

Reducing Fuzzy Relations of Fuzzy Time Series Model Using QR Factorization Method and Its Application to Forecasting Interest Rate of Bank Indonesia Certificate

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Abstract

Fuzzy time series is a dynamic process with linguistic values as its observations. Modelling fuzzy time series developed by some researchers used the discrete membership functions and table lookup scheme (Wang's method) from training data. The Wang's method is a simple method that can be used to overcome the conflicting rule by determining each rule degree. The weakness of fuzzy time series model based on the method is that the fuzzy relations may not be complete so the fuzzy relations can not cover all values in the domain. Generalization of the Wang's method has been developed to construct completely fuzzy relations. But this method causes complexly computations. This paper presents a method to reduce fuzzy relations to improve accuracy of prediction. Then, this method is applied to forecast interest rate of Bank Indonesia Certificate (BIC). The prediction of interest rate of BIC using the proposed method has a higher accuracy than that using the Wang's method and generalized Wang's method.

Keywords: fuzzy relation, fuzzy time series, generalized Wang's method, QR factorization, interest rate of BIC.

1. Introduction

Fuzzy time series is a dynamic process with linguistic values as its observations. In recently, fuzzy time series model was developed by some researchers. Song and Chissom developed fuzzy time series by fuzzy relational equation using Mamdani's method [12]. In this modeling, determining the fuzzy relations needs large computation. Then, Song and Chissom constructed first order fuzzy time series for time invariant and time variant case [13], [14]. This model needs complexity computation for fuzzy relational equation. Furthermore, to overcome the weakness of the model, Chen designed fuzzy time series model by clustering of fuzzy relations [6]. Hwang

constructed fuzzy time series model to forecast the enrollment in Alabama University [10]. Fuzzy time series model based on heuristic model gave more accuracy than its model designed by some previous researchers [9]. Then, forecasting for enrollment in Alabama University based on high order fuzzy time series resulted more accuracy prediction [7]. First order fuzzy time series model was also developed by Sah and Degtiarev [11] and Chen and Hsu [8].

Abadi [1] constructed fuzzy time series model using table lookup scheme (Wang's method) to forecast interest rate of Bank Indonesia certificate (BIC) and the result gave high accuracy. Then, forecasting inflation rate using singular value decomposition method had a higher accuracy than that using Wang's method [2], [3]. The weakness of the constructing fuzzy relations based on the Wang's method is that the fuzzy relations may not be complete so the fuzzy relations can not cover all values in the domain. To overcome this weakness, Abadi [4] designed generalized Wang's method. Furthermore, Abadi [5] constructed complete fuzzy relations of fuzzy time series model based on training data. Too many fuzzy relations result complex computations and too few fuzzy relations cause less powerful of fuzzy time series model in prediction accuracy.

In this paper, we will design optimal fuzzy relations of fuzzy time series model using QR factorization method to improve the prediction accuracy. Then, its result is used to forecast interest rate of BIC. The rest of this paper is organized as follows. In section 2, we briefly review the basic definitions of fuzzy time series. In section 3, we present a procedure to reduce fuzzy relations to improve prediction accuracy. In section 4, we apply the proposed method to forecasting interest rate of BIC. We also compare the proposed method with the Wang's method and the generalized Wang's method in the forecasting interest rate of BIC. Finally, some conclusions are discussed in section 5.

2. Fuzzy time series

In this section, we introduce the following definitions and properties of fuzzy time series referred from Song and Chissom [12].

Definition 1. Let $Y(t) \subset \mathbf{R}$, $t = \dots, 0, 1, 2, \dots$, be the universe of discourse on which fuzzy sets $f_i(t)$ ($i = 1, 2, 3, \dots$) are defined and $F(t)$ is the collection of $f_i(t)$, $i = 1, 2, 3, \dots$, then $F(t)$ is called **fuzzy time series** on $Y(t)$, $t = \dots, 0, 1, 2, 3, \dots$

In the Definition 1, $F(t)$ can be considered as a linguistic variable and $f_i(t)$ as the possible linguistic values of $F(t)$. The value of $F(t)$ can be different depending on time t so $F(t)$ is function of time t . The following procedure gives how to construct fuzzy time series model based on fuzzy relational equation.

Definition 2. Let I and J be indices sets for $F(t-1)$ and $F(t)$ respectively. If for any $f_j(t) \in F(t)$, $j \in J$, there exists $f_i(t-1) \in F(t-1)$, $i \in I$ such that there exists a fuzzy relation $R_{ij}(t, t-1)$ and $f_j(t) = f_i(t-1) \circ R_{ij}(t, t-1)$, $R(t, t-1) = \bigcup_{i,j} R_{ij}(t, t-1)$ where \cup is union operator, then $R(t, t-1)$ is called **fuzzy relation between $F(t)$ and $F(t-1)$** . This fuzzy relation can be written as

$$F(t) = F(t-1) \circ R(t, t-1). \quad (1)$$

where \circ is max-min composition.

In the equation (1), we must compute all values of fuzzy relations $R_{ij}(t, t-1)$ to determine value of $F(t)$. Based on above definitions, concept for first order and m -order of fuzzy time series can be defined.

Definition 3. If $F(t)$ is caused by $F(t-1)$ only or by $F(t-1)$ or $F(t-2)$ or ... or $F(t-m)$, then the fuzzy relational equation

$$F(t) = F(t-1) \circ R(t, t-1) \text{ or} \\ F(t) = (F(t-1) \cup F(t-2) \cup \dots \cup F(t-m)) \circ R_0(t, t-m) \quad (2)$$

is called first order model of $F(t)$.

Definition 4. If $F(t)$ is caused by $F(t-1), F(t-2), \dots$ and $F(t-m)$ simultaneously, then the fuzzy relational equation

$$F(t) = (F(t-1) \times F(t-2) \times \dots \times F(t-m)) \circ R_a(t, t-m) \quad (3)$$

is called m -order model of $F(t)$.

From equations (2) and (3), the fuzzy relations $R(t, t-1), R_a(t, t-m), R_o(t, t-m)$ are important factors to design fuzzy time series model. Furthermore for the first order model of $F(t)$, for any $f_j(t) \in F(t)$, $j \in J$, there exists $f_i(t-1) \in F(t-1)$, $i \in I$ such that there exists fuzzy relations $R_{ij}(t, t-1)$ and $f_j(t) = f_i(t-1) \circ R_{ij}(t, t-1)$. This is equivalent to "if $f_i(t-1)$, then $f_j(t)$ ", and then we have the fuzzy relation $R_{ij}(t, t-1) = f_i(t-1) \times f_j(t)$. Because of $R(t, t-1) = \bigcup_{i,j} R_{ij}(t, t-1)$, then

$$R(t, t-1) = \text{maks}_{i,j} \{ \min(f_j(t), f_i(t-1)) \}. \quad (4)$$

For the relation $R_o(t, t-m)$ of the first order model, we get

$$R_o(t, t-m) = \text{maks}_p \{ \text{maks}_k \{ \min_{ik,j} (f_{ik}(t-k), f_j(t)) \} \}. \quad (5)$$

Based on m -order model of $F(t)$, we have

$$R_a(t, t-m) = \text{maks}_p \{ \min_{j,i_1,i_2,\dots,i_m} (f_{i_1}(t-1) \times f_{i_2}(t-2) \times \dots \times f_{i_m}(t-m) \times f_j(t)) \} \quad (6)$$

From equations (4), (5) and (6), we can compute the fuzzy relations using max-min composition.

Definition 5. If for $t_1 \neq t_2$, $R(t_1, t_1-1) = R(t_2, t_2-1)$ or $R_a(t_1, t_1-m) = R_a(t_2, t_2-m)$ or $R_o(t_1, t_1-m) = R_o(t_2, t_2-m)$, then $F(t)$ is called time-invariant fuzzy time series. Otherwise it is called time-variant fuzzy time series.

Time-invariant fuzzy time series models are independent of time t . Those imply that in applications, the time-invariant fuzzy time series models are simpler than the time-variant fuzzy time series models. Therefore it is necessary to derive properties of time-invariant fuzzy time series models.

Theorem 1. If $F(t)$ is fuzzy time series and for any t , $F(t)$ has only finite elements $f_i(t)$, $i = 1, 2, 3, \dots, n$, and $F(t) = F(t-1)$, then $F(t)$ is a time-invariant fuzzy time series.

Theorem 2. If $F(t)$ is a time-invariant fuzzy time series, then

$$R(t, t-1) = \dots \cup f_{i_1}(t-1) \times f_{j_0}(t) \cup f_{i_2}(t-2) \times f_{j_1}(t-1) \cup \dots \cup f_{i_m}(t-m) \times f_{j_{m-1}}(t-m+1) \cup \dots$$

where m is a positive integer and each pair of fuzzy sets is different.

Based on the Theorem 2, we should not calculate fuzzy relations for all possible pairs. We need only to use one possible pair of the element of $F(t)$ and $F(t-1)$ with all possible t 's. This implies that to construct time-invariant fuzzy time series model, we need only one observation for every t and we set fuzzy relations for every pair of observations in the different of time t . Then union of the fuzzy relations results a fuzzy relation for the model. Theorem 2 is very useful because we sometime have only one observation in every time t .

Let $F_1(t)$ be fuzzy time series on $Y(t)$. If $F_1(t)$ is caused by $(F_1(t-1), F_2(t-1)), (F_1(t-2), F_2(t-2)), \dots, (F_1(t-n), F_2(t-n))$, then the fuzzy logical relationship is presented by $(F_1(t-n), F_2(t-n)), \dots, (F_1(t-2), F_2(t-2)), (F_1(t-1), F_2(t-1)) \rightarrow F_1(t)$ and it is called two-factor n -order fuzzy time series forecasting model, where $F_1(t), F_2(t)$ are called the main factor and the secondary factor fuzzy time series respectively. If a fuzzy logical relationship is presented as

$$(F_1(t-n), F_2(t-n), \dots, F_m(t-n)), \dots, (F_1(t-2), F_2(t-2), \dots, F_m(t-2)), (F_1(t-1), F_2(t-1), \dots, F_m(t-1)) \rightarrow F_1(t) \quad (7)$$

then the fuzzy logical relationship is called m -factor n -order fuzzy time series forecasting model, where $F_1(t)$ are called the main factor fuzzy time series and $F_2(t), \dots, F_m(t)$ are called the secondary factor fuzzy time series.

Let $A_{1,k}(t-i), \dots, A_{N_i,k}(t-i)$ be N_i fuzzy sets with continuous membership function that are normal and complete in fuzzy time series $F_k(t-i)$, $i=1, 2, 3, \dots, n$, $k=1, 2, \dots, m$, then the fuzzy rule:

$$R^j: IF (x_1(t-n) \text{ is } A_{\eta_1,1}^j(t-n) \text{ and } \dots \text{ and } x_m(t-n) \text{ is } A_{\eta_m,m}^j(t-n)) \text{ and } \dots \\ \text{and } (x_1(t-1) \text{ is } A_{\eta_1,1}^j(t-1) \text{ and } \dots \text{ and } x_m(t-1) \text{ is } A_{\eta_m,m}^j(t-1)), THEN x_1(t) \text{ is } A_{\eta_1,1}^j(t) \quad (8)$$

is equivalent to the fuzzy logical relationship (7) and vice versa. So (8) can be viewed as fuzzy relation in $U \times V$ where $U = U_1 \times \dots \times U_m \subset R^{m \times n}$, $V \subset R$ with

$$\mu_A(x_1(t-n), \dots, x_1(t-1), \dots, x_m(t-n), \dots, x_m(t-1)) = \mu_{A_{\eta_1,1}}(x_1(t-n)) \dots \mu_{A_{\eta_1,1}}(x_1(t-1)) \dots \mu_{A_{\eta_m,m}}(x_m(t-n)) \dots \mu_{A_{\eta_m,m}}(x_m(t-1)),$$

where $A = A_{\eta_1,1}(t-n) \times \dots \times A_{\eta_1,1}(t-1) \times \dots \times A_{\eta_m,m}(t-n) \times \dots \times A_{\eta_m,m}(t-1)$.

Designing m -factor one-order time invariant fuzzy time series model using generalized Wang's method can be seen in [5]. But this method can be generalized to m -factor n -order fuzzy time series model. Suppose we are given the following N training data: $(x_{1p}(t-1), x_{2p}(t-1), \dots, x_{mp}(t-1); x_{1p}(t))$, $p=1, 2, 3, \dots, N$. Based on a method developed in [5], we have complete fuzzy logical relationships designed from training data:

$$R^l: (A_{\eta_1,1}^l(t-1), A_{\eta_2,2}^l(t-1), \dots, A_{\eta_m,m}^l(t-1)) \rightarrow A_{\eta_1,1}^l(t), l=1, 2, 3, \dots, M \quad (9)$$

If we are given input fuzzy set $A^l(t-1)$, then the membership function of the forecasting output $A^l(t)$ is

$$\mu_{A^l(t)}(x_1(t)) = \max_{l=1}^M (\sup_{x \in U} (\mu_{A^l}(x(t-1)) \prod_{j=1}^m \mu_{A_{\eta_j,j}^l(t-1)}(x_j(t-1)) \mu_{A_{\eta_1,1}^l}(x_1(t)))) \quad (10)$$

For example, if given the input fuzzy set $A^l(t-1)$ with Gaussian membership function

$$\mu_{A^l(t-1)}(x(t-1)) = \exp\left(-\sum_{i=1}^m \frac{(x_i(t-1) - x_i^*(t-1))^2}{a_i^2}\right),$$

then the forecasting real output with center average defuzzifier is

$$x_1(t) = f(x_1(t-1), \dots, x_m(t-1)) = \frac{\sum_{j=1}^M y_j \exp\left(-\sum_{i=1}^m \frac{(x_i(t-1) - x_i^{*j}(t-1))^2}{a_i^2 + \sigma_{i,j}^2}\right)}{\sum_{j=1}^M \exp\left(-\sum_{i=1}^m \frac{(x_i(t-1) - x_i^{*j}(t-1))^2}{a_i^2 + \sigma_{i,j}^2}\right)} \quad (11)$$

where y_j is center of the fuzzy set $A_{\eta_1,1}^j(t)$.

3. Reducing fuzzy relations of fuzzy time series model

If the number of training data is large, then the number of fuzzy logical relationships may be large too. So increasing the number of fuzzy logical relationships will add the complexity of computations. To overcome that, first we construct complete fuzzy logical relationships using generalized Wang's method referred from [5] and then we will apply QR factorization method to reduce the fuzzy logical relationships using the following steps.

Step 1. Set up the firing strength of the fuzzy logical relationship (9) for each training datum $(x; y) = (x_1(t-1), x_2(t-1), \dots, x_m(t-1); x_1(t))$ as follows

$$L_l(x,y) = \frac{\prod_{f=1}^m \mu_{A_{f,j}(t-1)}(x_f(t-1)) \mu_{A_{i,1}}(x_1(t))}{\sum_{k=1}^M \prod_{f=1}^m \mu_{A_{f,j}(t-1)}(x_f(t-1)) \mu_{A_{k,1}}(x_1(t))}$$

Step 2. Construct $N \times M$ matrix $L = \begin{pmatrix} L_1(1) & L_2(1) & \cdots & L_M(1) \\ L_1(2) & L_2(2) & \cdots & L_M(2) \\ \vdots & \vdots & \vdots & \vdots \\ L_1(N) & L_2(N) & \cdots & L_M(N) \end{pmatrix}$.

Step 3. Compute singular value decomposition of L as $L = USV^T$, where U and V are $N \times N$ and $M \times M$ orthogonal matrices respectively, S is $N \times M$ matrix whose entries $s_{ij} = 0, i \neq j, s_{ii} = \sigma_i, i = 1, 2, \dots, r$ with $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r \geq 0, r \leq \min(N, M)$.

Step 4. Determine the number of fuzzy logical relationships that will be taken as r with $r \leq \text{rank}(L)$.

Step 5. Partition V as $V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$, where V_{11} is $r \times r$ matrix, V_{21} is $(M-r) \times r$ matrix, and construct $\bar{V}^T = \begin{pmatrix} V_{11}^T & V_{21}^T \end{pmatrix}$.

Step 6. Apply QR -factorization to \bar{V}^T and find $M \times M$ permutation matrix E such that $\bar{V}^T E = QR$ where Q is $r \times r$ orthogonal matrix, $R = [R_{11} \ R_{12}]$, and R_{11} is $r \times r$ upper triangular matrix.

Step 7. Assign the position of entries one's in the first r columns of matrix E that indicate the position of the r most important fuzzy logical relationships.

Step 8. Construct fuzzy time series forecasting model (10) or (11) using the r most important fuzzy logical relationships.

4. Application of the proposed method

In this section, we apply the proposed method to forecast interest rate of BIC based on one-factor two-order fuzzy time series model. The data are taken from January 1999 to February 2003. The data from January 1999 to December 2001 are used to training and the data from January 2002 to February 2003 are used to testing. We apply the procedure in Section 3 to predict interest rate of BIC of k^{th} month using data of $(k-2)^{\text{th}}$ and $(k-1)^{\text{th}}$ months. We use [10, 40] as universe of discourse of two inputs and one output and we define seven fuzzy sets A_1, A_2, \dots, A_7 with Gaussian membership function on each universe of discourse of input and output. We apply the generalized Wang's method, designed in [5], to yield forty nine fuzzy logical relationships shown in Table 1.

We apply the QR factorization method to get optimal fuzzy relations. The singular values of firing strength matrix are shown in Figure 1. After we take the r most important fuzzy logical relationships, we get fifteen fuzzy logical relationships that are the optimal number of fuzzy logical relationships. The positions of the fifteen most important fuzzy logical relationships are known as 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 17, 18, 25 (bold rules in Table 1). The resulted fuzzy logical relationships are used to design fuzzy time series forecasting model (10) or (11).

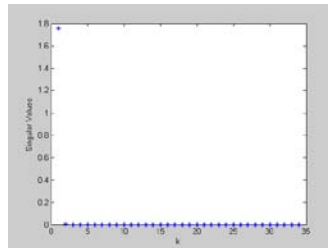


Figure 1. Distribution of singular values of firing strength matrix

Table 1. Fuzzy logical relationship groups for interest rate of BIC using generalized Wang's method

| Rule | $(x(t-2), x(t-1)) \rightarrow x(t)$ | Rule | $(x(t-2), x(t-1)) \rightarrow x(t)$ | Rule | $(x(t-2), x(t-1)) \rightarrow x(t)$ |
|------|-------------------------------------|------|-------------------------------------|------|-------------------------------------|
| 1 | (A1, A1) → A1 | 17 | (A3, A3) → A3 | 33 | (A5, A5) → A3 |
| 2 | (A1, A2) → A2 | 18 | (A3, A4) → A3 | 34 | (A5, A6) → A7 |
| 3 | (A1, A3) → A2 | 19 | (A3, A5) → A2 | 35 | (A5, A7) → A7 |
| 4 | (A1, A4) → A2 | 20 | (A3, A6) → A2 | 36 | (A6, A1) → A2 |
| 5 | (A1, A5) → A3 | 21 | (A3, A7) → A7 | 37 | (A6, A2) → A2 |
| 6 | (A1, A6) → A3 | 22 | (A4, A1) → A2 | 38 | (A6, A3) → A2 |
| 7 | (A1, A7) → A3 | 23 | (A4, A2) → A2 | 39 | (A6, A4) → A3 |
| 8 | (A2, A1) → A1 | 24 | (A4, A3) → A2 | 40 | (A6, A5) → A3 |
| 9 | (A2, A2) → A2 | 25 | (A4, A4) → A2 | 41 | (A6, A6) → A5 |
| 10 | (A2, A3) → A3 | 26 | (A4, A5) → A2 | 42 | (A6, A7) → A6 |
| 11 | (A2, A4) → A3 | 27 | (A4, A6) → A7 | 43 | (A7, A1) → A2 |
| 12 | (A2, A5) → A3 | 28 | (A4, A7) → A7 | 44 | (A7, A2) → A2 |
| 13 | (A2, A6) → A3 | 29 | (A5, A1) → A2 | 45 | (A7, A3) → A3 |
| 14 | (A2, A7) → A7 | 30 | (A5, A2) → A2 | 46 | (A7, A4) → A3 |
| 15 | (A3, A1) → A2 | 31 | (A5, A3) → A2 | 47 | (A7, A5) → A3 |
| 16 | (A3, A2) → A2 | 32 | (A5, A4) → A2 | 48 | (A7, A6) → A5 |
| | | | | 49 | (A7, A7) → A6 |

Based on the Table 2, the average forecasting errors of interest rate of BIC using the Wang's method, the proposed method and the generalized Wang's method are 3.8568%, 2.4881%, 2.7698%, respectively. So we can conclude that forecasting interest rate of BIC using the proposed method results more accuracy than that using the Wang's method and the generalized Wang's method.

Table 2. Comparison of average forecasting errors of interest rate of BIC from the different methods

| Method | Number of fuzzy relations | MSE of training data | MSE of testing data | Average forecasting errors (%) |
|---------------------------|---------------------------|----------------------|---------------------|--------------------------------|
| Wang's method | 12 | 0.98759 | 0.46438 | 3.8568 |
| Proposed method | 15 | 4.59840 | 0.19867 | 2.4881 |
| Generalized Wang's method | 49 | 0.91623 | 0.24134 | 2.7698 |

The comparison of prediction and true values of interest rate of BIC using the Wang's method, the generalized Wang's method and the proposed method is shown in Figure 2.

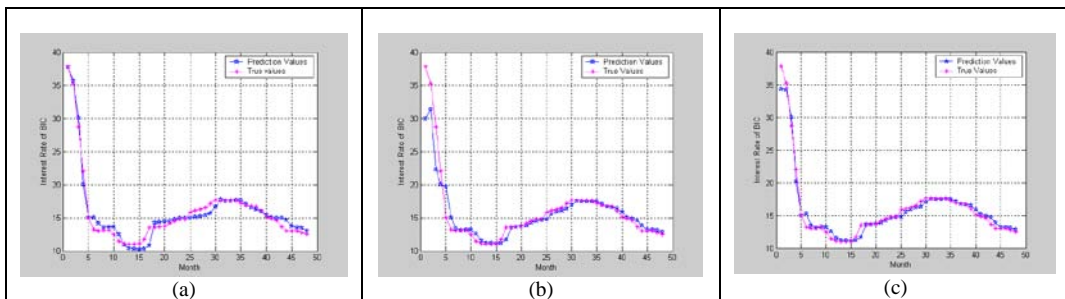


Figure 2. Prediction and true values of interest rate of BIC using: (a) Wang's method, (b) proposed method, (c) generalized Wang's method

5. Conclusions

In this paper, we have presented a method to reduce fuzzy relations of fuzzy time series model based on the training data. The method is used to get optimal number of fuzzy relations. We applied the proposed method to forecast the interest rate of BIC. The result is that forecasting interest rate of BIC using the proposed method has a higher accuracy than that using the Wang's method and generalized Wang's method. The precision of forecasting depends also on taking factors as input variables. In the future works, we will construct a procedure to select the important variables to improve prediction accuracy.

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References

- [1] A.M. Abadi, Subanar, Widodo and S. Saleh, *Forecasting Interest Rate of Bank Indonesia Certificate Based on Univariate Fuzzy Time Series*, International Conference on Mathematics and Its applications SEAMS, Gadjah Mada University, Yogyakarta, 2007
- [2] A.M. Abadi, Subanar, Widodo and S. Saleh, *Constructing Complete Fuzzy Rules of Fuzzy Model Using Singular Value Decomposition*, Proceeding of International Conference on Mathematics, Statistics and Applications (ICMSA), Syiah Kuala University, Banda Aceh, 2008a
- [3] A.M. Abadi, Subanar, Widodo and S. Saleh, *Designing Fuzzy Time Series Model and Its Application to Forecasting Inflation Rate*, 7th World Congress in Probability and Statistics, National University of Singapore, Singapore, 2008b
- [4] A.M. Abadi, Subanar, Widodo and S. Saleh, *A New Method for Generating Fuzzy Rule from Training Data and Its Application in Financial Problems*, The Proceedings of The 3rd International Conference on Mathematics and Statistics (ICoMS-3), Institut Pertanian Bogor, Bogor, 2008c
- [5] A.M. Abadi, Subanar, Widodo and S. Saleh, *Designing Fuzzy Time Series Model Using Generalized Wang's Method and Its Application to Forecasting Interest Rate of Bank Indonesia Certificate*, Proceedings of The First International Seminar on Science and Technology, Islamic University of Indonesia, Yogyakarta, 2009
- [6] S.M. Chen, *Forecasting Enrollments Based on Fuzzy Time Series*, Fuzzy Sets and Systems, 81(1996), 311-319
- [7] S.M. Chen, *Forecasting Enrollments Based on High-Order Fuzzy Time Series*, Cybernetics and Systems Journal, 33(2002), 1-16
- [8] S.M. Chen and C.C. Hsu, *A New Method to Forecasting Enrollments Using Fuzzy Time Series*, International Journal of Applied Sciences and Engineering, 2:3(2004), 234-244
- [9] K. Huarng, *Heuristic Models of Fuzzy Time Series for Forecasting*, Fuzzy Sets and Systems, 123(2001), 369-386

- [10] J.R. Hwang, S.M. Chen and C.H. Lee, *Handling Forecasting Problems Using Fuzzy Time Series*, Fuzzy Sets and Systems, 100(1998), 217-228
- [11] M. Sah and K.Y. Degtiarev, *Forecasting Enrollments Model Based on First-Order Fuzzy Time Series*, Transaction on Engineering, Computing and Technology VI, Enformatika VI(2004), 375-378
- [12] Q. Song and B.S. Chissom, *Forecasting Enrollments with Fuzzy Time Series, Part I*, Fuzzy Sets and Systems, 54(1993a), 1-9
- [13] Q. Song and B.S. Chissom, *Fuzzy Time Series and Its Models*, Fuzzy Sets and Systems, 54(1993b), 269-277
- [14] Q. Song and B.S. Chissom, *Forecasting Enrollments with Fuzzy Time Series, Part II*, Fuzzy Sets and Systems, 62(1994), 1-8
- [15] L.X. Wang, *A Course in Fuzzy Systems and Control*, Prentice-Hall, Inc., Upper Saddle River, 1997