22

Electrical Circuits and Instruments

Simple electrical circuits can be solved by applying Ohm's law. For circuits that are not too complicated, we can use the method of conservation of energy or the definition of potential difference between two points in a circuit. Ultimately we must depend upon the more advanced method of Kirchhoff's rules, as will be discussed here.

We shall see the application of these rules not only to solve circuit problems, but also to many electrical instruments as well. The measurement of different electrical quantities is done by using a voltmeter, ammeter, Wheatstone bridge, potentiometer, or other instruments. These will be discussed in this chapter. In the end we shall discuss charging and discharging of a capacitor in a RC series circuit.
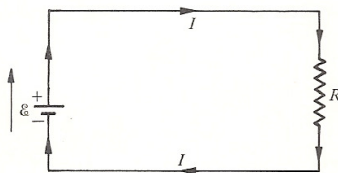
22.1. Single-Loop Circuit

To start we shall consider a simple single-loop circuit, and find the relation between the current, potential drop, and emf. Figure 22.1, containing a source of emf $\mathcal{E}$ and resistance R, is a single-loop circuit. According to energy conservation, the electrical power P_S supplied by the source must be equal to the power dissipated, P_R, as thermal energy (Joule heat) in the resistor. That is, if the current in the circuit is I,

FIGURE 22.1. *Single-loop circuit containing R and $\mathcal{E}$.*

$$P_S = P_R \quad \text{or} \quad \mathcal{E}I = I^2R \tag{22.1}$$

Thus,

$$\mathcal{E} = IR, \quad I = \frac{\mathcal{E}}{R} \tag{22.2}$$

which is a *single-loop equation*.

We have assumed that the source has zero resistance, the connecting wires have negligible resistance, and R is the only resistance present in the circuit. In reality this is not true for the source resistance, although it is often true for the resistance of the connecting wires. The source always has some resistance, say r, however small, as shown in Figure 22.2. The shaded block implies that r is not separate from the source, r being inside the source itself and is called the *internal resistance*. Some power P_r must be lost as thermal energy in the resistor r as well. Thus, Equation 22.1 will take the following form:

FIGURE 22.2. *Simple single-loop circuit containing R, $\mathcal{E}$, and r (internal resistance).*

$$P_S = P_R + P_r \quad \text{or} \quad \mathcal{E}I = I^2R + I^2r$$

Thus,

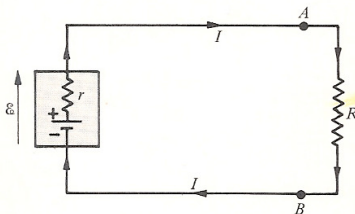$$\mathcal{E} = I(R + r), \quad I = \frac{\mathcal{E}}{R + r} \tag{22.3}$$

The application of the energy conservation principle is not always as simple as discussed above. An alternative is to make use of the definition of electrical potential. Accordingly, in the direct-current circuits, the potential at any point in the circuit must have the same value at any time. This is only possible if the following is true: *In a complete traversal of a circuit, the algebraic sum of all the potential differences encountered must add up to zero.* This is actually the second of the two Kirchhoff's rules which we shall discuss in Section 3.

Before we use this rule, we must decide on a convenient sign convention stated below and demonstrated in Figure 22.3. Let us keep in mind that in each of the four cases shown, point A is at a higher potential than the corresponding
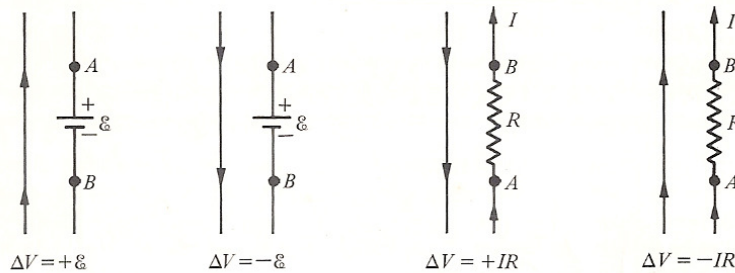

FIGURE 22.3. *Sign convention while traversing a circuit.*

point B. This is because the positive terminal is at a higher potential than the negative. The direction of the current is from a higher to lower potential.

1. If we travel through a source from − to +, the change in potential $\Delta V = +\mathcal{E}$, and in the opposite direction from + to −, $\Delta V = -\mathcal{E}$.

2. If we travel through a resistor in the direction opposite that of the current, the change in potential $\Delta V = +IR$, and in the same direction as the current, $\Delta V = -IR$. (This change is also called the *IR* drop.)

Let us apply these rules to the simple circuit in Figure 22.2. Starting with point A, where the potential is V_A, going clockwise we encounter resistor R, across which the potential change will be $-IR$. Next we meet the source where the potential drop will be $+\mathcal{E}$, while across r it will be $-Ir$. Finally, we arrive at A. The sum of the potentials above must be zero. That is,

$$-IR + \mathcal{E} - Ir = 0$$

or

$$\mathcal{E} = I(R + r) \qquad (22.4)$$

which is the result already derived in Equation 22.3.

This method has another advantage. Suppose that we want to find the potential difference between points A and B in Figure 22.2. Let V_A and V_B be the potentials at points A and B. Starting from A and going clockwise to point B, we may write

$$V_A - IR = V_B$$

or

$$\Delta V = V_A - V_B = IR$$

Substituting for I from Equation 22.3, we get

$$\Delta V = \mathcal{E}\frac{R}{R + r} \tag{22.5}$$

We can arrive at the same result if we start from A and follow the circuit in a counterclockwise direction. Thus,

$$V_A + Ir - \mathcal{E} = V_B$$
$$V_A - V_B = \mathcal{E} - Ir$$

Substituting for $I = \mathcal{E}/(R + r)$ and simplifying, we get the same value of ΔV as given by Equation 22.5, which is called the *terminal voltage* V_t.

EXAMPLE 22.1 A typical 12-V automobile battery has a resistance of $0.012\ \Omega$. What is the terminal voltage of this battery when the starter draws a current of 100 A? Also, calculate R, P_S, P_R, and P_r.

Since $\mathcal{E} = 12$ V, the terminal voltage V_{AB} from Equation 22.4 may be written

$$V_t = IR = \mathcal{E} - Ir = 12\text{ V} - (100\text{ A})(0.012\ \Omega) = (12 - 1.2)\text{ V} = 10.8\text{ V}$$

Also, from this relation

$$R = \frac{\mathcal{E} - Ir}{I} = \frac{10.8\text{ V}}{100\text{ A}} = 0.108\ \Omega$$

To calculate P,

$$P_r = I^2 r = (100\text{ A})^2(0.012\ \Omega) = 120\text{ W}$$
$$P_R = I^2 R = (100\text{ A})^2(0.108\ \Omega) = 1080\text{ W}$$
$$P_S = \mathcal{E}I = (12\text{ V})(100\text{ A}) = 1200\text{ W}$$

Note that $P_S = P_R + P_r$, as it must be.

EXERCISE 22.1 A typical 12-V automobile battery of internal resistance $0.01\ \Omega$ is connected to an external circuit of resistance $0.10\ \Omega$. Calculate the terminal voltage, current in the circuit, P_r, P_R, and P_S. [*Ans.*: $V_t = 10.9$ V, $I = 109$ A, $P_r = 119$ W, $P_R = 1188$ W, $P_S = 1308$ W.]

22.2. Series and Parallel Circuits

Let us now analyze circuits containing series and parallel combinations of resistors and sources of emf.

Series Circuits

Figure 22.4 contains three resistors, R_1, R_2, and R_3, which are connected end to end and are said to be in series. The characteristic describing this series
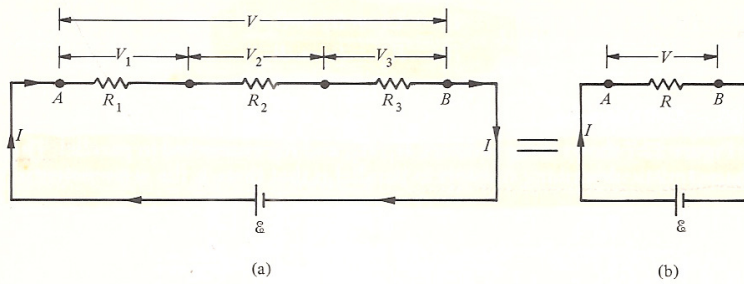
FIGURE 22.4. *Resistors in series.*

circuit is that there is the same current I through each resistor. Let V be the potential difference between A and B. Let V_1, V_2, and V_3 be the potential drops across the resistors R_1, R_2, and R_3, respectively, so that

$$V = V_1 + V_2 + V_3 \qquad (22.6)$$

where $V_1 = IR_1$, $V_2 = IR_2$, and $V_3 = IR_3$. Let R be the resistance that is equivalent to this series combination as far as external connections are concerned, so that, as shown in Figure 22.4(b), $R = V/I$ or $V = IR$. Substituting the values of V, V_1, V_2, and V_3 in Equation 22.6, we get

$$IR = IR_1 + IR_2 + IR_3$$

or

$$R = R_1 + R_2 + R_3$$

If there were more than three resistors in series, a single resistance equivalent to this combination will be

$$R = R_1 + R_2 + R_3 + \cdots \qquad (22.7)$$

That is in a series circuit a single equivalent resistance R is equal to the sum of the individual resistors.

In order to connect sources of emf in series, the positive terminal of one source is connected to the negative terminal of the next one, as shown in Figure 22.5. A common example is a flashlight, where two or three dry cells are connected in series. The law of conservation of energy requires that a single source of emf which must be equivalent to several sources of emf connected in series, as shown in Figure 22.5, is given by

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + \cdots \qquad (22.8)$$

where $\mathcal{E}$ is equal to the total work done in moving a unit positive charge through all the sources.

Finally, Figure 22.6 shows three resistors R_1, R_2, and R_3, which are in series, and three sources of emf $\mathcal{E}_1$, $\mathcal{E}_2$, and $\mathcal{E}_3$, which are in series also. In this case the equivalent resistance is $R = R_1 + R_2 + R_3$ and the equivalent emf is $\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3$. The current through the circuit will be
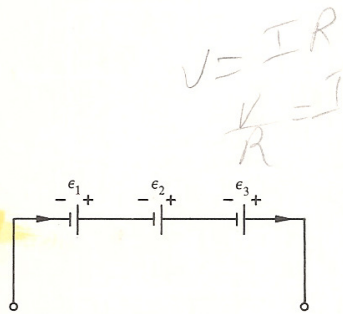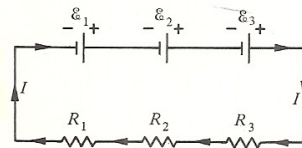

FIGURE 22.5. *Sources of emfs in series.*

FIGURE 22.6. *Resistors and sources of emfs in series.*

$$I = \frac{\mathcal{E}}{R} = \frac{\mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3}{R_1 + R_2 + R_3} \tag{22.9}$$

Parallel Circuits

Figure 22.7 shows three resistors, R_1, R_2, and R_3, connected in parallel. The characteristic describing resistors in parallel is that there is the same potential
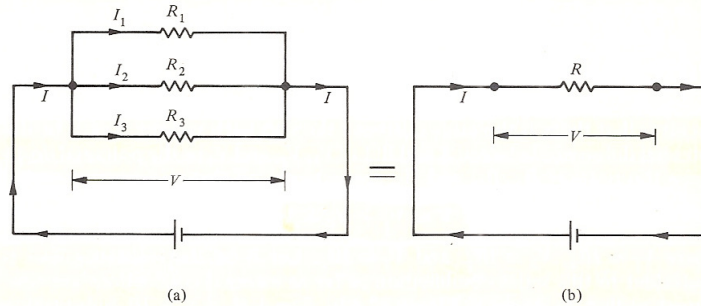

(a)   (b)

FIGURE 22.7. *Resistors in parallel.*

difference V across each of the resistors. The current I in the circuit must be equal to the sum of the currents I_1, I_2, and I_3 in the three resistors. Thus,

$$I = I_1 + I_2 + I_3 \tag{22.10}$$

where $I_1 = V/R_1$, $I_2 = V/R_2$, and $I_3 = V/R_3$. Let R be the resistance which is equivalent to the three resistors in parallel so that, as shown in Figure 22.7(b), $I = V/R$. Substituting the values of I, I_1, I_2, and I_3 in Equation 22.10,

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

or

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In general,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \tag{22.11}$$

That is, the reciprocal of the single equivalent resistor R is equal to the sum of the reciprocals of the resistances R_1, R_2, R_3, . . . , which are connected in parallel.

Figure 22.8(a) shows how to connect sources of emf in parallel combination—all the positive terminals are connected together and all the negative terminals are connected together. All sources of emf in parallel should always be of the same voltage (otherwise, some will behave as sources while others behave as sinks). Thus, the voltage of the system is equal to the voltage across any one of them, as shown in Figure 22.8(b). Thus the total current is equal to
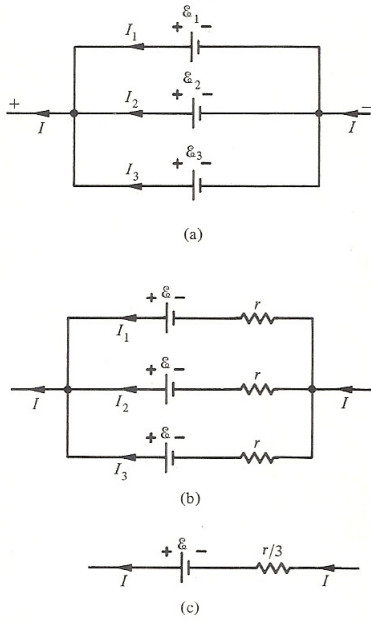

(a)

(b)

(c)

FIGURE 22.8. *Sources of emfs in parallel.*

the sum of the currents through each of the sources, that is, $I = I_1 + I_2 + I_3$ while $\mathcal{E} = \mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_3$. Thus, using this combination it is possible to increase the current without changing the emf. Also if each has an internal resistance r, the net resistance will be $r/3$, as shown in Figure 22.8(c).

EXAMPLE 22.2   For the network shown in the accompanying figure, calculate the total resistance and current I_1.
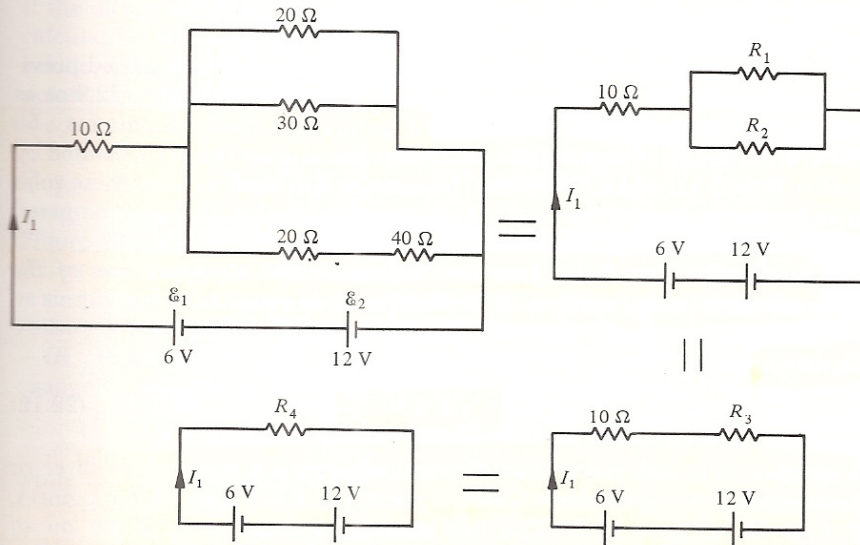
Ex. 22.2

Since the 20-Ω and 30-Ω are in parallel, equivalent resistance R_1 is:

$$\frac{1}{R_1} = \frac{1}{20} + \frac{1}{30} = \frac{5}{60} = \frac{1}{12} \quad \text{or} \quad R_1 = 12\ \Omega$$

The 40 Ω and 20 Ω are in series; therefore, their combined resistance R_2 is

$$R_2 = 40\ \Omega + 20\ \Omega = 60\ \Omega$$

Now R_1 and R_2 are in parallel. Therefore, the combined resistance R_3 is

$$\frac{1}{R_3} = \frac{1}{12} + \frac{1}{60} = \frac{6}{60} = \frac{1}{10} \quad \text{or} \quad R_3 = 10\ \Omega$$

R_3 and 10 Ω are in series. Hence, the combined resistance R_4 is

$$R_4 = R_3 + 10\ \Omega = 10\ \Omega + 10\ \Omega = 20\ \Omega$$

The combined voltage of the sources $\mathcal{E}_1$ and $\mathcal{E}_2$ is
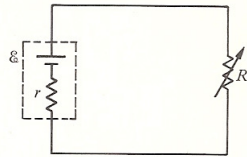
$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 = 6\ \text{V} + 12\ \text{V} = 18\ \text{V}$$

Thus, the whole circuit reduces to an equivalent circuit containing a net

resistance of 20 Ω and a source of 18 V. Thus, I_1 will be

$$I_1 = \frac{V}{R} = \frac{18\ \text{V}}{20\ \Omega} = 0.9\ \text{A}$$

PROBLEMS

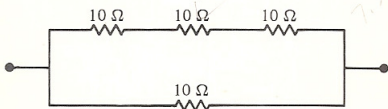1. Calculate the current through a single-loop circuit if $\mathcal{E} = 120$ V, $R = 1000\ \Omega$, and the internal resistance $r = 0.01\ \Omega$.

2. An external resistor $R = 100\ \Omega$ is connected to a 12-V battery having an internal resistance of $r = 0.1\ \Omega$. Calculate I, P_S, P_R, and P_r.

3. A typical 12-V automobile battery has a resistance of $0.015\ \Omega$. What is the terminal voltage of this battery when the starter draws a current of 150 A?

4. Consider a source of emf $\mathcal{E}$ with an internal resistance r connected to a variable resistor R as shown. (a) Calculate the power P supplied to R in terms of $\mathcal{E}$, r, and R. (b) Make a plot of P versus R for $R = \frac{1}{4}r, \frac{2}{4}r, \frac{3}{4}r$. From this graph find the value of R in terms of r for which P is maximum. Show that the maximum value is $\mathcal{E}^2/4r$.

5. An automobile battery of 12 V has an internal resistance of $0.04\ \Omega$ and when fully charged has available a charge in the amount of 150 A-h. To charge this battery it is connected to a 120-V dc source through a variable resistance R so as to limit the charging current to 10 A. (a) Show the connections for charging the battery; (b) the value of R; (c) the electrical power converted into chemical potential energy; and (d) the cost of charging the battery at the rate of 3 cents/kWh, neglecting the waste through R.
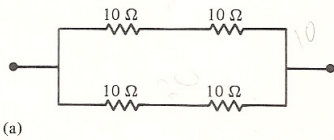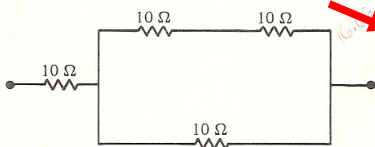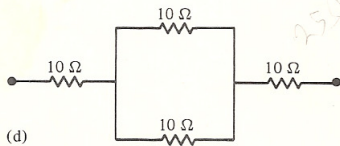
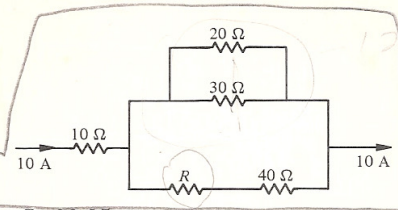6. A 12-V battery with an internal resistance of $0.008\ \Omega$ is short-circuited by connecting its terminals to a wire of very low resistance, say $2 \times 10^{-4}\ \Omega$. Calculate the current through the circuit. Suppose that the wire is made of copper and has a mass of 1 g. How much energy is dissipated every second in the Joule heating of this wire? How long will it take before the wire melts? (The melting point of copper is $1083°$C and its heat of fusion is 49.0 cal/g.)

7. Consider three 12-Ω resistors. What is the total resistance when these are connected (a) in series; and (b) in parallel?

8. Consider N resistors, each of $R\ \Omega$. Show that the total net resistance is NR if they are connected in series and R/N if connected in parallel.

9. How many 5-Ω resistors should be connected in parallel so that the total resistance will be $1\ \Omega$?

10. Show that when two or more resistors are connected in parallel, the total resistance is smaller than the smallest of all the resistors.

11. Four 10-Ω resistors are connected as shown. What is the net resistance in each case?

12. Calculate the net resistance between A and B in the accompanying figure.

P. 22.4

P. 22.11

P. 22.12

(a)
(b)
(c)
(d)
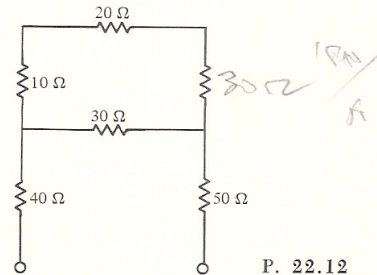
13. Two resistors of 15 Ω and 30 Ω are connected in parallel. What should be the value of R to be connected in series with the other two so that the net resistance will be 20 Ω?

14. Calculate the net resistance of a circuit if (a) a 10-Ω resistor is connected in series with a parallel combination of two resistors, each 15 Ω. (b) A 10-Ω resistor is connected in parallel with a series combination of two resistors, each 15 Ω.

15. What should be the value of R so that the total resistance of the network shown is $2R$?

16. Four cells each of 1.5 V and having an internal resistance of 0.01 Ω are connected (a) in series; and (b) in parallel. Calculate the current in each case.

17. Each cell of a storage battery has an internal resistance of 0.2 Ω and an emf of 2 V. Four of these are connected in (a) series; and (b) in parallel with a 10-Ω resistor. Calculate the current and the potential difference across the resistor in each case.

18. A 100-Ω resistor, when connected to the terminals of a battery, results in a current of 1.5 A. If the resistor is replaced by 150 Ω, the current is 1 A. Calculate the emf and the internal resistance of the battery.
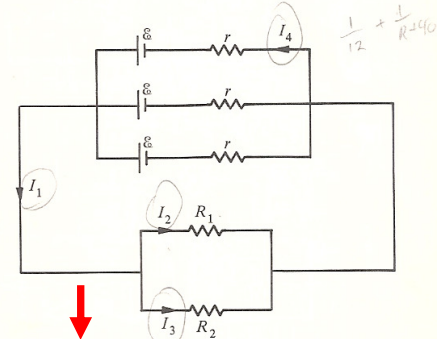
19. Three resistors, 20 Ω, 30 Ω, and 60 Ω, are connected in parallel. If the current through the 30-Ω resistor is 2 A, calculate the current through the other two resistors.

20. In the arrangement shown, $\mathcal{E} = 1.5$ V, $r = 0.01\ \Omega$, $R_1 = 10\ \Omega$, and $R_2 = 20\ \Omega$. Calculate the currents I_1, I_2, I_3, and I_4.
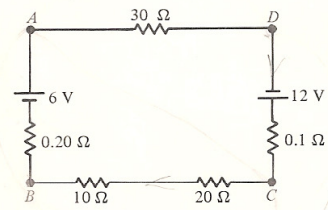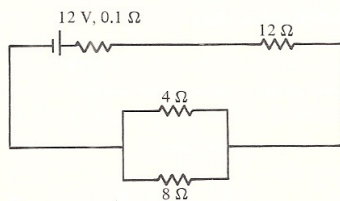
21. In the circuit shown, calculate the potential difference (a) across AB; (b) across AC; (c) across AD; and (d) across BD.

22. A 12-V battery has an internal resistance of 0.1 Ω. A 1000-Ω resistor and another 20,000-Ω resistor are connected in series with the battery. What is the current through the circuit? Suppose that a voltmeter of $5 \times 10^4\ \Omega$ is connected across the 1000-Ω resistor. Calculate the current through this resistor and the voltmeter.

23. Calculate the combined resistance of two bulbs 50 W (120 V) and 100 W (120 V) when they are connected (a) in series; and (b) in parallel.

24. Two 60-W bulbs are connected with a 120-V line source (a) in series; and (b) in parallel. Calculate the total power used in each case.

25. The total power consumed by two equal resistors connected in series is 100 W. Calculate the power consumed if these are connected in parallel.

26. Consider the arrangement shown. Calculate (a) the current through each resistor; (b) the power lost as heat in each resistor; and (c) the power lost by the source. Does this agree with the sum in part (b)?

P. 22.15

P. 22.20

P. 22.21

P. 22.26

8