PROGRAM PASCASARJANA UNY
UJIAN AKHIR SEMESTER GENAP 2013/2014

MATA KULIAH : ANALISIS KOMPLEKS I
HARI/TANGGAL : RABU/18 JUNI 2014
PRODI : PMAT S2
RUANG : R. 306 A GEDUNG LAMA
WAKTU : 10.15 – 11.55
DOSEN : HARTONO
SIFAT : OPEN BOOKS

SOAL 1 (30 POIN):
Selidiki apakah fungsi kompleks berikut ini analitik atau tidak, atau apabila analitik di wilayah tertentu pada bidang kompleks maka tentukanlah wilayah tersebut.

$$f(z) = 2xy + i(x^2 - y^2), \quad \text{dim } ana \quad z = x + iy$$

SOAL 2 (30 POIN):
Hitunglah integral kontur $\int_C f(z)dz$, dimana $f(z) = \frac{2}{z}$ dan kontur C adalah kontur pada bidang kompleks (dengan arah positip) sepanjang sisi-sisi segiempat dengan titik-titik sudutnya (0 , 1), (0 , 2), (2 , 0) dan (1 , 0). Apakah nilai integralnya sama dengan nol? Berikan argumen atas hasil perhitungan Anda.

SOAL 3 (40 POIN) :
(a) Nyatakan deret Maclaurin dari $f(z) = \frac{z+1}{z-1}$ dan tentukan di wilayah mana penderetan tersebut valid
(b) Nyatakan deret Laurent dari fungsi pada (a) untuk domain $1 < |z| < \infty$.

*** apakah suatu dosa kalau aku bekerjasama dalam ujian? ***

(A) Analisis Kompleks I 2014

PPs UNY

Ujian :	(Gsl/Gnp) TA :	Nama MHS :	Kunci
Hari / Tgl : /		No. MHS :	001
Mata Ujian :		Prodi / Sem :	/
Nama Dosen :		TTD :	
Kode :		Nilai :	

① $f(z) = 2xy + i(x^2 - y^2)$, $z = x + iy$

$u(x,y) = 2xy$, $v(x,y) = x^2 - y^2$

$u_x = 2y$, $v_x = 2x$

$u_y = 2x$ $\quad v_y = -2y$

$u_x \neq v_y$ dan $u_y \neq -v_x$ kecuali di $x = y = 0$.

Ini tidak memenuhi C-R, jadi tidak analitik dimanapun.

$f'(0,0) = 0$?

Definim

(B) I

$$I_1 = \int_{C_1} \frac{2}{x+yi}\, d(x+yi) = \int_{C_1} \frac{2}{x+yi} \cdot \frac{x-yi}{x-yi}\, d(x+yi)$$

$$= \int_{C_1} \left(\frac{2x}{x^2+y^2} - i\,\frac{2y}{x^2+y^2} \right)(dx+i\,dy)$$

$$= \int_{C_1} \frac{2x}{x^2+y^2}\,dx - i\int_{C_1} \frac{2y}{x^2+y^2}\,dx + i\int_{C_1} \frac{2x}{x^2+y^2}\,dy + \int_{C_1} \frac{2y\,dy}{x^2+y^2}$$

(1) (2) (3) (4)

C_1: $\boxed{y = -x+1 \;,\; 0 \le x \le 1}$

$$\int_{C_1} \frac{2}{x+yi}\, d(x+yi) = \int_0^1 \frac{2x}{x^2+(-x+1)^2}\,dx + \int_0^1 \frac{2(-x+1)-dx}{x^2+(-x+1)^2} +$$

(1) (4)

$$i\int_0^1 \frac{2x\cdot -dx}{x^2+(-x+1)^2} - i\int_0^1 \frac{2(-x+1)\,dx}{x^2+(-x+1)^2}$$

(3) (2)

$$= \int_0^1 \frac{4x-2}{2x^2-2x+1}\,dx + i\int_0^1 \frac{-2\,dx}{2x^2-2x+1}$$

(1)+(4) (3)+(2)

$$= \left[\ln(2x^2-2x+1)\right]_0^1 - 2i\int_0^1 \frac{dx}{2\left(x^2-x+\frac{1}{2}\right)}$$

$$= \ln 1 - \ln 1 - i\int_0^1 \frac{dx}{\left(x-\frac{1}{2}\right)^2+\frac{1}{4}}$$

(c) II

$$\int_0^1 \frac{dx}{(x-\frac{1}{2})^2+\frac{1}{4}} = ?$$

Misalkan $(x-\frac{1}{2}) = \frac{1}{2}\tan\theta$, $x=0 \rightarrow \theta = -\frac{\pi}{4}$

$dx = \frac{1}{2}\sec^2\theta\, d\theta$ $\quad x=1 \rightarrow \theta = \frac{\pi}{4}$

$$\int_0^1 \frac{dx}{(x-\frac{1}{2})^2+\frac{1}{4}} = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\frac{1}{2}\sec^2\theta\, d\theta}{\frac{1}{4}\tan^2\theta+\frac{1}{4}} = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\frac{1}{2}\sec^2\theta\, d\theta}{\frac{1}{4}(\tan^2\theta+1)}$$

$$= \int_{-\pi/4}^{\pi/4} 2\, d\theta = 2\theta\Big]_{-\pi/4}^{\pi/4} = \frac{\pi}{2}+\frac{\pi}{2} = \pi$$

$\therefore \int_{C_1} \frac{2}{x+yi}\, d(x+yi) = 0-\pi i$, jadi $\boxed{I_1 = -\pi i}$

(D) III

$$I_2 = \int_{C_2} \frac{2}{x+yi}\, d(x+yi) \quad , \; C_2 : y=0, \; 1 \le x \le 2$$

$$I_2 = \int_1^2 \frac{2x}{x^2}\, dx \quad - 0 \quad + 0 \quad + 0 \quad \text{(berdasarkan hal I)}$$

① ② ③ ④

$$= \ln x^2 \Big]_1^2 = \ln 4 - \ln 1 = \ln 4$$

Jadi $\boxed{I_2 = \ln 4}$

• $I_3 = \int_{C_3} \frac{2}{x+yi}\, d(x+yi) \quad , \quad C_3 : \; y = -x+2 \; , \; 2 \ge x \ge 0$

$$I_3 = \int_2^0 \frac{2x\, dx}{x^2+(-x+2)^2} + \int_2^0 \frac{2(-x+2)\, d(x)}{x^2+(-x+2)^2} +$$

① ④

$$i \int_2^0 \frac{2x(-dx)}{x^2+(-x+2)^2} \; i \int_2^0 \frac{2(-x+2)\, dx}{x^2+(-x+2)^2}$$

③ ②

$$I_3 = \int_2^0 \frac{4x-4}{2x^2-4x+4}\, dx + i \int_2^0 \frac{-4\, dx}{2x^2-4x+4}$$

① + ④ ③ + ②

$$= \ln(2x^2-4x+4)\Big|_2^0 + i \int_2^0 \frac{-4\, dx}{2((x-1)^2+1)}$$

$$= \ln 4 - \ln 4 - 2i \int_2^0 \frac{dx}{(x-1)^2+1}$$

(E) IV

$$\int_2^0 \frac{dx}{(x-1)^2+1} = ?$$

Misalkan $x-1 = \tan\theta$, $x=2 \rightarrow +1 = \tan\theta \rightarrow \theta = \frac{\pi}{4}$

$dx = \sec^2\theta\, d\theta$ $\quad x=0 \rightarrow -1 = \tan\theta \rightarrow \theta = -\frac{\pi}{4}$

$(x-1)^2+1 = \tan^2\theta + 1 = \sec^2\theta$

$$\int_2^0 \frac{dx}{(x-1)^2+1} = \int_{\pi/4}^{-\pi/4} \frac{\sec^2\theta\, d\theta}{\sec^2\theta} = \theta\Big]_{\pi/4}^{-\pi/4} = -\pi/4 - \pi/4 = -\pi/2$$

Shg $I_3 = \ln 4 - \ln 4 - 2i\left(-\frac{\pi}{2}\right)$

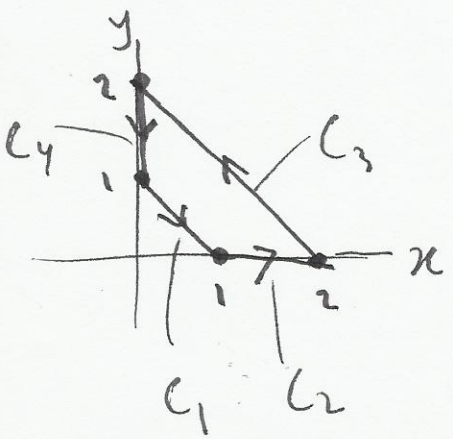
$$\boxed{I_3 = \pi i}$$

$$I_4 = \int_{C_4} \frac{2}{x+yi}\, d(x+yi) \quad , \quad C_4 : 2 \geq y \geq 1 \ , \ x = 0$$

$$I_4 = \underset{①}{0} + \underset{②}{0} + i\int_2^1 \underset{③}{0}\, dy + \int_2^1 \underset{④}{\frac{2y\, dy}{y^2}}$$

$$I_4 = \ln y^2\Big]_2^1 = \ln 1 - \ln 4 = -\ln 4$$

Jadi $I_4 = -\ln 4$



$$\boxed{I_1 + I_2 + I_3 + I_4 = -\pi i + \ln 4 + \pi i - \ln 4 = 0}$$

Analisis Kompleks I 2014

(F)

PPs UNY	Ujian : (Gsl/Gnp) TA :	Nama MHS : Kunci	
	Hari / Tgl : /	No. MHS :	
	Mata Ujian :	Prodi / Sem : /	
	Nama Dosen :	TTD :	
	Kode :	Nilai :	

(3) a) $f(z) = \frac{z+1}{z-1} = \frac{z-1+2}{z-1} = 1 + \frac{2}{z-1} = 1 + 2\left(\frac{1}{z-1}\right)$

$= 1 + 2\left(\frac{1}{1-z}\right)$

$= 1 - 2\sum_{n=0}^{\infty} z^n$

$= 1 - 2 + 2\sum_{n=1}^{\infty} z^n$

$= -1 - 2\sum_{n=1}^{\infty} z^n$

$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n$, $|z| < 1$

atau

b) $f(z) = \frac{z+1}{z-1} = \frac{z\left(1+\frac{1}{z}\right)}{z\left(1-\frac{1}{z}\right)} = \frac{1+\frac{1}{z}+\frac{2}{z}}{1-\frac{1}{z}} = 1 + \frac{2}{z}\cdot\left(\frac{1}{1-\frac{1}{z}}\right)$

$= 1 + \frac{2}{z}\cdot\sum_{n=0}^{\infty}\left(\frac{1}{z}\right)^n$, $0 < \left|\frac{1}{z}\right| < 1$ $\Leftrightarrow$ $\infty > |z| > 1$

$= 1 + \frac{2}{z}\left(1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots\right)$

$= 1 + 2\cdot\frac{1}{z}\left(1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots\right)$

$= 1 + 2\left(\frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \frac{1}{z^4} + \cdots\right)$

$= 1 + 2\sum_{n=1}^{\infty}\frac{1}{z}$

$\left|\frac{1}{z}\right| < 1$

$\frac{1}{|z|} < 1 \longrightarrow |z| > 1$