max\{0.4, 0.2, 0.5, 0.7\}$$
$$= 0.7$$

Using max-product composition, we obtain:

$$\mu_{R_1 \circ R_2}(2, a) = \max\{0.4 * 0.9, 0.2 * 0.2, 0.8 * 0.5, 0.9 * 0.7\}$$
$$= \max\{0.36, 0.04, 0.40, 0.63\}$$
$$= 0.63$$

Fuzzy if-then rules (3.3)

Linguistic Variables

- Conventional techniques for system analysis are intrinsically unsuited for dealing with systems based on human judgment, perception & emotion

- Principle of incompatibility

  - As the complexity of a system increases, our ability to make precise & yet significant statements about its behavior decreases until a fixed threshold

  - Beyond this threshold, precision & significance become almost mutually exclusive characteristics [Zadeh, 1973]

Fuzzy if-then rules (3.3) (cont.)

- The concept of linguistic variables introduced by Zadeh is an alternative approach to modeling human thinking

- Information is expressed in terms of fuzzy sets instead of crisp numbers

- **Definition:** A linguistic variable is a quintuple (x, T(x), X, G, M) where:

  - x is the name of the variable
  - T(x) is the set of linguistic values (or terms)
  - X is the universe of discourse
  - G is a syntactic rule that generates the linguistic values
  - M is a semantic rule which provides meanings for the linguistic values

Fuzzy if-then rules (3.3) (cont.)

– Example:

A numerical variable takes numerical values

Age = 65

A linguistic variables takes linguistic values

Age is old

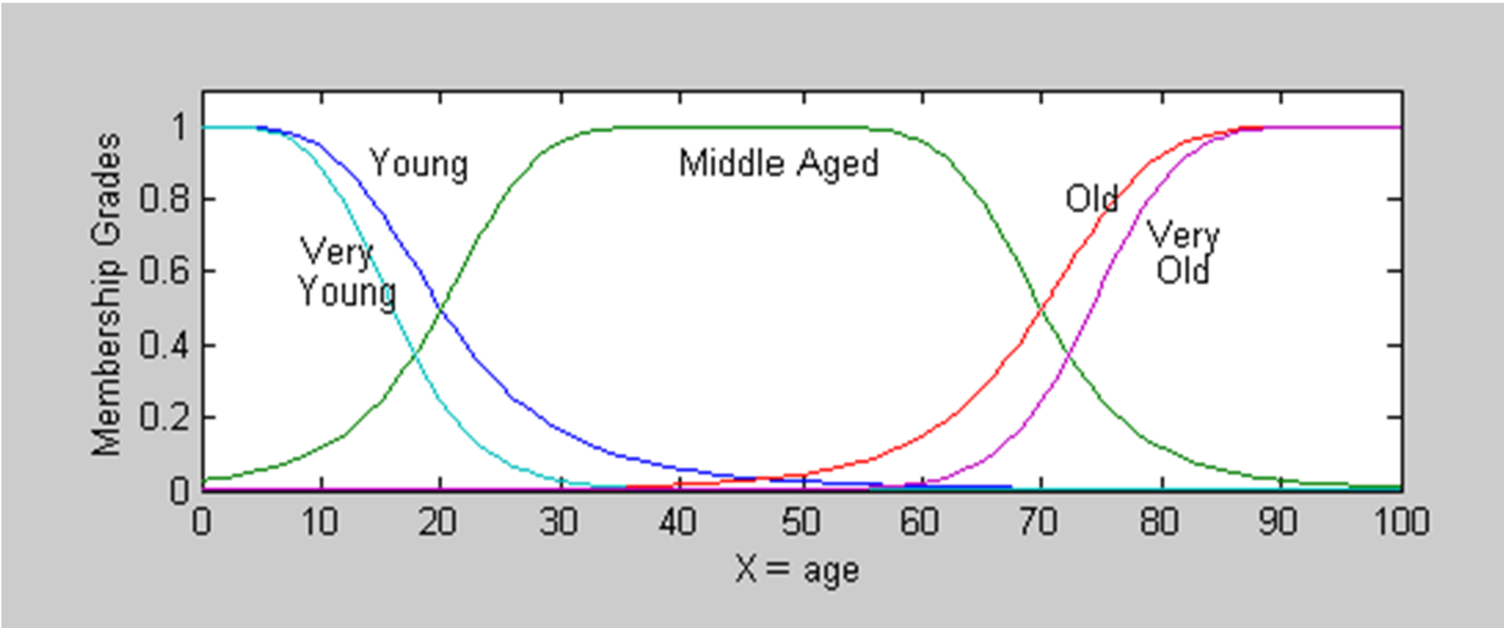
A linguistic value is a fuzzy set

All linguistic values form a term set

T(age) = {young, not young, very young, ...
middle aged, not middle aged, ...
old, not old, very old, more or less old, ...
not very yound and not very old, ...}

Fuzzy if-then rules (3.3) (cont.)

- Where each term T(age) is characterized by a fuzzy set of a universe of discourse X= = [0,100]



Fuzzy if-then rules (3.3) (cont.)

– The syntactic rule refers to the way the terms in T(age) are generated

– The semantic rule defines the membership function of each linguistic value of the term set

– The term set consists of primary terms as (young, middle aged, old) modified by the negation ("not") and/or the hedges (very, more or less, quite, extremely,…) and linked by connectives such as (and, or, either, neither,…)

Fuzzy if-then rules (3.3) (cont.)

– Concentration & dilation of linguistic values

- Let A be a linguistic value described by a fuzzy set with membership function $\mu_A(.)$

  – $$A^k = \int_X [\mu_A(x)]^k / x$$

  is a modified version of the original linguistic value.

  – A^2 = CON(A) is called the concentration operation

  – $\sqrt{A}$ = DIL(A) is called the dilation operation

  – CON(A) & DIL(A) are useful in expression the hedges such as "very" & "more or less" in the linguistic term A

  – Other definitions for linguistic hedges are also possible

Fuzzy if-then rules (3.3) (cont.)

– Composite linguistic terms

Let's define:

$$\mathbf{NOT(A) = \neg A = \int_X [1 - \mu_A(x)]/x,}$$

$$\mathbf{A\ and\ B = A \cap B = \int_X [\mu_A(x) \wedge \mu_B(x)]/x}$$

$$\mathbf{A\ or\ B = A \cup B = \int_X [\mu_A(x) \vee \mu_B(x)]/x}$$

where A, B are two linguistic values whose semantics are respectively defined by $\mu_A(.)$ & $\mu_B(.)$

Composite linguistic terms such as: "not very young", "not very old" & "young but not too young" can be easily characterized

Fuzzy if-then rules (3.3) (cont.)

– Example: Construction of MFs for composite linguistic terms

Let's $$\mu_{young}(x) = bell(x,20,2,0) = \frac{1}{1+\left(\frac{x}{20}\right)^4}$$

$$\mu_{old}(x) = bell(x,30,3,100) = \frac{1}{1+\left(\frac{x-100}{30}\right)^6}$$

Where x is the age of a person in the universe of discourse [0, 100]

- More or less = DIL(old) = √old = $\int_X \sqrt{\frac{1}{1+\left(\frac{x-100}{30}\right)^6}} \Big/ x$

Fuzzy if-then rules (3.3) (cont.)

- Not young and not old = ¬young ∩ ¬old =

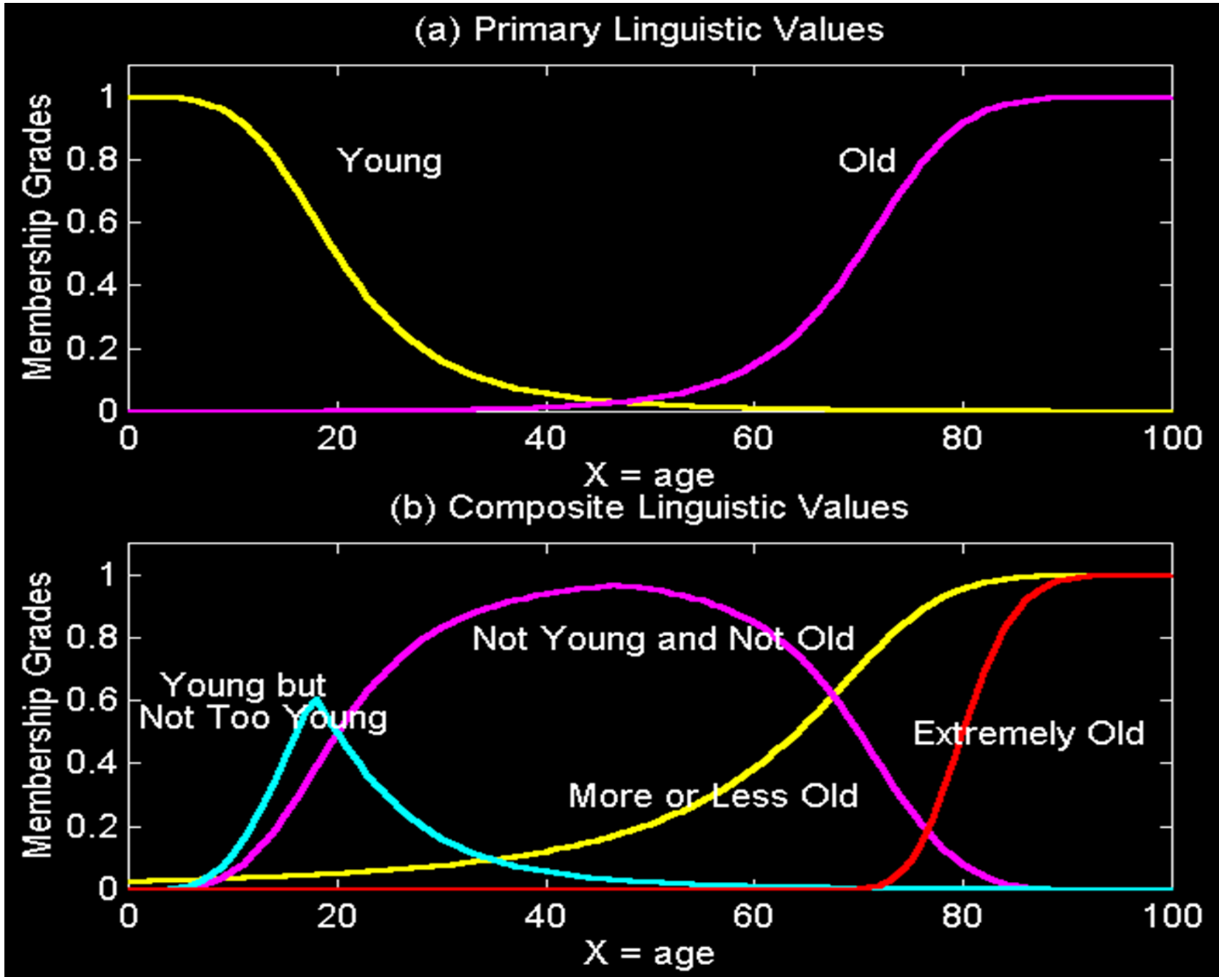
$$\int_X \left[1-\frac{1}{1+\left(\frac{x}{20}\right)^4}\right] \wedge \left[1-\frac{1}{1+\left(\frac{x-100}{30}\right)^6}\right] / x$$

- Young but not too young = young ∩ ¬young^2 (too = very) =

$$\int_x \left[\frac{1}{1+\left(\frac{x}{20}\right)^4}\right] \wedge \left[1-\left(\frac{1}{1+\left(\frac{x}{20}\right)^4}\right)^2\right] / x$$

- Extremely old ≡ very very very old = CON (CON(CON(old))) =

$$\int_x \left[\frac{1}{1+\left(\frac{x-100}{30}\right)^6}\right]^8 / x$$



(a) Primary Linguistic Values
Membership Grades
1
0.8
0.6
0.4
0.2
0
Young
Old
0
20
40
60
80
100
X = age
(b) Composite Linguistic Values
Membership Grades
1
0.8
0.6
0.4
0.2
0
Not Young and Not Old
Young but
Not Too Young
Extremely Old
More or Less Old
0
20
40
60
80
100
X = age

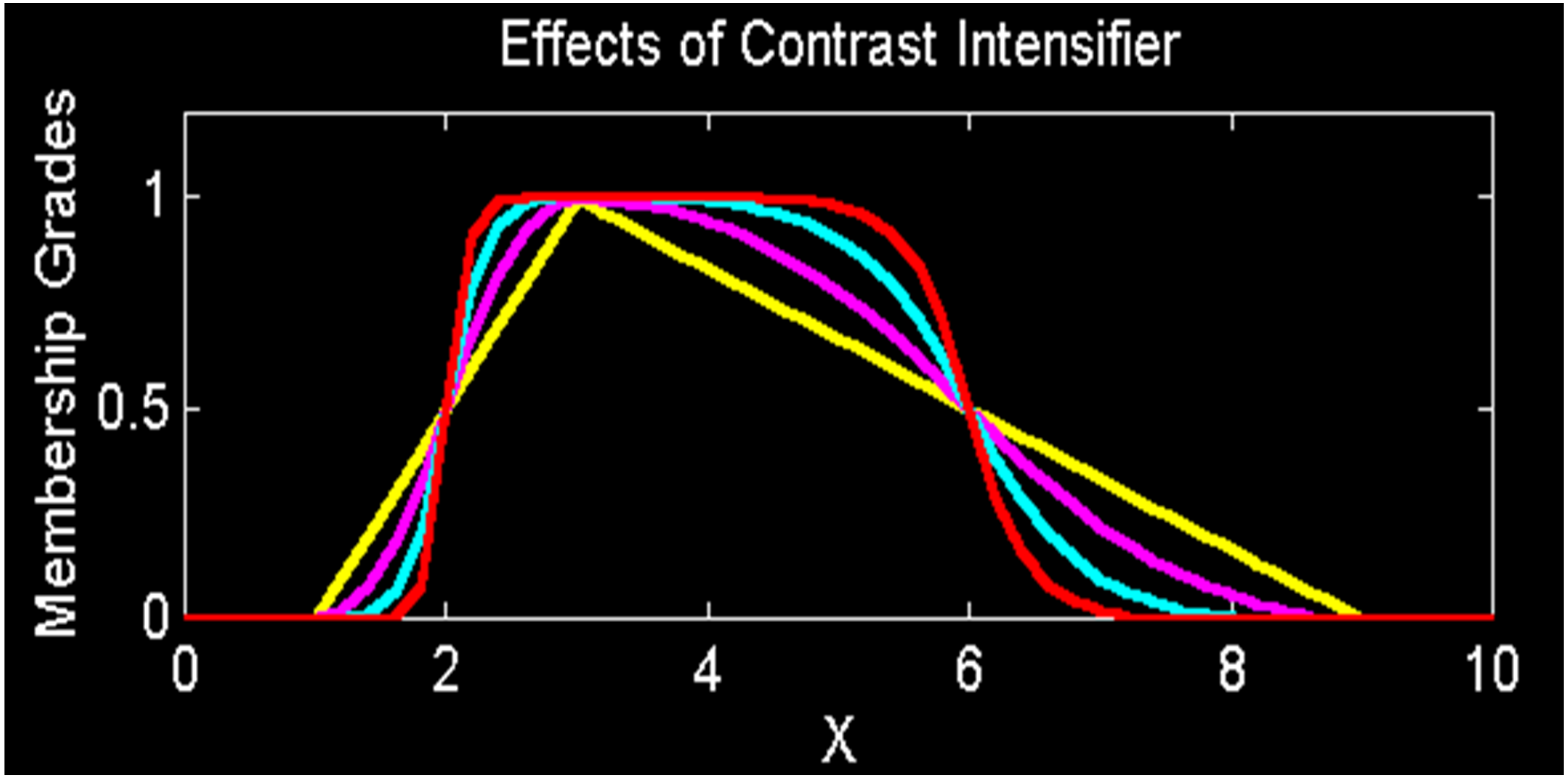
Fuzzy if-then rules (3.3) (cont.)

– Contrast intensification

the operation of contrast intensification on a linguistic value A is defined by

$$\mathrm{INT}(A) = \begin{cases} 2A^2 & \text{if } 0 \le \mu_A(x) \le 0.5 \\ \neg 2(\neg A)^2 & \text{if } 0.5 \le \mu_A(x) \le 1 \end{cases}$$

- INT increases the values of $\mu_A(x)$ which are greater than 0.5 & decreases those which are less or equal that 0.5
- Contrast intensification has effect of reducing the fuzziness of the linguistic value A


Effects of Contrast Intensifier
Membership Grades
1
0.5
0
0
2
4
6
8
10
X