The Properties of Left Simple and Right Simple Ordered Bilinear Form Semigroups

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Abstract

Research and development of Bilinear Form Semigroups have been introduced by Rajendran and Nambooripad. This research has been developed by Karyati and Wahyuni. The characteristics of the Fuzzy Bilinear Form Subsemigroup also has been developed by Karyati, at al. Many topics of research have been done by Karyati, at al. These are about fuzzy ideals, fuzzy relations, fuzzy congruence, fuzzy Green relation on Bilinear form semigroups.

Inspired by the paper which is written by Kehayopulu and Tsengelis, who have studied about partial ordered semigroup, the aim of this research is to find the characteristics of the partial ordered bilinear form semigroup in term of their fuzzy right and fuzzy left ideals. We obtain some characteristics of the partial ordered bilinear form semigroup, i.e: A bilinear form semigroup $S(B)$ is a left simple if and only if $(S(B)a] = S(B)$, for every $\tilde{a} \in S(B)$. A bilinear form semigroup $S(B)$ is a right simple if and only if $(aS(B)] = S(B)$, for every $\tilde{a} \in S(B)$. An ordered bilinear form semigroup is a left and right simple if and only if every fuzzy quasi-ideal of the bilinear form semigroup is a constant function. An ordered bilinear form semigroup $S(B)$ is a completely regular if and only if for every quasi-ideal fuzzy α of a semigroup $S(B)$ we have $\alpha(\tilde{a}) = \alpha(\tilde{a}^2)$ for every $\tilde{a} \in S(B)$.

Keywords: *partial ordered semigroup, bilinear form semigroup, fuzzy quasi-ideal, left simple, right simple, completely regular.*

1 Introduction and Prerequisites

Theory of fuzzy subset has been established by Zadeh. The developing of this theory has been done by many researchers. Rosenfeld has been developed this theory to the fuzzy subgroupoid theory. Zimmerman[20] has consider the application of the fuzzy subsets. Mordeson & Malik[16] has developed the fuzzy subset theory on fuzzy semigroup. Karyati, at al[9] have been developed

the theory of Subsemigroup fuzzy into the special semigroup called fuzzy bilinear form subsemigroup. The new theory has been establish, i.e.: the characteristics of fuzzy right/left ideal, the fuzzy principle ideal, fuzzy relation and Green relation on bilinear form semigroups. Kehayopulu, at al[13] have established the theory of the partial ordered semigroup and groupoid.

A semigroup $(S, .)$ with a *partial order* operation $' \leq '$, such that $(S, \leq)$ is a partial ordered set (poset) and for every $x, y, z \in S$, with $x \leq y$, we have $zx \leq zy$ and $xz \leq yz$, then $(S, ., \leq)$ is called partial ordered semigroup. Many researchers have reseach about this topic. Defining a partial order into a semigroup has many consequences. These are related to the defining of (right/left) ideal, right/left) quasi-ideal, fuzzy (right/left) ideal and fuzzy (right/left) quasi-ideal. Based on these definitions, we can develope to get the new theories related to the partial ordered semigroups. In this paper, we will find the characteristics of the partial bilinear form semigroup in term their right and left ideals.

2. Theoretical Review

On this section, we give many definitions, theorems, lemmas, propositions and corollaries to support this research.

2.1 Partial ordered Semigroup ($po\_semigrup$)

A semigroup is an algebra structure with an associative binary operation.

Definition 1. *Let S be a non empty set. The set S with a binary operation $'.'$ is called a semigroup if:*

i. $(\forall x, y \in S) \ x.y \in S$
ii. $(\forall x, y, z \in S) \ (x.y).z = x.(y.z)$

Let S be a semigroup and $a \in S$. The element a is called a regular element if there exist $a' \in S$ such that $a = aa'a$. A semigroup S is called a regular semigroup if and only if every element of S is a regular element.

The following definition give a definition of the partial partial ordered.

Definition 2. *A non empty set P is called partial ordered $' \leq '$ if and only if:*

i. *Reflective : $(\forall x \in P) x \leq x$*
ii. *Anti symmetry : $(\forall x, y \in P) \ x \leq y$ and $y \leq x \implies x = y$*
iii. *Transitive : $(\forall x, y, z \in P) \ x \leq y$ and $y \leq z \implies y \leq z$*

The partial partial ordered set is called poset. The following definition give a definition about a partial ordered semigroup:

Definition 3 . Let S be a non empty set. The set S with a binary operation $'.'$ and a partial ordered $'\leq'$ is called a partial ordered semigroup if and only if:

i. $(S,.)$ is a semigroup
ii. $(S,\leq)$ is a partial ordered set
iii. $(\forall a,b,x \in S) a \leq b \Longrightarrow xa \leq xb$ and $ax \leq bx$

Definition 4. Let $(S,.,\leq)$ be a partial ordered semigroup. Then a non empty subset I is called an ideal of a semigroup S if :

i. $(\forall a \in S)(\forall b \in I) a \leq b \Longrightarrow a \in I$
ii. $IS \subseteq I$ and $SI \subseteq I$

2.2. Bilinear Form Semigroups

A bilinear form semigroup is a special semigroup. We give the following theory how to construct a bilinear form semigroup. Let $\mathcal{L}(X)$ and $\mathcal{L}(Y)$ be a set of all linear operator X and Y, respectively. If $f \in \mathcal{L}(X)$, then we get a vector subspace of X:

$N(f) = \{u \in X \mid f(u) = 0\}$ and $R(f) = \{v \in X \mid f(x) = v,$ for any $x \in X \}$

An element $f \in \mathcal{L}(X)$ is called an adjoin pair with $g \in \mathcal{L}(Y)$ with respect to the bilinear form B, and vice versa, if and only if $B(x,g(y)) = B(f(x),y)$ for every $x \in X$ and $y \in Y$. The next, we will denote the following sets:

$$\mathcal{L}'(X) = \{f \in \mathcal{L}(X) \mid N(B_*) \subseteq N(f),\ R(f) \cap N(B_*) = \{0\}\}$$
$$\mathcal{L}'(Y) = \{g \in \mathcal{L}(Y) \mid N(B^*) \subseteq N(g),\ R(g) \cap N(B^*) = \{0\}\}$$
$$S(B) = \{(f,g) \in \mathcal{L}'(X) \times \mathcal{L}'(Y)^{op} \mid (f,g)\ \text{an adjoin pair}\}$$

Karyati at al, (2002) have proved that the set $S(B)$ is a semigroup with respect to the binary operation which is defined as $(f,g)(f',g') = (ff',g'g)$, [4]. This semigroup $S(B)$ is called a bilinear form semigroup.

The properties of this semigroup has been establish by Rajendran & Nambboripad, [18]. Based on this properties, Karyati at al, [5], [6], [7], [8], [9], [10], [11] have developed this theory included the fuzzy version.

2.3. Fuzzy Subsemigroups

Refer to the papers which are written by Asaad [1], Kandasamy [3], Mordeson & Malik [16], Shabir [19], we have a definition of a fuzzy subset α of a semigroup S is a mapping from S into $[0,1]$,i.e. $\alpha: S \to [0,1]$.

Definition 5. Let S be a semigroup. A mapping $\alpha: S \to [0,1]$ is called a fuzzy subsemigroup if and only if $\alpha(xy) \geq \min \{\alpha(x),\alpha(y)\}$ for every $x,y \in S$.

Definition 6. [15] Let α be a fuzzy subsemigrup of a semigroup S. Then:
(i) α is a fuzzy left ideal if $(\forall x,y \in S) \alpha(xy) \geq \alpha(y)$

(ii) α is a fuzzy rigth idea lif $(\forall x, y \in S)\alpha(xy) \geq \alpha(x)$
(iii) α is a fuzzy ideal if α is a fuzzy left ideal and a fuzzy right ideal, i.e.:
$(\forall x, y \in S) \, \alpha(xy) \geq max\{\alpha(x), \alpha(y)\}$

Let S be a partial ordered semigroup. Then the definition of a fuzzy left ideal, fuzzy right ideal and fuzzy ideal (two sided) of S are defined as follow:
Definition 7. [15] *Let $(S, ., \leq)$ be a partial ordered semigroup . Then a fuzzy suset α of the partial ordered semigroup S is called fuzzy left ideal if :*
i. $(\forall x, y \in S) \, \alpha(xy) \geq \alpha(y)$
ii. $(\forall x, y \in S) \, x \leq y \Longrightarrow \alpha(x) \geq \alpha(y)$

Definition 8. [15] *Let $(S, ., \leq)$ be a partial ordered semigroup . Then a fuzzy subset α of the partial ordered semigroup S is called fuzzy right ideal if :*
i. $(\forall x, y \in S) \, \alpha(xy) \geq \alpha(x)$
ii. $(\forall x, y \in S) \, x \leq y \Longrightarrow \alpha(x) \geq \alpha(y)$

2.4. Partial Ordered Bilinear Form Semigroup in Term of The Fuzzy Subset

Based on the paper written by Calais [12], one of the characteristics of a regular semigroup S: A semigroup S is a regular semigroup if and only if the right and left ideals of S are idempotent. Iseki [12] proved that a semigroup S is regular if and only if for every right ideal A and every left ideal B, $A \cap B = AB$. As a consequence, if S is a commutative semigroup then S is a regular semigroup if and only if every ideal of S is idempotent.

In this paper, $\langle a \rangle_R$ and $\langle a \rangle_L$ denote a right ideal and a left ideal of S generated by $a \in S$, respectively. We always have $\langle a \rangle_R = \{a\} \cup \{aS\} = (a \cup aS]$ and $\langle a \rangle_L = \{a\} \cup \{Sa\} = (a \cup Sa]$. The partial ordered semigruop $(S, \leq)$ is called regular if and only if for every $a \in S$ there exist $x \in S$ such that $a \leq axa$. If $A \subseteq S$, then we denote $(A] = \{t \in S | t \leq h \text{ for any } h \in A\}$. Based on this notation, so we have $A \subseteq (A]$. If $A \subseteq B$, then $(A] \subseteq (B]$, $(A](B] \subseteq (AB]$ and $\big((A]\big] = (A]$. A fuzzy subset of a semigroup S is defined as a mapping $\alpha : S \longrightarrow [0,1]$. For a fuzzy subset α of a partial ordered semigroup $(S, \leq)$, we denote $A_a = \{(y, z) \in S \times S | a \leq yz\}$. Let α, β be fuzzy subsets of a semigroup S. Then $\alpha \leqslant \beta$ if and only if $\alpha(x) \leq \beta(x)$ for all $x \in S$. For two fuzzy subsets α and β of a semigroup S, we define:

$$(\alpha \circ \beta)(a) = \begin{cases} \bigvee_{(y,z) \in A_a} min\{\alpha(y), \beta(z)\}, & A_a \neq \emptyset \\ 0, & A_a = \emptyset \end{cases}$$

We denote by $F(S)$ the set of all fuzzy set of all fuzzy subsets of S. On $F(S)$ we defined other binary operation $\preccurlyeq$ defined as follow:

For every $\alpha, \beta \in F(S)$, $\alpha \preccurlyeq \beta$ if and only if $\alpha(a) \leq \beta(a)$, for every $a \in S$. The set $F(S)$ is a partial ordered set with respect to the operation '$\preccurlyeq$'.

The following propositions will be developed to establish many characteristics of the partial ordered bilinear form semigroups.

Proposition 1. [11] *If* $(S(B), \leq)$ *is a partial ordered groupoid and* $\alpha_1, \alpha_2, \beta_1, \beta_2$ *are fuzzy subsets of* $S(B)$ *such that* $\alpha_1 \preccurlyeq \beta_1$ *and* $\alpha_2 \preccurlyeq \beta_2$, *then* $\alpha_1 \circ \alpha_2 \preccurlyeq \beta_1 \circ \beta_2$.

By Proposition 1, the set $F(S)$ of all fuzzy subsets of S endowed with the multiplication "$\circ$" and the order "$\preccurlyeq$" is a partial ordered groupoid.

Lemma 1.[11] *A partial ordered bilinear form semigruop* $(S(B), \leq)$ *is regular if and only if* $\langle \tilde{a} \rangle_R \cap \langle \tilde{a} \rangle_L = (\langle \tilde{a} \rangle_R \langle \tilde{a} \rangle_L]$, *for every* $\tilde{a} \in S(B)$

Let $(S(B), \leq)$ be an partial ordered bilinear form semigroup which have a unity element and $A \subseteq S(B)$. Then a fuzzy subset C_A of $S(B)$ is a characteristics function of A defined by:

$$C_A: S(B) \longrightarrow [0,1]$$
$$C_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

A fuzzy subset α of a semigroup S is called a fuzzy right ideal if: i) $\alpha(xy) \geq \alpha(x)$, for every $x, y \in S$, ii) If $x \leq y$, maka $\alpha(x) \geq \alpha(y)$. A fuzzy subset α of a semigroup S is called a fuzzy left ideal of a semigroup S if: i) $\alpha(xy) \geq \alpha(y)$ for every $x, y \in S$, ii) If $x \leq y$, then $\alpha(x) \geq \alpha(y)$. A fuzzy subset α of a semigroup S is called fuzzy ideal (two sided) of S if α is a fuzzy right and left ideal of S. This is is equivalence with α is a fuzzy ideal (two sided) of a semigroup S if and only if: i) $\alpha(xy) \geq \alpha(x)$, for every $x, y \in S$, ii) If $x \leq y$, then $\alpha(x) \geq \alpha(y)$.

Lemma 2.[6] *Let S be a semigroup with an identity (unit) element. Then a non empty subset L of a semigroup S is a left ideal of a semigroup S if and only if the characteristics function C_L is a fuzzy left ideal of S.*

Lemma 3.[6] *Let S be a semigroup with an identity element . Then a non empty subset R of a semigroup is a right ideal of S if and only if the characteristics fungtion C_R is a right ideal of S.*

Proposition 2. [11] *Let $(S(B), \leq)$ be a partial ordered bilinear form semigroup with a unit element. If α is a right of $S(B)$ and β is a fuzzy left ideal of $S(B)$, then $\alpha \circ \beta \leqslant \alpha \wedge \beta$.*

1. Main Results

Based on Proposition 2, we can weak the condition for β become a fuzzy subset and without a unit element. Then we get the following proposition:

Proposition 3. *Let $(S(B), \leq)$ be a partial ordered bilinear form semigroup. Then for every fuzzy right ideal α and every fuzzy subset β of $S(B)$, we have $\alpha \wedge \beta \leqslant \alpha \circ \beta$.*

Proof. Let α be a fuzzy right ideal and β be a fuzzy left ideal of $S(B)$. Then we must prove that $(\alpha \wedge \beta)(\tilde{a}) \leqslant (\alpha \circ \beta)(\tilde{a})$, for every $\tilde{a} \in S(B)$. Since $S(B)$ is a regular, there exist $\tilde{x} \in S(B)$ such that $\tilde{a} \leq \tilde{a}\tilde{x}\tilde{a} = (\tilde{a}\tilde{x})\tilde{a}$. Then $(\tilde{a}\tilde{x}, \tilde{a}) \in A_{\tilde{a}}$. Since $A_{\tilde{a}} \neq \emptyset$, we have:
$$(\alpha \circ \beta)(\tilde{a}) = \bigvee_{(\tilde{y},\tilde{z}) \in A_{\tilde{a}}} \min\{\alpha(\tilde{y}), \beta(\tilde{z})\}$$
Besides $(\alpha \wedge \beta)(\tilde{a}) = \min\{\alpha(\tilde{a}), \beta(\tilde{a})\}$. Since α is a fuzzy right ideal of $S(B)$, we have: $\alpha(\tilde{a}\tilde{x}) \geq \alpha(\tilde{a})$. Then $\min\{\alpha(\tilde{a}\tilde{x}), \beta(\tilde{a})\} \geq \min\{\alpha(\tilde{a}), \beta(\tilde{a})\}$. Thus we have: $(\alpha \wedge \beta)(\tilde{a}) \leq \min\{\alpha(\tilde{a}\tilde{x}), \beta(\tilde{a})\}$. Since $(\tilde{a}\tilde{x}, \tilde{a}) \in A_{\tilde{a}}$, we have:
$$\min\{\alpha(\tilde{a}\tilde{x}), \beta(\tilde{a})\} \leq \bigvee_{(\tilde{y},\tilde{z}) \in A_{\tilde{a}}} \min\{\alpha(\tilde{y}), \beta(\tilde{z})\}$$
Hence we have:
$(\alpha \circ \beta)(\tilde{a}) = \bigvee_{(\tilde{y},\tilde{z}) \in A_{\tilde{a}}} \min\{\alpha(\tilde{y}), \beta(\tilde{z})\} \geq \min\{\alpha(\tilde{a}\tilde{x}), \beta(\tilde{a})\} \geq (\alpha \wedge \beta)(\tilde{a})$.
Therefore $\alpha \wedge \beta \leqslant \alpha \circ \beta$.

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Proposition 4. *Let $(S(B), \leq)$ be a partial ordered bilinear form semigroup. Then for every fuzzy subset α and every fuzzy left ideal β of $S(B)$, we have $\alpha \wedge \beta \leqslant \alpha \circ \beta$.*

Proof. The proof of this proposition is similar with the proof of the previous proposition.

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Theorem 1. *A partial ordered bilinear form semigroup $(S(B), \leq)$ is regular if and only if for every fuzzy right ideal α and every fuzzy left ideal β of $(S(B), \leq)$, we have :*

$$\alpha \wedge \beta \leqslant \alpha \circ \beta, \text{ equivalently, } \alpha \wedge \beta = \alpha \circ \beta$$

Proof.
($\Longrightarrow$)
Let $(S(B), \leq)$ be a regular semigroup, α be a fuzzy right ideal and β be a fuzzy left ideal of $S(B)$. Based on Proposition 3, we have $\alpha \wedge \beta \leqslant \alpha \circ \beta$. On the other hand, based on Proposition 2, we have $\alpha \circ \beta \leqslant \alpha \wedge \beta$. Then we have $\alpha \wedge \beta = \alpha \circ \beta$.

($\Longleftarrow$)
Suppose $\alpha \wedge \beta \leqslant \alpha \circ \beta$ for every fuzzy right ideal α and every fuzzy left ideal β of $(S(B), \leq)$. Based on Lemma 1, we have:
$$\langle \tilde{a} \rangle_R \cap \langle \tilde{a} \rangle_L \subseteq (\langle \tilde{a} \rangle_R \langle \tilde{a} \rangle_L], \forall \tilde{a} \in S(B)$$
Let $\tilde{a} \in S(B)$, $\tilde{b} \in \langle \tilde{a} \rangle_R \cap \langle \tilde{a} \rangle_L$. Then $\tilde{b} \in (\langle \tilde{a} \rangle_R \langle \tilde{a} \rangle_L]$. Since $\langle \tilde{a} \rangle_R$ is a right ideal of $S(B)$, by Lemma 3, the characteristics function $\alpha_{\langle \tilde{a} \rangle_R}$ is a fuzzy right ideal of $S(B)$. Based on Lemma 2, the characteristics function $\alpha_{\langle \tilde{a} \rangle_L}$ is a fuzzy left ideal of $S(B)$. Then, by hypothesis , we have:
$$\left(\alpha_{\langle \tilde{a} \rangle_R} \wedge \alpha_{\langle \tilde{a} \rangle_L}\right)(\tilde{b}) \leq \left(\alpha_{\langle \tilde{a} \rangle_R} \circ \alpha_{\langle \tilde{a} \rangle_L}\right)(\tilde{b})$$
Since $\left(\alpha_{\langle \tilde{a} \rangle_R} \wedge \alpha_{\langle \tilde{a} \rangle_L}\right)(\tilde{b}) = \min\{\alpha_{\langle \tilde{a} \rangle_R}(\tilde{b}), \alpha_{\langle \tilde{a} \rangle_L}(\tilde{b})\}$, so we have:
$$\min\{\alpha_{\langle \tilde{a} \rangle_R}(\tilde{b}), \alpha_{\langle \tilde{a} \rangle_L}(\tilde{b})\} \leq \left(\alpha_{\langle \tilde{a} \rangle_R} \circ \alpha_{\langle \tilde{a} \rangle_L}\right)(\tilde{b})$$
Since $\tilde{b} \in \langle \tilde{a} \rangle_R$ and $\tilde{b} \in \langle \tilde{a} \rangle_L$, so we get $\langle \tilde{a} \rangle_R(\tilde{b}) = 1$ and $\langle \tilde{a} \rangle_L(\tilde{b}) = 1$, then we have $\min\{\alpha_{\langle \tilde{a} \rangle_R}(\tilde{b}), \alpha_{\langle \tilde{a} \rangle_L}(\tilde{b})\} = 1$ and
$$1 \leq \left(\alpha_{\langle \tilde{a} \rangle_R} \circ \alpha_{\langle \tilde{a} \rangle_L}\right)(\tilde{b}) \tag{1}$$
If $A_{\tilde{b}} = \emptyset$, then $\left(\alpha_{\langle \tilde{a} \rangle_R} \circ \alpha_{\langle \tilde{a} \rangle_L}\right)(\tilde{b}) = 0$, which is impossible by (1). So we have $A_{\tilde{b}} \neq \emptyset$.

We prove that there exist $(\tilde{y}, \tilde{z}) \in A_{\tilde{b}}$ such that $\tilde{y} \in \langle \tilde{a} \rangle_R$ and $\tilde{z} \in \langle \tilde{a} \rangle_L$. Then we have $\tilde{b} \leq \tilde{y}\tilde{z} \in \langle \tilde{a} \rangle_R \langle \tilde{a} \rangle_L$ and $\tilde{b} \in (\langle \tilde{a} \rangle_R \langle \tilde{a} \rangle_L]$.
Suppose for each $(\tilde{y}, \tilde{z}) \in A_{\tilde{b}}$ we have $\tilde{y} \notin \langle \tilde{a} \rangle_R$ or $\tilde{z} \notin \langle \tilde{a} \rangle_L$. Then
$$\min\{\alpha_{\langle \tilde{a} \rangle_R}(\tilde{y}), \alpha_{\langle \tilde{a} \rangle_L}(\tilde{z})\} = 0, \forall(\tilde{y}, \tilde{z}) \in A_{\tilde{b}} \tag{2}$$

Let $(\tilde{y}, \tilde{z}) \in A_{\tilde{b}}$, if $\tilde{y} \notin \langle \tilde{a} \rangle_R$ then $\alpha_{\langle \tilde{a} \rangle_R}(\tilde{y}) = 0$. Since $\tilde{z} \in S(B)$, we have $\alpha_{\langle \tilde{a} \rangle_L}(\tilde{z}) \geq 0$. Hence we have $\min\{\alpha_{\langle \tilde{a} \rangle_R}(\tilde{y}), \alpha_{\langle \tilde{a} \rangle_L}(\tilde{z})\} = 0$. Based on the equation (2), we have $\bigvee_{(\tilde{y},\tilde{z}) \in A_{\tilde{a}}} \min\{\alpha_{\langle \tilde{a} \rangle_R}(\tilde{y}), \alpha_{\langle \tilde{a} \rangle_L}(\tilde{z})\} = 0$. Since $A_{\tilde{a}} \neq \emptyset$, we have: $\left(\alpha_{\langle \tilde{a} \rangle_R} \circ \alpha_{\langle \tilde{a} \rangle_L}\right)(\tilde{b}) = \bigvee_{(\tilde{y},\tilde{z}) \in A_{\tilde{a}}} \min\{\alpha_{\langle \tilde{a} \rangle_R}(\tilde{y}), \alpha_{\langle \tilde{a} \rangle_L}(\tilde{z})\}$

Then we have $\left(\alpha_{\langle \tilde{a} \rangle_R} \circ \alpha_{\langle \tilde{a} \rangle_L}\right)(\tilde{b}) = 0$. Based on (1), it is impossible.

∎

Corollary 1. *A partial ordered bilinear form semigroup $(S(B), \leq)$ is regular if and only if for every fuzzy right ideal α and every fuzzy subset β of $S(B)$, we have: $\alpha \wedge \beta \leqslant \alpha \circ \beta$.*
Proof: Based on Proposition 3 and Theorem 1 we can prove this corollary.

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Corollary 2. *A partial ordered bilinear form semigroup $(S(B), \leq)$ is regular if and only if for every fuzzy subset α and every fuzzy left ideal β of $S(B)$, we have: $\alpha \wedge \beta \leqslant \alpha \circ \beta$.*

Proof: Based on Proposition 4 and Theorem 1 we can prove this corollary.

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In case of a partial ordered semigroup, a right or left ideal is called idempotent if $A = (A^2]$.
Theorem 2. *Let $(S(B), \leq)$ be a partial ordered bilinear form semigroup with an identity element and α a fuzzy right ideal of $(S(B), \leq)$. Then $\alpha \circ 1 = \alpha$.*

Proof. Let α be a right ideal of $(S(B), \leq)$. The first we have $1 \in F(S)$ i.e. 1 is a fuzzy subset of $(S(B), \leq)$. Let $\tilde{a} \in S(B)$. Then $(\alpha \circ 1)(\tilde{a}) \leq \alpha(\tilde{a})$, i.e.:
If $A_{\tilde{a}} = \emptyset$, then $(\alpha \circ 1)(\tilde{a}) = 0$. Since α is a fuzzy subset of $S(B)$, we have $\alpha(\tilde{a}) \geq 0$. So $(\alpha \circ 1)(\tilde{a}) \leq \alpha(\tilde{a})$.
If $A_{\tilde{a}} \neq \emptyset$. Then $(\alpha \circ 1)(\tilde{a}) = \bigvee_{(\tilde{y},\tilde{z}) \in A_{\tilde{a}}} \min\{\alpha(\tilde{y}), 1(\tilde{z})\}$. We have
$$\min\{\alpha(\tilde{y}), 1(\tilde{z})\} \leq \alpha(\tilde{a}), \qquad \forall (\tilde{y}, \tilde{z}) \in A_{\tilde{a}}$$
Let $(\tilde{y}, \tilde{z}) \in A_{\tilde{a}}$. Since $\tilde{a} \leq \tilde{y}\tilde{z}$ and α is a fuzzy right ideal of $S(B)$, we have $\alpha(\tilde{a}) \geq \alpha(\tilde{y}\tilde{z}) \geq \alpha(\tilde{y})$. Since α is a fuzzy subset in $S(B)$, we have $\alpha(\tilde{y}) \leq 1$. Since $\alpha(\tilde{z}) = 1$, we have $\min\{\alpha(\tilde{y}), 1(\tilde{z})\} = \alpha(\tilde{y}) \leq \alpha(\tilde{a})$. Hence we have $(\alpha \circ 1)(\tilde{a}) \leq \alpha(\tilde{a})$

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Corollary 3. *Let $(S(B), \leq)$ be a partial ordered bilinear form semigroup with identity element and β a fuzzy left ideal of $(S(B), \leq)$. Then $1 \circ \beta = \beta$.*

Proof. The proof is similar with the proof of Theorem 2.

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Theorem 3. *Let $(S(B), \leq)$ be a partial ordered bilinear form semigroup with an identity element and α a fuzzy right ideal of $(S(B), \leq)$. Then $\alpha \circ \alpha \leqslant \alpha$.*

Proof. Let α be a fuzzy right ideal of $S(B)$. Since $\alpha \leqslant 1$ and $\alpha \leqslant \alpha$ and based on Proposition 1, we have $\alpha \circ \alpha \leqslant \alpha \circ 1$. On the other hand, based on Theorem 2, we have $\alpha \circ 1 = \alpha$. Thus we have $\alpha \circ \alpha \leqslant \alpha$.

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Corollary 4. *Let $(S(B), \leq)$ be a partial ordered bilinear form semigroup with an identity element and β a fuzzy left ideal of $(S(B), \leq)$. Then $\beta \circ \beta \leqslant \beta$.*

Proof. The proof is similar with the proof of Theorem 3.

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Theorem 4. *Let $(S(B), \leq)$ be a regular partial ordered bilinear form semigroup and α be a fuzzy right ideal of $(S(B), \leq)$. Then $\alpha \leqslant \alpha \circ \alpha$.*

Proof. Let $\tilde{a} \in S(B)$, then we must prove that $\alpha(\tilde{a}) \leq (\alpha \circ \alpha)(\tilde{a})$. Since $S(B)$ is regular, there exist $\tilde{x} \in S(B)$ such that $\tilde{a} \leq \tilde{a}\tilde{x}\tilde{a}$. Then $(\tilde{a}\tilde{x}, \tilde{a}) \in A_{\tilde{a}}$. Since $A_{\tilde{a}} \neq \emptyset$, we have:
$(\alpha \circ \alpha)(\tilde{a}) = \bigvee_{(\tilde{y}, \tilde{z}) \in A_{\tilde{a}}} \min\{\alpha(\tilde{y}), \alpha(\tilde{z})\} \geq \min\{\alpha(\tilde{y}), \alpha(\tilde{z})\}, \quad \forall(\tilde{y}, \tilde{z}) \in A_{\tilde{a}}$
Since $(\tilde{a}\tilde{x}, \tilde{a}) \in A_{\tilde{a}}$, we obtain $(\alpha \circ \alpha)(\tilde{a}) \geq \min\{\alpha(\tilde{a}\tilde{x}), \alpha(\tilde{a})\}$. Since $\tilde{a} \leq \tilde{a}\tilde{x}\tilde{a}$ and α is a fuzzy right ideal of $(S(B), \leq)$, then we have:
$$\alpha(\tilde{a}) \geq \alpha((\tilde{a}\tilde{x})\tilde{a}) \geq \alpha(\tilde{a}\tilde{x}) \geq \alpha(\tilde{a})$$
Hence we have $\alpha(\tilde{a}\tilde{x}) = \alpha(\tilde{x})$, so $\min\{\alpha(\tilde{a}\tilde{x}), \alpha(\tilde{a})\} = \alpha(\tilde{a})$ and $\alpha(\tilde{a}) \leq (\alpha \circ \alpha)(\tilde{a})$.

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Corollary 5. *Let $(S(B), \leq)$ be a regular partial ordered bilinear form semigroup and β be a fuzzy left ideal of $(S(B), \leq)$. Then $\beta \leqslant \beta \circ \beta$.*

Proof. The proof is similar with the proof of Theorem 5.

■

A fuzzy subset α of a semigroup is called idempotent if and only if $\alpha \circ \alpha = \alpha$.

Corollary 6. *Let* $(S(B), \leq)$ *be a regular partial ordered bilinear form semigroup. Then the fuzzy right ideals and the fuzzy left ideals are idempotent.*

Proof. Let α be an arbitrary fuzzy right ideal of $(S(B), \leq)$. Based on Theorem 3, we have $\alpha \circ \alpha \leqslant \alpha$. And based on Theorem 4 we have $\alpha \leqslant \alpha \circ \alpha$. So we get $\alpha \circ \alpha = \alpha$ or it proves that α is idempotent. Similarly, for an arbitrary fuzzy left ideal β of $(S(B), \leq)$, we get $\beta \circ \beta = \beta$

■

3 Conclusion

In this paper, we considered characterizations of partial ordered bilinear form semigroups in term their fuzzy right and left ideals. We obtained several properties of this semigroup. These properties are the following:

1. An ordered semigroup $S(B)$ is called left simple if and only if $(S(B)\tilde{a}] = S(B)$ for every $\tilde{a} \in S(B)$.
2. An ordered semigroup $S(B)$ is called right simple if and only if $(\tilde{a}S(B)] = S(B)$ for every $\tilde{a} \in S(B)$.
3. An ordered semigroup $(S(B), ., \leq)$ is regular, right and left simple if and only if every fuzzy quasy-ideal of $S(B)$ is a constant function.

References

1. Asaad,M.: Group and Fuzzy Subgroup. Fuzzy Sets and systems 39 , pp: 323 - 328. (1999).
2. Howie, J.M.: An Introduction to Semigroup Theory. Academic Press. London(1976).
3. Kandasamy, W.B.V. : Smarandache Fuzzy Algebra. American Research Press and W.B. Vasantha Kandasamy Rehoboth. USA. (2003).
4. Karyati. :Semigrup yang Dikonstruksikan dari Bentuk Bilinear. Tesis: Program Pascasarjana Universitas Gadjah Mad. Yogyakarta. (2002)
5. Karyati, Wahyuni, S.: The Properties of Non-degenerate Bilinear Form. Proceeding of SEAMS-GMU: International Conference on Mathematics and Its Applications. (2003).
6. Karyati, Wahyuni, S, Surodjo, B, Setiadji,: Beberapa Sifat Ideal Fuzzy Semigrup yang Dibangun oleh Subhimpunan Fuzzy, Prosiding Seminar Nasional Matematika, Universitas Negeri Jember. (2009).
7. Karyati, Wahyuni, S, Surodjo, B, Setiadji. : Quotient Semigroups Induced by Fuzzy Congruence Relations, Proceeding IndoMS International Conference on Mathematics and Its Application (IICMA) , GMU, Yogyakarta, pp: 102-111. (2009).

8. Karyati, Wahyuni, S, Surodjo, B, Setiadji. : Subsemigrup S(B) Fuzzy. Prosiding Seminar Nasional PIPM, Jurusan Pendidikan Matematika, FMIPA, UNY. (2009).

9. Karyati, Wahyuni, S, Surodjo, B, Setiadji. :The Fuzzy Regularity of Bilinear Form Semigroups, Proceedings of ''The 6th SEAMS-UGM Conference 2011'' (2012).

10. Karyati, Wahyuni, S, Surodjo, B, Setiadji. Membangun Suatu Relasi Fuzzy pada Semigrup Bentuk Bilinear. Prosiding Seminar Nasional Jurusan Matematika, Universitas Sebelas Maret. (2013).

11. Karyati, Dhoriva, U.W: Semigrup Bentuk Bilinear Terurut Parsial dalam Batasan Subhimpunan Fuzzy. Seminar Nasional MAtematika dan Pendidikan Matematika, PPs Universitas Sebelas Maret . (2013)

12. Kehayopulu, N : Ideals and Green Relations in Partial ordered Semigroups, International Journal of Mathematics and Mathematical Sciences, Volume 26, pp: 1-8. . (2005).

13. Kehayopulu, N.: Left Regular Partial ordered Semigroups in which the Fuzzy Left Ideals are Two-Sided, International Journal of Algebra, Vol 6, no.10, pp:493-499. (2012).

14. Klir, G.J, Clair, U.S, Yuan, B. :Fuzzy Set Theory: Foundation and Applications. Prentice-Hall, Inc. USA. (1997).

15. Mohanraj, G, Krishnaswamy, D and Hema, R. : On Generalized Redefined Fuzzy Prime Ideals of Partial ordered Semigroups, Annals of Fuzzy Mathematics and Informatics, Volume X, No 10, pp: 1- 9. (2011).

16. Mordeson, J.N, Malik, D.S. :Fuzzy Commutative Algebra. World Scientifics Publishing Co. Pte. Ltd. Singapore, (1998,)

17. Murali, V.: Fuzzy Equivalence Relation. Fuzzy Sets and System 30 , pp: 155-163. (1998)

18. Rajendran, D, Nambooripad, K.S.S. :Bilinear Form and a Semigroup of Linear Transformations. Southeast Asian Bulletin of Mathematics 24, p: 609-616 . (2000)

19. Shabir, M, Khan, A : Characterizations of Partial ordered Semigroups by the Properties of Their Fuzzy Ideals, Computers and Mathematics with Applications, Volume 59, pp: 539 – 549. (2010)

20. Zimmermann, H.J. : Fuzzy Set Theory and Its Applications. Kluwer Academic Publishers. USA. (1991)