Kuliah PD

Pertemuan ke-2:

Pokok Bahasan: PD Eksak

PD $M(x,y)dx + N(x,y)dy = 0$ dikatakan eksak jika ada fungsi F(x,y) sehingga $dF(x,y) = M(x,y)dx + N(x,y)dy$.

Tes PD eksak:

Jika $M(x,y)$ dan $N(x,y)$ fungsi-fungsi kontinu dan mempunyai turunan parsial pertama yang kontinu pada daerah persegi panjang, maka PD $M(x,y)dx + N(x,y)dy = 0$ eksak jika dan hanya jika $\dfrac{\partial M(x,y)}{\partial y} = \dfrac{\partial N(x,y)}{\partial x}$.

(bukti lihat Boyce, 2010, hal: 96).

Penyelesiaan PD eksak:

Misalkan PD $M(x,y)dx + N(x,y)dy = 0$ eksak ……………………………..(1)

dan misalkan PU nya berbentuk F(x,y) = c, maka

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(2)$$

Dari (1) dan (2) diperoleh $\dfrac{\partial F}{\partial x} = M(x,y)$ dan $\dfrac{\partial F}{\partial y} = N(x,y)$.

Sehingga $F(x,y) = \displaystyle\int M(x,y)d\ x + g(y)$

Selanjutnya F(x,y) ini diturunkan terhadap y, diperoleh

$$\frac{\partial F(x,y)}{\partial y} = \frac{\partial \int M(x,y)dx}{\partial y} + g'(y)$$

$$N(x,y) = \frac{\partial \int M(x,y)dx}{\partial y} + g'(y)$$

Contoh 1: Selesaikan PD berikut.

$3x(xy - 2\ dx +)(x^3 + 2y)dy = 0$

Jawab: $M(x,y) = 3x(xy - 2)$ $N(x,y) = x^3 + 2y$, $\dfrac{\partial}{\partial y} M(x,y) = 3x^2$, $\dfrac{\partial}{\partial x} N(x,y) = 3x^2$ dan

$\dfrac{\partial M(x,y)}{\partial y} = \dfrac{\partial N(x,y)}{\partial x}$. Jadi PD tersebut eksak. Misalkan PU nya F(x,y) = c, maka

$F(x,y) = \int 3x(xy - 2)dx + g(y)$

$\qquad = x^3 y - 3x^2 + g(y)$

Karena $\dfrac{\partial F}{\partial y} = N(x,y)$, maka $\dfrac{\partial F}{\partial y} = x^3 + g'(y) = x^3 + 2y$, maka $g'(y) = 2y$ sehingga

$g(y) = y^2$. Jadi F(x,y) = $x^3 y - 3x^2 + g(y)$ = c

Diperoleh PU: $x^3 y - 3x^2 + y^2 = c$.

Latihan 1:

Tunjukkan bahwa PD berikut PD eksak kemudian selesaikan PD tersebut.

1. $(y + 2xy^3)dx + (1 + 3x^2 y^2 + x)dy = 0$

2. $(t^2 - x)dt - t\, dx = 0$

3. $(xy + 1)dx + (\dfrac{1}{2}x^2 - 1)dy = 0$

4. $(y\sin x + xy\cos x)dx + (x\sin x + 1)dy = 0$

5. $\dfrac{-y^2}{t^2}dt + \dfrac{2y}{t}dy = 0$

6. $(x + 2y)dx + (2x + y)dy = 0$

7. $(2xy - 3x^2)dx + (x^2 + 2y)dy = 0$

8. $(y^2 - 2xy + 6x)dx - (x^2 - 2xy + 2)dy = 0$

9. $(\cos x + x\cos t)dt + (\sin t - t\sin x)dx = 0$

10. $e^{x^3}(3x^2 y - x^2)dx + e^{x^3}dy = 0$