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THE FUZZY VERSION OF THE FUNDAMENTAL THEOREM OF SEMIGROUP HOMOMORPHISM

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Abstract . A semigroup is an algebra structure which is more general than a group. The fuzzy version of the fundamental theorem of group homomorphism has been established by the previous researchers. In this paper we are going to establish about the fuzzy version of the fundamental theorem of semigroup homomorphism as a general form of group homomorphism. Using the properties of level subset of a fuzzy subsemigroup, we have proved that the fuzzy version of the fundamental theorem of semigroup homomorphism is held. If we have a surjective semigroup homomorphism, then the fuzzy subsemigroup quotient is isomorphic with the homomorphic image.

Keywords: Fuzzy subsemigroup, Level subset, Fuzzy subsemigroup quotient

1. Introduction

The basic concept of fuzzy sets was introduced by Zadeh since 1965 (in Aktas; 2004). This concept has developed in many areas including in algebra structure such as fuzzy subgroup, fuzzy subring, fuzzy submodule and soon .

A fuzzy subgroup is constructed by generalized a group based on the concept of an ordinary set G to a fuzzy set μ on G , i.e. a mapping $\mu : G \rightarrow [0,1]$ fulfills these axioms:

(i) for every $x, y \in G$, $\mu(xy) \geq \min \mu(x), \mu(y)$, (ii) for every $x \in G$, $\mu(x^{-1}) = \mu(x)$. Asaad (1991) has proved, if there is a surjective homomorphism group, then the quotient subgroup fuzzy is isomorphic to the homomorphic image.

Every a group is a semigroup. So we can say that a semigroup is a generalization of a group. Based on the set, we can generalize a group to a fuzzy subgroup which is defined as above. Kandasamy (2003) introduced how to generalize a semigroup to a fuzzy subsemigroup. If S is a semigroup, so a fuzzy subsemigroup is a mapping $\eta : S \rightarrow [0,1]$ such that fulfill this axiom: $\eta(xy) = \min \eta(x), \eta(y)$ for every $x, y \in S$. In this paper we are going to establish the fuzzy version of the fundamental theorem of semigroup homomorphism.

2. Theoretical Review

Let S be an ordinary non empty set, μ is called a fuzzy subset of S if μ is a mapping from S into interval $[0,1]$. Asaad (1991), Kandasamy (2003) and Mordeson & Malik (1998) have introduced how to

construct a fuzzy subgroup and a fuzzy normal subgroup. They can prove that the level subset μ_t and strong level subset $\mu_t^>$, $t \in [0, \mu(e)]$ of a fuzzy subset μ is an ordinary subgroup of G , with e is an identity element of a group G .

Ajmal (1994) defined a homomorphism of a fuzzy subgroup, establishes their properties related to the homomorphism, constructs the fuzzy quotient subgroup and proves the fundamental theorem of group homomorphism in fuzzy version.

Semigroup

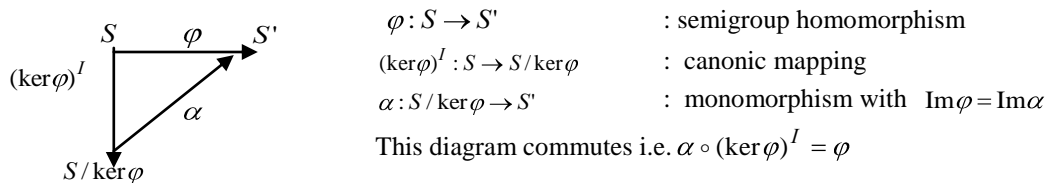
A semigroup is a non empty set with an associative binary operation is defined into this set. A semigroup S has an identity element if there is $e \in S$ such that $ex = x$ and $xe = x$. The set $A \subseteq S$ is called subsemigroup if A is a semigroup of S relative to the same binary operation which is defined in S .

Lemma 2.1. (Howie :p.7) *The non empty of the intersection of all subsemigroup S is a semigroup.*

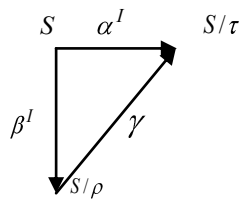
Proof:

Let S_i be a subsemigroup of S , for $i = 1, 2, 3, \dots$. Take $x, y \in \bigcap_i S_i$, so $x, y \in S_i$ for every $i = 1, 2, 3, \dots$. Since S_i is a subsemigroup of S for $i = 1, 2, 3, \dots$, then $xy \in S_i$ for every $i = 1, 2, 3, \dots$. The consequence is $xy \in \bigcap_i S_i$, hence $\bigcap_i S_i$ is a semigroup. ■

Let S, S' be semigroups. The mapping $\varphi: S \rightarrow S'$ is called a semigroup homomorphism if $\varphi(xy) = \varphi(x)\varphi(y)$ for every $x, y \in S$. The kernel of φ is defined as $\ker \varphi = \{x, y \in S \times S \mid \varphi(x) = \varphi(y)\}$. It is a congruency relation, so it divides S into congruency classes. The set of congruency classes denoted by $S/\ker \varphi$. This set is a semigroup under the following operation: $x \ker \varphi \cdot y \ker \varphi = (xy) \ker \varphi$. See the following diagram:



If φ is surjective, then $\text{Im } \varphi = S'$. The consequence is $\text{Im } \alpha = S'$, in other word α is surjective. On the other hand α is monomorphism, so α is isomorphism. If ρ and τ are congruency on S with $\rho \subseteq \tau$, so the mapping from S/ρ onto S/τ is surjective and the following diagram commutes (Howie:p.23).



Lemma 2.2. *If $\varphi: S \rightarrow S'$ is a semigroup homomorphism, then $\varphi(S)$ is a subsemigroup of S'*

Proof: Let $y, y' \in \varphi(S)$, so there is $x, x' \in S$ such that $y = \varphi(x)$ and $y' = \varphi(x')$. The next we must prove that $yy' \in \varphi(S)$. Construct $z = xx'$, so we have $z \in S$, such that: $\varphi(z) = \varphi(xx') = \varphi(x)\varphi(x') = yy'$. Since there is $z \in S$ such that $\varphi(z) = yy'$, therefore $yy' \in \varphi(S)$. Hence $\varphi(S)$ is a subsemigroup of S' . ■

2.2. Subsemigroup Fuzzy

A semigroup is the a simple algebra structure. The first we will give a definition of a fuzzy subsemigroup and the other basic definitions which refer to Asaad (1991), Kandasamy (2003), Mordeson & Malik (1998), Ajmal (1994), Shabir (2005).

Definition 2.1. Let S be a set, a fuzzy subset μ of S is a mapping from S into $[0,1]$, i.e. $\mu: S \rightarrow [0,1]$

Definition 2.2. Let μ be a fuzzy subset of S and $t \in [0,1]$, then the level subset μ_t dan the strong level subset $\mu_t^>$ of the fuzzy subset μ is defined as follow:

$$(i) \mu_t = \{x \in S \mid \mu(x) \geq t\}$$

$$(ii) \mu_t^> = \{x \in S \mid \mu(x) > t\}$$

Definition 2.3. Let S be a semigroup, then the mapping $\mu: S \rightarrow [0,1]$ is called a fuzzy semigroup if $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ for every $x, y \in S$

3. Results

As the initial result, we give the property of a fuzzy subsemigroup as follow:

Proposition 3.1. Let S be a semigroup and μ be a fuzzy subset of S . The fuzzy subset μ is a fuzzy subsemigroup if and only if for every non empty level subset is a subsemigroup of S

Proof:

(\Rightarrow)

Let $x, y \in \mu_t$, so $\mu(x) \geq t$ and $\mu(y) \geq t$. We know that $xy \in \mu_t$ if and only if $\mu(xy) \geq t$. On the other hand, $\mu(xy) \geq \min\{\mu(x), \mu(y)\} \geq \min\{t, t\} = t$. Hence $\mu(xy) \geq t$ or $xy \in \mu_t$

(\Leftarrow)

It is known that μ_t is a subsemigroup of S , so for every $\mu(x) \geq t$ and $\mu(y) \geq t$, so $\mu(xy) \geq t$. The consequence is: $\mu(xy) \geq t = \min\{\mu(x), \mu(y)\}$ ■

Proposition 3.2. Let S be a semigroup and μ be a fuzzy subset of S . The fuzzy subset μ is a fuzzy subsemigroup if and only if for every non empty strong level subset $\mu_t^>$ is a subsemigroup of S .

Proof:

(\Rightarrow)

Take $x, y \in \mu_t^>$, so $\mu(x) > t$ and $\mu(y) > t$. We know that $xy \in \mu_t^>$ if and only if $\mu(xy) > t$. We have: $\mu(xy) \geq \min\{\mu(x), \mu(y)\} > \min\{t, t\} = t$. Hence, we have $\mu(xy) > t$ or $xy \in \mu_t^>$

(\Leftarrow)

The strong level subset $\mu_t^>$ is a subsemigroup of S , so for every $\mu(x) > t$ and $\mu(y) > t$, we have $\mu(xy) > t$. The consequence is: $\mu(xy) \geq t = \min \{ \mu(x), \mu(y) \}$. ■

Definition 3.1. Let f be a mapping from a semigroup S to semigroup S' , $\mu: S \rightarrow [0,1]$ and $\eta: S' \rightarrow [0,1]$ be fuzzy subsets of S and S' , respectively. The Image homomorphic, denoted by $f(\mu)$, is defined as follow:

$$f(\mu)(y) = \sup_{x \in f^{-1}(y)} \mu(x). \text{ The pre-image } f^{-1}(\eta) \text{ is defined as } f^{-1}(\eta)(x) = \eta(f(x)).$$

If f is an isomorphism, so $f(\mu)$ is called isomorphic to μ . A fuzzy subsemigroup μ is called isomorphic to η if there is an isomorphism from S onto S' , or vice versa, such that $\mu = f(\eta)$ and $\eta = f(\mu)$.

Proposition 3.3. If f is a mapping from a semigroup S to a semigroup S' and μ is a fuzzy subsemigroup of S , then $f(\mu)_t = \bigcap_{t > \varepsilon > 0} f(\mu_{t-\varepsilon})$, for $t \in \mathbb{Q}_1^-$.

Proof:

We remember : $f(\mu)_t = \{ x \in S' \mid f(\mu)(y) \geq t \}$ and $f(\mu_{t-\varepsilon}) = \{ x \in S \mid \mu(x) \geq t-\varepsilon \}$, so $\mu_t \subseteq \mu_{t-\varepsilon}$.

Let $y = f(x)$ be in S' which fulfill $y \in f(\mu)_t$. Then the consequence is $f(\mu)f(x) = \sup_{x \in f^{-1}(f(x))} \mu(x) \geq t$. So,

for $\varepsilon > 0$, we have $y = f(x_0) \in f(\mu_{t-\varepsilon})$. Hence, $y \in f(\mu_{t-\varepsilon})$.

Finally, we have $f(\mu)_t \subseteq \bigcap_{t > \varepsilon > 0} f(\mu_{t-\varepsilon})$ (1)

Take $y \in \bigcap_{t > \varepsilon > 0} f(\mu_{t-\varepsilon})$, so $y \in f(\mu_{t-\varepsilon})$ for every $\varepsilon > 0$. The consequence, there is $x_0 \in \mu_{t-\varepsilon}$ such that

$y = f(x_0)$. So, for every $\varepsilon > 0$ there is $x_0 \in f^{-1}(y)$ with $\mu(x_0) \geq t-\varepsilon$ such that :

$$f(\mu)(y) = \sup_{x_i \in f^{-1}(y)} \mu(x_i) \geq \sup_{t > \varepsilon > 0} \{ t-\varepsilon \} = t$$

Hence $f(\mu)(y) \geq t$ or $y \in \mu_t$. Based on this fact, we have : $\bigcap_{t > \varepsilon > 0} f(\mu_{t-\varepsilon}) \subseteq f(\mu)_t$ (2)

From equation (1) and (2), we get $f(\mu)_t = \bigcap_{t > \varepsilon > 0} f(\mu_{t-\varepsilon})$ ■

Proposition 3.4. Let f be a semigroup homomorphism from S into S' and μ be a fuzzy subsemigroup of S , then $f(\mu)$ is a fuzzy subsemigroup of S' .

Proof:

Method I: We must prove that $f(\mu)_t$ is a subsemigroup of S' . For $t = 0$, then

$f(\mu)_t = f(\mu)_0 = \{ x \in S' \mid f(\mu)(y) \geq 0 \} = S'$, so $f(\mu)_t$ is a semigroup. For $t \in \mathbb{Q}_1^-$, $f(\mu)_t = \bigcap_{t > \varepsilon > 0} f(\mu_{t-\varepsilon})$. We know

that for $\varepsilon > 0$, $\mu_{t-\varepsilon}$ is a subsemigroup of S , since μ is a fuzzy subsemigroup of S . And we know f is a homomorphism, so its image homomorphic, denoted by $f(\mu_{t-\varepsilon})$, is a subsemigroup of S' . Hence, the intersection of all $f(\mu_{t-\varepsilon})$ is a subsemigroup. The consequence $f(\mu)_t$ is a subsemigroup of S' .

Method II:

It is known that μ is a fuzzy subsemigroup of S , so μ_t is a subsemigroup of S . The next we must prove that $f(\mu)_t$ is a subsemigroup of S' :

Take $a, b \in f(\mu)_t$, so $f(\mu)(a) \geq t$ and $f(\mu)(b) \geq t$. We get $\sup_{x \in f^{-1}(a)} \mu(x) \geq t$ dan $\sup_{x' \in f^{-1}(b)} \mu(x') \geq t$. The next step,

we must prove $ab \in f(\mu)_t$, i.e. $\sup_{x'' \in f^{-1}(ab)} \mu(x'') \geq t$:

If $x \in f^{-1}(a)$, then $f(x) = a$ and if $x' \in f^{-1}(b)$, then $f(x') = b$. The next we get $ab = f(x)f(x') = f(xx')$.

Construct $x'' = xx'$, so $x'' \in f^{-1}(ab)$ and we get :

$$\sup_{x'' \in f^{-1}(ab)} \mu(x'') = \sup_{xx' \in f^{-1}(ab)} \mu(xx') \geq \sup_{xx' \in f^{-1}(ab)} \min \{ \mu(x), \mu(x') \} \geq t$$

Hence $ab \in f(\mu)_t$, or in the other word $f(\mu)_t$ is a subsemigroup. It means that $f(\mu)$ is a fuzzy subsemigroup of S' ■

Corrolary 3.1. Let f be a homomorphism from semigroup S into semigroup S' . If η is a fuzzy subsemigroup of S' , then $f^{-1}(\eta)$ is a fuzzy subsemigroup of S

Proposition 3.5. Let $f: S \rightarrow S'$ be a semigroup homomorphism and $\eta: S' \rightarrow [0,1]$ be a fuzzy subsemigroup, then $f^{-1}(\eta_t) = f^{-1}(\eta)_t$

Proof :

We have $f^{-1}(\eta_t) = \{x \in S \mid \eta(f(x)) \geq t\} = \{x \in S \mid \eta(f^{-1}(y)) \geq t\}$. For any $y \in S'$ there is $x \in S$ such that $x = f^{-1}(y)$, with $\eta(f^{-1}(y)) \geq t$, so we get $f^{-1}(\eta_t) = \{x \in S \mid \eta(f^{-1}(y)) \geq t\} = f^{-1}(\eta)_t$ ■

Proposition 3.6. If f is a semigroup homomorphism from S to S' and μ is a fuzzy subsemigroup of S , then f is a homomorphism from μ_t to $f(\mu)_t$

Proof:

We know that μ_t is a subsemigroup of S and $f(\mu)_t$ is a subsemigroup of S' . So we get:

$$f|_{\mu_t} : \mu_t \rightarrow f(\mu)_t \text{ is a homomorphism.}$$
■

Proposition 3.7. If $f: S \rightarrow S'$ is an epimorphism semigroup and μ is a fuzzy subsemigroup of S , then the mapping $\alpha: \mu_t \rightarrow f(\mu)_t$ is an epimorphism.

Proof:

Based on Proposition 3.6, so α is a homomorphism which is induced by f . The next, we must prove that $\alpha: \mu_t \rightarrow f(\mu)_t$ is surjective. Take $y \in f(\mu)_t$, so we have $f(\mu)(y) \geq t$ or $\sup_{x \in f^{-1}(y)} \mu(x) \geq t$. We know that

$\mu(x) \geq t$, with $y = f(x)$. Since f is surjective, so for every $y \in S'$ has an element in pre image S . Since $f(\mu)_t = f(\mu_t) \subseteq S'$, so for every $y \in f(\mu)_t$, there is $x \in S$ and $\mu(x) \geq t$ such that $\alpha(x) = \sup_{x \in f^{-1}(y)} \mu(x) \geq f(\mu)(y)$

3.1. Fuzzy Quotient Subsemigroup

If we discuss about semigroup, we always talk about the quotient semigroup i.e. if we have a homomorphism $\varphi: S \rightarrow S'$ with kernel K , then we have a set S/K , $S/K = \{xK \mid x \in S\}$. This set is a semigroup with respect to a binary operation which is defined by $xK \cdot yK = xyK$. This semigroup is called a quotient semigroup. The following proposition talks about fuzzy quotient subsemigroup.

Proposition 3.8. Let $f: S \rightarrow S'$ be a semigroup homomorphism with kernel K and μ be a fuzzy subsemigroup of S . A mapping $\mu|_K: S/K \rightarrow [0,1]$ is defined as $\mu|_K(xK) = \sup_{k \in K} \{\mu(xk)\}$, is a fuzzy subsemigroup

Proof:

a. $\mu|_K$ is a mapping:

Let $xK, yK \in S/K$ with $xK = yK$. If the consequence is $\mu|_K(xK) > \mu|_K(yK)$, so we have :

$\sup_{k \in K} \{\mu(xk)\} > \sup_{k \in K} \{\mu(yk)\}$. So, there is $k_0 \in K$, such that $\mu(xk_0) > \mu(yk)$ for every $k \in K$. Finally, for every

$k \in K$, $yk \notin \mu_t$ if $t = \mu(xk_0)$. The consequence is $\mu_t \cap yK = \emptyset$. But it is known that $xk_0 \in \mu_t$ and $xk_0 \in xK$. It means that $xk_0 \in yK$. It contradicts with the fact that $\mu_t \cap yK = \emptyset$. So we have

$$\mu|_K(xK) = \mu|_K(yK)$$

b. $\mu|_K$ is a fuzzy subsemigroup

Take $xK, yK \in S/K$, so we have :

$$\alpha_K(xK \cdot yK) = \mu|_K(xyK) = \sup_{k \in K} \{xyk\} = \sup_{k \in K} \{\mu(xkyk)\} \geq \sup_{k \in K} \left[\min \left\{ \mu(xk), \mu(yk) \right\} \right]$$

$$\geq \min \left(\sup_{k \in K} \{\mu(xk)\}, \sup_{k \in K} \{\mu(yk)\} \right) = \min \left[\mu|_K(xK), \mu|_K(yK) \right]$$

■

This semigroup $\mu|_K$ is called fuzzy quotient subsemigroup.

Proposition 3.9. Let $f: S \rightarrow S'$ be a semigroup homomorphism with kernel K and μ be a fuzzy subsemigroup of S , then $\mu|_K \circ \varphi = \mu_t / K$

Proof:

Take $xK \in \mu|_K \circ \varphi$, so $\mu|_K(xK) \geq t$ or $\sup_{k \in K} \mu(xk) \geq t$. As the consequence, there is $k_0 \in K$ such that $\mu(xk_0) \geq t$, so

we have $xk_0 \in \mu_t$. The next, if we multiply it by K , the result is $xk_0K \in \mu_t / K$. Since $k_0K = K$, then $xK \in \mu_t / K$.

By reverses this proof, we get the proof of this proposition completely.

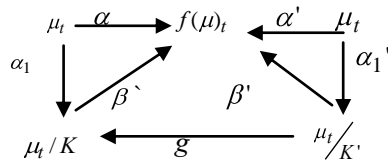
■

Proposition 3.10. Let $f: S \rightarrow S'$ be a semigroup homomorphism with kernel K and μ be a fuzzy subsemigroup of S and $\alpha: \mu_t \rightarrow f(\mu)_t$. Then $\mu_t / K \cong \mu_t / \mu_t \cap K$

Proof:

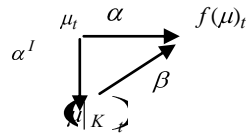
See the following diagrams:

In the following diagram, $\mu_t \cap K$ denoted by K'



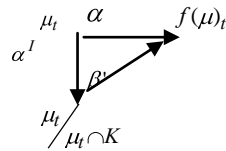
By the fundamental theorem of homomorphism semigroup, we obtain β and β' are monomorphism. We know that K, K' are congruencies, with $K' \subset K$. Therefore g is surjective. The next, we prove that g is injective. It is easy to verify that $\beta \circ g = \beta'$. The first, take $g(xK') = g(x'K')$. Since β is a mapping, so we get $\beta \circ g(xK') = \beta \circ g(x'K')$. Since $\beta \circ g = \beta'$, so we obtain $\beta'(xK') = \beta'(x'K')$. Finally, we have $xK' = x'K'$ since β' is monomorphism. Hence g is injective and $\mu_t/K \cong \mu_t/(\mu_t \cap K)$ ■

Theorem 3.1. Let $f : S \rightarrow S'$ be a semigroup homomorphism with kernel K and μ be a fuzzy subsemigroup of S , then for every $t \in [0,1]$ the following diagram commutes and the mapping $\beta : \mu_t/K \rightarrow f(\mu)_t$ is monomorphism with $\text{Im}\alpha = \text{Im}\beta$



Proof:

See the following diagram:



The kernel of α is $\mu_t \cap K$. Based on the fundamental theorem of semigroup homomorphism, so β' is monomorphism with $\text{Im}\alpha = \text{Im}\beta'$. Refer to the Proposition 3.10, we obtain $\mu_t/K \cong \mu_t/(\mu_t \cap K)$. The consequence, β is monomorphism with $\text{Im}\alpha = \text{Im}\beta$. ■

Corollary 3.2. Let $f : S \rightarrow S'$ be a semigroup epimorphism with kernel K , and μ be a fuzzy subsemigroup of S , then for every $t \in [0,1]$ the set μ_t/K is isomorphic with $f(\mu)_t$

Proof:

Using Theorem 3.1 and Proposition 3.7, we get a mapping $\beta : \mu_t/K \rightarrow f(\mu)_t$ is a monomorphism with $\text{Im}\alpha = \text{Im}\beta$ and $\beta \circ \alpha^I = \alpha$. Since f epimorphism, then based on Proposition 3.7 α is epimorphism. As the consequence, β is an isomorphism. Finally, μ_t/K is isomorphic with $f(\mu)_t$. ■

Theorem 3.2. Let $f: S \rightarrow S'$ be a semigroup epimorphism with kernel K and μ be fuzzy subsemigroup of S , then for every $t \in [0,1]$ the set $\mu|_K$ is isomorphic with $f(\mu)$.

Proof:

Using Corollary 3.2 we obtain an isomorphism $\beta: \mu|_K \rightarrow f(\mu)$. The next we prove that $f(\mu) = \beta(\mu|_K)$:

$$f(\mu)(y) = \sup_{x \in f^{-1}(y)} \mu(x) \quad (3.1)$$

For $x \in f^{-1}(y)$, then $y = f(x)$ for any $x \in S$. We can see that $y = \beta(xK)$ for any $xK \in \mu|_K$. In the other word, $xK \in \beta^{-1}(y)$. Substitute $xK \in \beta^{-1}(y)$ to the equation (3.1), we obtain:

$$f(\mu)(y) = \sup_{x \in f^{-1}(y)} \mu(x) = \sup_{xK \in \beta^{-1}(y)} \mu(xK) = \beta(\mu|_K)(y)$$

So, we get $f(\mu) = \beta(\mu|_K)$ ■

Conclusion

Based on our discussion, we can prove that the fundamental theorem of the fuzzy subsemigroup homomorphism is hold. The side result is: if $f: S \rightarrow S'$ is an epimorphism semigroup with kernel K and μ is a fuzzy subsemigroup of S , then for every $t \in [0,1]$, the set $\mu|_K$ is isomorphic with $f(\mu)$.

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