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FUZZY MODEL FOR FORECASTING INTEREST RATE OF BANK INDONESIA CERTIFICATE

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Abstract. In fuzzy modelling, Wang's method is a simple method that can be used to overcome the conflicting rule by determining each rule degree. The weakness of fuzzy model based on the method is that the fuzzy relations may not be complete so the fuzzy relations can not cover all values in the domain. Generalization of the Wang's method has been developed to construct completely fuzzy relations. But this method causes complexly computations. Furthermore, prediction accuracy depends not only on fuzzy relations but also on input variables. This paper presents a method to select input variables and reduce fuzzy relations to improve accuracy of prediction. Then, this method is applied to forecast interest rate of Bank Indonesia Certificate (BIC). The prediction of interest rate of BIC using the proposed method has a higher accuracy than that using generalized Wang's method.

Keywords: fuzzy relation, singular value decomposition, QR factorization, interest rate of BIC.

1. Introduction

Fuzzy time series is a dynamic process with linguistic values as its observations. Song and Chissom developed fuzzy time series by fuzzy relational equation using Mamdani's method (Song and Chissom, 1993a). In this modeling, determining the fuzzy relations needs large computation. Then, Song and Chissom (1993b, 1994) constructed first order fuzzy time series for time invariant and time variant case. This model needs complexity computation for fuzzy relational equation. Furthermore, to overcome the weakness of the model, Chen designed fuzzy time series model by clustering of fuzzy relations (Chen, 1996). Hwang, et.al (1998) constructed fuzzy time series model to forecast the enrollment in Alabama University. Fuzzy time series model based on heuristic model gave more accuracy than its model designed by some previous researchers (Huarng, 2001). Then, forecasting for enrollment in Alabama University based on high order fuzzy time series resulted more accuracy prediction (Chen, 2002). First order fuzzy time series model was also developed by Sah and Degtiarev (2004) and Chen and Hsu (2004).

Abadi, et al (2007) constructed fuzzy time series model using table lookup scheme (Wang's method) to forecast interest rate of Bank Indonesia certificate (BIC) and the result gave high accuracy. Then, forecasting inflation rate using singular value decomposition method had a higher accuracy than that using Wang's method (Abadi, et al, 2008a, 2008b). The weakness of the constructing fuzzy relations based on the Wang's method is that the fuzzy relations may not be complete so the fuzzy relations can not cover all values in the domain. To overcome this weakness, Abadi, et al (2008c) designed generalized Wang's method. Furthermore, Abadi, et al (2009) constructed complete fuzzy relations of fuzzy time series model based on training data. Too many fuzzy relations result complex computations and too few fuzzy relations cause less powerful of fuzzy time series model in prediction accuracy. Then, prediction accuracy depends not only on fuzzy relations but also on input variables.

In this paper, we will design optimal input variables and fuzzy relations of fuzzy time series model using singular value decomposition method to improve the prediction accuracy. Then, its result is used to forecast interest rate of BIC. The rest of this paper is organized as follows. In section 2, we briefly review the basic definitions of fuzzy time series. In section 3, we present a procedure to select input variables. In section 4, we present a method to reduce fuzzy relations to improve prediction accuracy. In section 5, we apply the proposed method to forecasting interest rate of BIC. We also compare the proposed method with the generalized Wang's method in the forecasting interest rate of BIC. Finally, some conclusions are discussed in section 6.

2. Fuzzy Time Series

In this section, we introduce the following definitions and properties of fuzzy time series referred from Song and Chissom (1993a).

Definition 1. *Let $Y(t) \subset \mathbf{R}$, t = ..., 0, 1, 2, ..., , be the universe of discourse on which fuzzy sets $f_i(t)$ (i = 1, 2, 3,...) are defined and $F(t)$ is the collection of $f_i(t)$, i = 1, 2, 3,...,then $F(t)$ is called **fuzzy time series** on $Y(t)$, t = ..., 0, 1, 2, 3, ...*

In the Definition 1, $F(t)$ can be considered as a linguistic variable and $f_i(t)$ as the possible linguistic values of $F(t)$. The value of $F(t)$ can be different depending on time *t* so $F(t)$ is function of time *t*. The following procedure gives how to construct fuzzy time series model based on fuzzy relational equation.

Definition 2. *Let I and J be indices sets for $F(t-1)$ and $F(t)$ respectively. If for any $f_j(t) \in F(t)$, $j \in J$, there exists $f_i(t-1) \in F(t-1)$, $i \in I$ such that there exists a fuzzy relation $R_{ij}(t,t-1)$ and $f_j(t) = f_i(t-1) \circ R_{ij}(t,t-1)$, $R(t,t-1) = \bigcup_{i,j} R_{ij}(t,t-1)$ where $\cup$ is union operator, then $R(t,t-1)$ is called fuzzy relation between $F(t)$ and $F(t-1)$. This fuzzy relation can be written as*

$$F(t) = F(t-1) \circ R(t,t-1). \tag{1}$$

where $\circ$ is max-min composition.

In the equation (1), we must compute all values of fuzzy relations $R_{ij}(t,t-1)$ to determine value of $F(t)$. Based on above definitions, concept for first order and *m*-order of fuzzy time series can be defined.

Definition 3. *If $F(t)$ is caused by $F(t-1)$ only or by $F(t-1)$ or $F(t-2)$ or … or $F(t-m)$, then the fuzzy relational equation*

$$F(t) = F(t-1) \circ R\ (t,t-1) \text{ or}$$
$$F(t) = (F(t-1) \cup F(t-2) \cup ... \cup F(t-m)) \circ R_0(t,t-m) \tag{2}$$

is called first order model of $F(t)$.

Definition 4. *If $F(t)$ is caused by $F(t-1)$, $F(t-2)$, ... and $F(t-m)$ simultaneously, then the fuzzy relational equation*

$$F(t) = (F(t-1) \times F(t-2) \times ... \times F(t-m)) \circ R_a(t,t-m) \tag{3}$$

is called m-order model of $F(t)$.

From equations (2) and (3), the fuzzy relations $R(t,t-1)$, $R_a(t,t-m)$, $R_o(t,t-m)$ are important factors to design fuzzy time series model. Furthermore for the first order model of $F(t)$, for any $f_j(t) \in F(t)$, $j \in J$, there exists $f_i(t-1) \in F(t-1)$, $i \in I$ such that there exists fuzzy relations $R_{ij}(t,t-1)$ and $f_j(t) = f_i(t-1) \circ R_{ij}(t,t-1)$. This is equivalent to "if $f_i(t-1)$, then $f_j(t)$", and then we have the fuzzy relation $R_{ij}(t,t-1) = f_i(t-1) \times f_j(t)$. Because of $R(t,t-1) = \bigcup_{i,j} R_{ij}(t,t-1)$, then

$$R(t,t-1) = \text{maks}_{i,j}\{\min(f_j(t), f_i(t-1))\}. \tag{4}$$

For the relation $R_o(t,t-m)$ of the first order model, we get

$$R_o(t,t-m) = \underset{p}{\text{maks}}\{\ \underset{k}{\text{maks}}\{\underset{ik,j}{\min}(f_{ik}(t-k), f_j(t))\}\ \}. \tag{5}$$

Based on *m*-order model of $F(t)$, we have

$$R_a(t,t-m) = \underset{p}{\text{maks}}\{\ \underset{j,i1,i2,...,im}{\min}(\ f_{i1}(t-1) \times f_{i2}(t-2) \times ... \times f_{im}(t-m) \times f_j(t)\ )\} \tag{6}$$

From equations (4), (5) and (6), we can compute the fuzzy relations using max-min composition.

Definition 5. *If for $t_1 \neq t_2$, $R(t_1,t_1-1) = R(t_2,t_2-1)$ or $R_a(t_1,t_1-m) = R_a(t_2,t_2-m)$ or $R_o(t_1,t_1-m) = R_o(t_2,t_2-m)$, then $F(t)$ is called time-invariant fuzzy time series. Otherwise it is called time-variant fuzzy time series.*

Time-invariant fuzzy time series models are independent of time *t*. Those imply that in applications, the time-invariant fuzzy time series models are simpler than the time-variant fuzzy time series models. Therefore it is necessary to derive properties of time-invariant fuzzy time series models.

Theorem 1. *If $F(t)$ is fuzzy time series and for any t, $F(t)$ has only finite elements $f_i(t)$, i = 1, 2, 3, ..., n, and $F(t) = F(t-1)$, then $F(t)$ is a time-invariant fuzzy time series.*

Theorem 2. *If $F(t)$ is a time-invariant fuzzy time series, then*

$R(t,t-1) = \ldots \cup f_{i1}(t-1)\times f_{j0}(t) \cup f_{i2}(t-2)\times f_{j1}(t-1) \cup \ldots \cup f_{im}(t-m)\times f_{jm-1}(t-m+1) \cup \ldots$

where m is a positive integer and each pair of fuzzy sets is different.

Based on the Theorem 2, we should not calculate fuzzy relations for all possible pairs. We need only to use one possible pair of the element of $F(t)$ and $F(t-1)$ with all possible *t*'s. This implies that to construct time-invariant fuzzy time series model, we need only one observation for every *t* and we set fuzzy relations for every pair of observations in the different of time *t*. Then union of the fuzzy relations results a fuzzy relation for the model. Theorem 2 is very useful because we sometime have only one observation in every time *t*. Let $F_1(t)$ be fuzzy time series on $Y(t)$. If $F_1(t)$ is caused by $(F_1(t-1), F_2(t-1))$, $(F_1(t-2), F_2(t-2)), \ldots, (F_1(t-n), F_2(t-n))$, then $(F_1(t-n), F_2(t-n)), \ldots, (F_1(t-2), F_2(t-2)), (F_1(t-1), F_2(t-1)) \to F_1(t)$ is the fuzzy relation and it is called two-factor *n*-order fuzzy time series forecasting model, where $F_1(t), F_2(t)$ are called the main factor and the secondary factor fuzzy time series respectively. If a fuzzy relation is presented as

$$(F_1(t-n), F_2(t-n), \ldots, F_m(t-n)), \ldots, (F_1(t-2), F_2(t-2), \ldots, F_m(t-2)), (F_1(t-1), F_2(t-1), \ldots, F_m(t-1)) \to F_1(t) \quad (7)$$

then the fuzzy relation is called *m*-factor *n*-order fuzzy time series forecasting model, where $F_1(t)$ are called the main factor fuzzy time series and $F_2(t), \ldots, F_m(t)$ are called the secondary factor fuzzy time series.

Let $A_{1,k}(t-i), \ldots, A_{N_i,k}(t-i)$ be N_i fuzzy sets with continuous membership function that are normal and complete in fuzzy time series $F_k(t-i)$, *i* =1, 2, 3,…, *n*, *k* = 1, 2, …, *m*, then the fuzzy rule:

$$R^j: \; IF \; (x_1(t-n) \; is \; A^j_{i_1,1}(t-n) \text{ and ...and } x_m(t-n) \text{ is } A^j_{i_m,m}(t-n)) \text{ and } \ldots$$
$$\text{and } (x_1(t-1) \; is \; A^j_{i_1,1}(t-1) \text{ and ...and } x_m(t-1) \text{ is } A^j_{i_m,m}(t-1)), \; THEN \; x_1(t) \text{ is } A^j_{i_1,1}(t) \quad (8)$$

is equivalent to the fuzzy relation (7) and vice versa. So (8) can be viewed as fuzzy relation in $U \times V$ where $U = U_1 \times \ldots \times U_{mn} \subset R^{mn}$, $V \subset R$ with $\mu_A(x_1(t-n), \ldots, x_1(t-1), \ldots, x_m(t-n), \ldots, x_m(t-1)) =$ $\mu_{A_{i_1,1}}(x_1(t-n)) \ldots \mu_{A_{i_1,1}}(x_1(t-1)) \ldots \mu_{A_{i_m,m}}(x_m(t-n) \ldots \mu_{A_{i_m,m}}(t-1)$,

where $A = A_{i_1,1}(t-n) \times \ldots \times A_{i_1,1}(t-1) \times \ldots \times A_{i_m,m}(t-n) \times \ldots \times A_{i_m,m}(t-1)$.

We refer to Abadi, et al (2009) to design *m*-factor one-order time invariant fuzzy time series model using generalized Wang's method. But this method can be generalized to *m*-factor *n*-order fuzzy time series model. Suppose we are given the following *N* training data: $(x_{1p}(t-1), x_{2p}(t-1), \ldots, x_{mp}(t-1); x_{1p}(t))$, $p = 1, 2, 3, \ldots, N$. Based on a method developed Abadi, et al (2009), we have complete fuzzy relations designed from training data:

$$R^l: \; (A^l_{j_1,1}(t-1), A^l_{j_2,2}(t-1), \ldots, A^l_{j_m,m}(t-1)) \to A^l_{j_1^*,1}(t), \; l = 1, 2, 3, \ldots, M. \quad (9)$$

If we are given input fuzzy set $A'(t-1)$, then the membership function of the forecasting output $A'(t)$ is

$$\mu_{A'(t)}(x_1(t)) = \max_{l=1}^{M}(\sup_{x \in U}(\mu_{A'}(x(t-1))\prod_{f=1}^{m}\mu_{A_{f,f(t-1)}}(x_f(t-1))\mu_{A^l_{i_1,1}}(x_1(t)))). \quad (10)$$

For example, if given the input fuzzy set $A'(t-1)$ with Gaussian membership function $\mu_{A'(t-1)}(x(t-1)) = \exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^*(t-1))^2}{a_i^2})$, then the forecasting real output with center average defuzzifier is

$$x_1(t) = f(x_1(t-1), \ldots, x_m(t-1)) = \frac{\sum_{j=1}^{M} y_j \exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^{*j}(t-1))^2}{a_i^2+\sigma_{i,j}^2})}{\sum_{j=1}^{M}\exp(-\sum_{i=1}^{m}\frac{(x_i(t-1)-x_i^{*j}(t-1))^2}{a_i^2+\sigma_{i,j}^2})} \quad (11)$$

where y_j is center of the fuzzy set $A^j_{i_1,1}(t)$.

3. Selection of Input Variables

3.1 Sensitivity of input variables

Given M fuzzy relations where the l^{th} fuzzy relation is expressed by:

"If x_1 is $A_1^{j_1}$ and x_2 is $A_2^{j_2}$ and ... and x_n *is* $A_n^{j_n}$, then y_i is B_i"

and the output of fuzzy model is defined by

$$y(t) = \frac{\sum_{r=1}^{M} w_r y_r(t)}{\sum_{r=1}^{M} w_r}$$

with y_r is output of r^{th} fuzzy relation, $w_r = A_1^r \times A_2^r \times \ldots \times A_n^r$, and

$$A_i^r(x_i) = \exp\left(-\frac{(x_i - \overline{x}_i^r)^2}{\sigma_{ir}^2}\right)$$

Saez, D, and Cipriano, A (2001) defined the sensitivity of input variable x_i by $\xi_i(x) = \frac{\partial y(t)}{\partial x_i}$.

If $\sigma_{ir}^2 = \sigma_i^2$ for every r, then

$$\xi_i(x) = \frac{\left(\sum_{r=1}^{M} \frac{2(\overline{x}_i^r)}{\sigma_i^2} y_r w_r\right)\sum_{r=1}^{M} w_r - \sum_{r=1}^{M}\left(\frac{2(\overline{x}_i^r)}{\sigma_i^2}\right) w_r \sum_{r=1}^{M} w_r y_r}{\left(\sum_{r=1}^{M} w_r\right)^2}$$

with $w_r = \exp\left(-\sum_{i=1}^{n} \frac{(x_i - \overline{x}_i^r)^2}{\sigma_{ir}^2}\right)$

Sensitivity $\xi_i(x)$ depends on input variable x and computing of the sensitivity based on training data. Thus, we compute $I_i = \mu_i^2 + \sigma_i^2$ for each variable where μ and σ are mean and standard deviation of sensitivity of variable x_i respectively. Then, input variable with the smallest value I_i is discarded. Based on this procedure, to choose the important input variables, we must take some variables having the biggest values I_i.

3.2 Sensitivity Matrix

In this section, we propose a method to select important input variables using sensitivity matrix. The important input variables can be detected by singular value decomposition and *QR* factorization of sensitivity matrix. Singular value decomposition and *QR* factorization methods are used to know strongly independence columns referring to Golub (1976). Suppose, given N training data and n input variables, then selection of input variables can be done by the following steps:

Step 1. Compute sensitivity of input variables x_i as $\xi_i(x)$.

Step 2. Construct sensitivity matrix N x n as

$$M_s = \begin{pmatrix} \xi_1(1) & \xi_2(1) & \cdots & \xi_n(1) \\ \xi_1(2) & \xi_2(2) & \cdots & \xi_n(2) \\ \vdots & \vdots & \vdots & \vdots \\ \xi_1(N) & \xi_2(N) & \cdots & \xi_n(N) \end{pmatrix}$$

Step 3. Compute singular value decomposition of M_s as $M_s = USV^T$, where U and V are N x N and n x n orthogonal matrices respectively, S is N x n matrix whose entries $s_{ij} = 0, i \neq j$, $s_{ii} = \sigma_i$ $i = 1, 2, \ldots, k$ with $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_k \geq 0$, $k \leq \min(N, n)$.

Step 4. Determine the biggest r singular values that will be taken as r with $r \leq \operatorname{rank}(M_s)$.

Step 5. Partition V as $V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$, where V_{11} is r x r matrix, V_{21} is (n-r) x r matrix.

Step 6. Construct $\bar{V}^T = \begin{pmatrix} V_{11}^T & V_{21}^T \end{pmatrix}$.

Step 7. Apply QR-factorization to $\bar{V}^T$ and find n x n permutation matrix E such that $\bar{V}^T E = QR$ where Q is r x r orthogonal matrix, $R = [R_{11}\ R_{12}]$, and R_{11} is r x r upper triangular matrix.

Step 8. Assign the position of entries one's in the first r columns of matrix E that indicate the position of the r most important input variables.

Step 9. Construct fuzzy time series forecasting model (10) or (11) using the r most important input variables.

Step 10. If the model is optimal, then stop. If It is not yet optimal, then go to Step 4.

4. Reduction of Fuzzy Relations

If the number of training data is large, then the number of fuzzy relations may be large too. So increasing the number of fuzzy relations will add the complexity of computations. To overcome that, first we construct complete fuzzy relations using generalized Wang's method referred from Abadi, et al (2009) and then we will apply QR factorization method to reduce the fuzzy relations using the following steps.

Step 1. Set up the firing strength of the fuzzy relation (9) for each training datum $(x;y)$ = $(x_1(t-1), x_2(t-1), ..., x_m(t-1); x_1(t))$ as follows

$$L_l\,(x;y) = \frac{\prod_{f=1}^{m} \mu_{A_{f,f(t-1)}}(x_f\,(t-1))\,\mu_{A_{1,1}^{l}}(x_1(t))}{\sum_{k=1}^{M}\prod_{f=1}^{m} \mu_{A_{f,f(t-1)}}(x_f\,(t-1))\,\mu_{A_{1,1}^{k}}(x_1(t))}$$

Step 2. Construct N x M matrix $L = \begin{pmatrix} L_1(1) & L_2(1) & \cdots & L_M(1) \\ L_1(2) & L_2(2) & \cdots & L_M(2) \\ \vdots & \vdots & \vdots & \vdots \\ L_1(N) & L_2(N) & \cdots & L_M(N) \end{pmatrix}$.

Step 3. Compute singular value decomposition of L as $L = USV^T$, where U and V are N x N and M x M orthogonal matrices respectively, S is N x M matrix whose entries $s_{ij} = 0, i \neq j$, s_{ii} = σ_i $i = 1, 2, ..., r$ with $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_r \geq 0$, $r \leq \min(N, M)$.

Step 4. Determine the biggest r singular values that will be taken as r with $r \leq \text{rank}(L)$.

Step 5. Partition V as $V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$, where V_{11} is r x r matrix, V_{21} is (M-r) x r matrix, and construct $\bar{V}^T = \begin{pmatrix} V_{11}^T & V_{21}^T \end{pmatrix}$.

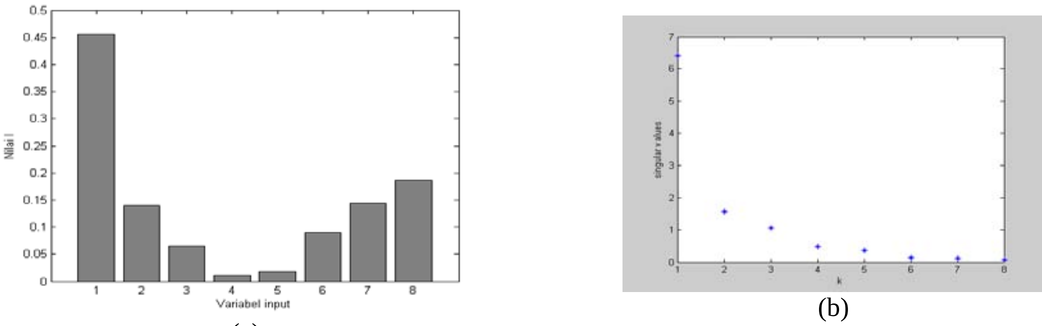
Step 6. Apply QR-factorization to $\bar{V}^T$ and find M x M permutation matrix E such that $\bar{V}^T E = QR$ where Q is r x r orthogonal matrix, $R = [R_{11}\ R_{12}]$, and R_{11} is r x r upper triangular matrix.

Step 7. Assign the position of entries one's in the first r columns of matrix E that indicate the position of the r most important fuzzy relations.

Step 8. Construct fuzzy time series forecasting model (10) or (11) using the r most important fuzzy relations.

5. Application of the Proposed Method

In this section, we apply the proposed method to forecast interest rate of BIC. First, we apply sensitivity matrix method to select input variables. Second, we apply singular value decomposition method to select the optimal fuzzy relations. We consider the initial fuzzy model with eight input variables (x(k-8), x(k-7), …, x(k-1)) from data of interest rate of BIC. The data are taken from January 1999 to February 2003. The data from January 1999 to May 2002 are used to training and the data from June 2002 to February 2003 are used to testing. We use [10, 40] as universe of discourse of eight inputs and one output and we define seven fuzzy sets $A_1, A_2, ..., A_7$ with Gaussian membership function on each universe of discourse of input and output. Then, we compute sensitivity of input variables and sensitivity matrix using the procedure in section 3.1 and section 3.2. Distributions of sensitivity of input variables and singular values of sensitivity matrix are showed in Figure 1. We choose the biggest two singular values and three singular values and then we apply QR factorization to get the significant input variables. Based on selecting the biggest two singular values and three singular values, the selected input variables are x(k-8), x(k-1) and x(k-8), x(k-3), x(k-1) respectively. Then, fuzzy time series model constructed by two input variables x(k-8) and x(k-1) has better prediction accuracy than fuzzy time series model constructed by three input variables x(k-8), x(k-3), x(k-1). So we choose x(k-8) and x(k-1) as input variables to predict value x(k). Then we apply the generalized Wang's method to yield forty nine fuzzy relations showed in Table 1.



(a) (b)

Figure 1. (a) Distribution of sensitivity of input variables, (b) Distribution of singular values of sensitivity matrix

We apply the singular value decomposition method in section 4 to get optimal fuzzy relations. The singular values of firing strength matrix are shown in Figure 2. After we take the *r* most important fuzzy relations, we get ten fuzzy relations that are the optimal number of fuzzy relations. The positions of the ten most important fuzzy relations are known as 1, 2, 8, 9, 10, 15, 17, 29, 37, 44 (blue colored rules in Table 1). The resulted fuzzy relations are used to design fuzzy time series forecasting model (10) or (11).

Table 1. Fuzzy relation groups for interest rate of BIC using generalized Wang's method

Rule	$(x(t-8), x(t-1)) \rightarrow x(t)$	Rule	$(x(t-8), x(t-1)) \rightarrow x(t)$	Rule	$(x(t-8), x(t-1)) \rightarrow x(t)$
1	(A1, A1) → A1	17	(A3, A3) → A2	33	(A5, A5) → A1
2	(A1, A2) → A2	18	(A3, A4) → A2	34	(A5, A6) → A2
3	(A1, A3) → A2	19	(A3, A5) → A2	35	(A5, A7) → A2
4	(A1, A4) → A3	20	(A3, A6) → A2	36	(A6, A1) → A2
5	(A1, A5) → A3	21	(A3, A7) → A2	37	(A6, A2) → A2
6	(A1, A6) → A3	22	(A4, A1) → A1	38	(A6, A3) → A2
7	(A1, A7) → A3	23	(A4, A2) → A1	39	(A6, A4) → A2
8	(A2, A1) → A1	24	(A4, A3) → A2	40	(A6, A5) → A2
9	(A2, A2) → A2	25	(A4, A4) → A2	41	(A6, A6) → A2
10	(A2, A3) → A3	26	(A4, A5) → A2	42	(A6, A7) → A2
11	(A2, A4) → A3	27	(A4, A6) → A2	43	(A7, A1) → A2
12	(A2, A5) → A3	28	(A4, A7) → A2	44	(A7, A2) → A2
13	(A2, A6) → A3	29	(A5, A1) → A1	45	(A7, A3) → A2
14	(A2, A7) → A3	30	(A5, A2) → A1	46	(A7, A4) → A2
15	(A3, A1) → A1	31	(A5, A3) → A1	47	(A7, A5) → A2
16	(A3, A2) → A2	32	(A5, A4) → A1	48	(A7, A6) → A2
				49	(A7, A7) → A2

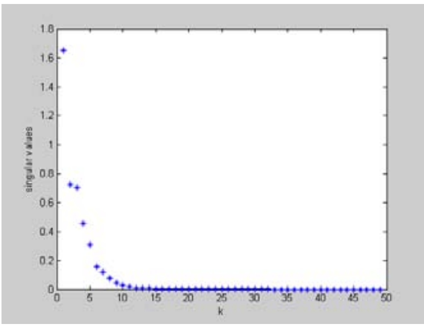


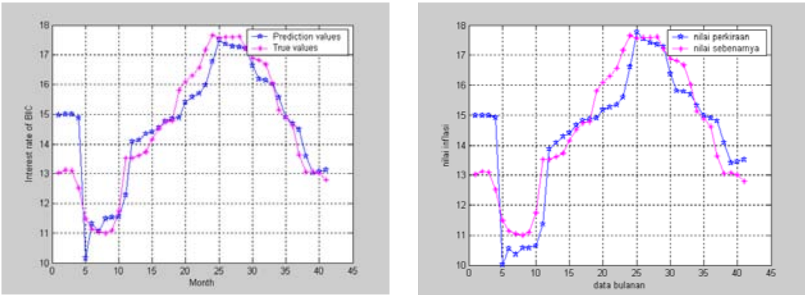
Figure 2. Distribution of singular values of firing strength matrix

Based on the Table 2, the average forecasting errors of interest rate of BIC using the proposed method and the generalized Wang's method are 1.8787%, 3.7750% respectively. So we can conclude that forecasting interest rate of BIC using the proposed method results more accuracy than that using the generalized Wang's method.

Table 2. Comparison of average forecasting errors of interest rate of BIC from the different methods

Method	Number of fuzzy relations	MSE of training data	MSE of testing data	Average forecasting errors (%)
Proposed method	10	0.85014	0.14180	1.8787
Generalized Wang's method	49	1.15250	0.38679	3.7750

The comparison of prediction and true values of interest rate of BIC using the generalized Wang's method and the proposed method is shown in Figure 3.



(a) (b)

Figure 3. Prediction and true values of interest rate of BIC using: (a) proposed method, (b) generalized Wang's method

6. Conclusions

In this paper, we have presented a method to select input variables and reduce fuzzy relations of fuzzy time series model based on the training data. The method is used to get significant input variables and optimal number of fuzzy relations. We applied the proposed method to forecast the interest rate of BIC. The result is that forecasting interest rate of BIC using the proposed method has a higher accuracy than that using generalized Wang's method. The precision of forecasting depends also on determining number of fuzzy sets and parameter of fuzzy sets. In the future works, we will construct a procedure to determine the optimal fuzzy sets to improve prediction accuracy.

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