# 21
Current Electricity

Now we pursue the study of charges in motion—current electricity. Starting with the sources that produce current, we shall look into the characteristics of the materials that affect the current through them. After getting familiar with the terminology, we investigate different relations between potential difference, current, resistance, electric field, and so on, leading to the statement of Ohm's law. We shall calculate the energy and power loss by moving charges in simple circuits.

21.1. Sources of Electromotive Force

A CONTINUOUS flow of charges across any cross-sectional area of a conductor is referred to as *current electricity*. If the average motion of the charges is always in the same direction, it is referred to as *direct current*. Consider two parallel plates A and B, A being positively charged while B is negatively charged, as shown in Figure 21.1. Thus, A is at a higher potential than B. Suppose that the two plates are connected by means of a
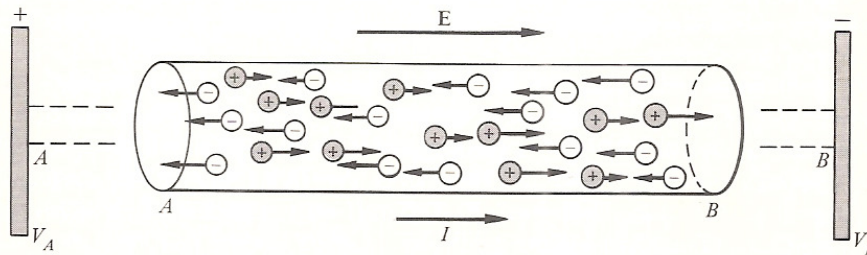
FIGURE 21.1. *Flow of charge carriers in a material with electric field E. Direction of current I is that of E.*

conductor, say a copper wire. An electric field is established from A to B inside the conductor, but this field does not last long. The electrons from plate B start flowing through the conductor to plate A. In a very short time, enough charge is transferred so that the two plates are at the same potential and the electric field in the copper wire becomes zero. There is no more flow of charges. During the short time interval the charges are flowing, a *transient current* is said to exist.

It is quite clear from the example above that in order to maintain a steady direct current, the potential difference between the two plates must be maintained at a constant value. This can be done only if the two plates are connected to some device such as a dry-cell battery or an electric generator which converts some other form of energy into electrical energy, thereby maintaining the required potential difference. This is done by transporting positive charges from a lower potential to the higher potential. Such a device, which converts nonelectrical energy into electrical energy, is called a source of

electromotive force (emf)—a totally misleading name because the source refers to energy and not to force. It should have been called *electromotive energy* (eme). Since emf is in common use, we will stick with it and represent the emf of a source by $\mathcal{E}$. We define a *source emf* $\mathcal{E}$ equal to the work done in carrying 1 coulomb of charge through the source. Suppose that q coulombs require an amount of work W joules; then $\mathcal{E}$ in volts in given by

$$\mathcal{E} = \frac{W}{q}$$ (21.1)

Thus, if $\mathcal{E} = 1.5$ V, it means that 1.5 J of work must be done to move a charge of 1 coulomb through the source.

There are many sources of emf. A few examples are: (1) batteries or cells convert chemical energy into electrical energy, (2) electric generators convert mechanical energy into electrical energy, and (3) thermocouples convert heat energy into electrical energy. Some of these sources will be discussed in this text while at present we confine ourselves to explaining how chemical energy is converted into electrical energy. It may be pointed out that it was the Italian physicist Alessandro Volta (1745–1827) who discovered this process of energy conversion, in 1800.

A simple chemical battery consists of two dissimilar metal rods A and B immersed in a dilute acid solution S, as shown in Figure 21.2(a). Let us concentrate on rod A. Like most metals, rod A dissolves in acid. When this happens, each atom leaves an electron on the rod while the atom itself enters the solution as a positive ion. As dissolving continues, more and more electrons are left on rod A, and the positive ions go into solution. Eventually, rod A becomes so negative that as many positive ions go into solution as are attracted back by the rod. We can say that *equilibrium* has been reached. Rod A, being negative, is at a lower potential with respect to the solution. Similar things happen to rod B. When equilibrium is reached, rod B is not only at a lower potential with respect to the solution S, but also with respect to rod A (because rod B dissolves faster in acid and has even more negative charge than rod A) as shown in Figure 21.3. Thus, there exists a potential difference between rods A and B, with A being at a higher potential than B. That is, A is a positive terminal while B is a negative terminal, as illustrated in Figure 21.3.

Outside the solution the electric field is from A to B. If terminals A and B are connected by a wire [Figure 21.2(b)], the positive charges will flow from A (the higher potential) to B (the lower potential) while the electrons will flow from B to A. Since in metals, the positive ions are fixed, only the electrons flowing from B to A contribute to the electric current. At the same time, as the electrons leave B, the rod becomes less negative. Some of the positive ions leave rod B and go into the solution. The arrival of electrons at rod A through the wire helps attract the positive ions in the solution which get deposited on rod A. Thus, the metal rod B keeps on dissolving while rod A, because of the positive ions depositing on it, gains mass. If rod A gets completely covered with the ions coming from rod B, A and B will be at the same potential; hence, there will be no current between them. Thus, rod A must be cleaned occasionally.

A common battery of this type uses Zn and Pb plates in a sulfuric acid solution, although any two different metal plates should work. A dry-cell
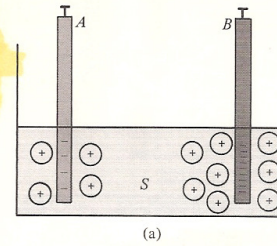
(a)
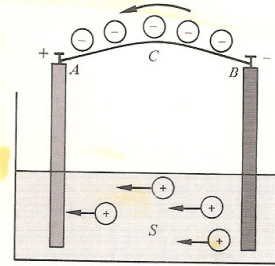
(b)

FIGURE 21.2. *Two dissimilar metal rods immersed in a dilute acid solution (a) before connecting A and B and (b) after connecting A and B by a conductor C.*
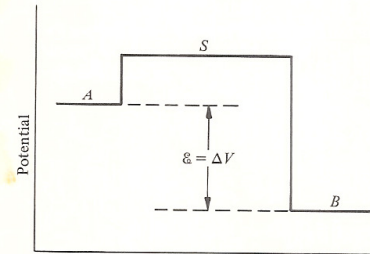

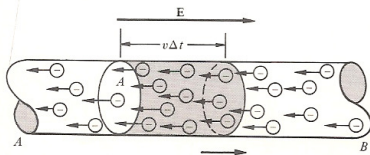**FIGURE 21.3.** *Potential of metal rods A and B with respect to the potential of solution S.*

battery consists of a carbon rod and a zinc can separated by a thick moist paste of ammonium chloride and manganese dioxide. Commercial batteries are much more involved and we shall not discuss them here.

21.2. Electric Current and Current Density

Let us consider a portion AB of a material connected to a voltage source as shown in Figure 21.1. An electric field $\mathbf{E}$ will be established inside the material. If there are free charges inside the material, the positive charges will drift from left to right—that is, in the direction of $\mathbf{E}$, while the negative charges will drift from right to left, that is, in the direction opposite $\mathbf{E}$.

If the material AB is a metallic conductor, the charge carriers are free electrons, as shown in Figure. 21.4; hence, the current is due to these drifting electrons. On the other hand, if the material AB is an electrolyte such as a salt–water solution, there are no free electrons, but the charge carriers are both positive and negative ions, drifting in opposite directions. If the material AB is an ionized gas, such as in a mercury vapor lamp, the positive charge carriers are positive ions while the negative charge carriers are free electrons. Note that if the material AB is an insulator, there will be no flow of charges; hence no current.

There is another class of materials, called semiconductors. The charge transfer in semiconductors is described by means of electrons as well as the motion of holes which behave almost like positive charges. We shall discuss this in Chapter 33.

In Figure 21.1 the positive charges moving to the right increase the positive charges on the right side of the conductor. The effect of the electrons moving to the left will be to decrease the negative charge on the right side of the conductor, or equivalently to increase the positive charge on the right. Thus, for all practical purposes, as far as current is concerned, a positive charge moving in one direction is equivalent to a negative charge moving in the opposite direction.

In order to define current quantitatively, we must state its magnitude and its sense of direction. *The conventional current direction is that in which the positive charges will drift, that is, in the direction of* $\mathbf{E}$. Thus, in Figure 21.1 or 21.4, even though the negative charges move to the left, the direction of current is to the right. *The magnitude of the current across any area is defined as the net charge flowing across the area per unit time.* Thus, if a net charge ΔQ flows across an area in a time interval Δt, the current I across the area is

$$I = \frac{\Delta Q}{\Delta t} \qquad (21.2)$$

where I has a sense of direction but is not a vector quantity. It is a scalar, as is clear from Equation 21.2. The SI unit of current is 1 *ampere* (1 A). It is defined as that current in which the charges flow across any cross section at the rate of 1 C/s. The definition of ampere in terms of magnetic forces will be given in Chapter 24. This unit was named in honor of the French scientist André Marie Ampère (1775–1836), whose contributions to the field of electricity and magnetism will also be discussed in Chapter 24. One ampere is a large unit; hence, smaller units that are commonly used are

FIGURE 21.4. *Charge carriers in a metallic conductor under the influence of* **E**.

drift velocity ~ 10^{-4} m/s

$$1 \text{ milliampere} = 1 \text{ mA} = 10^{-3} \text{ A}$$
$$1 \text{ microampere} = 1\mu\text{A} = 10^{-6} \text{ A}$$

The instrument used for detecting very small currents is called a *galvanometer* or, in its modified form, an *ammeter*.

Let us now express current I in terms of quantities such as drift velocity and the charge on the charge carrier. For simplicity let us consider a metallic conductor in which the free electrons will be the charge carriers, as shown in Figure 21.4. These electrons are ordinarily in random thermal motion. Typical speeds of electrons in metals are $\sim 10^{-4}$ m/s. The motion is so random that in a given time as many electrons cross any cross-sectional area to the left as to the right; hence, there is no net current. These electrons constantly collide with the atoms (or ions) of the metal and change direction as shown in Figure 21.5 by the continuous line. However, when an electric field is applied from one end of the conductor to the other, there will be a drift velocity superimposed on these electrons in the direction opposite **E**. Thus, the electrons' motion will be slightly biased toward the left, and they follow the dashed path shown in Figure 21.5(b). Note that, owing to random motion, in seven collisions the electron in the absence of an electric field moves from position 1 to X, while the effect of the electric field has been to move the electron to its final position, X'. Thus, the drift distance is only XX'. This explains why the drift velocity v is of the order of a fraction of a centimeter per second. It is this drift velocity that is used in calculating current.

Suppose that there are n free electrons per unit volume, each with a charge q and drift velocity v. In time Δt each electron will travel a distance $v\,\Delta t$. Thus, all the electrons within the shaded cylinder of length $L = v\,\Delta t$, as shown in Figure 21.6, will flow across the cross-sectional area A of the cylinder in time Δt. Since the volume of such a cylinder is $v\,\Delta t\,A$, the total number of negative charge carriers will be $nv\,\Delta t\,A$, each having a charge q. Thus, the charge ΔQ crossing the cross section A in time Δt will be $\Delta Q = qnv\,\Delta t\,A$, or the current I will be

$$I = \frac{\Delta Q}{\Delta t} = nqvA$$

In general, if the conductor contains different types of charge carriers both positive and negative with different charge densities and moving with different drift velocities, the current I is given by

$$I = n_1 q_1 v_1 A + n_2 q_2 v_2 A + n_3 q_3 v_3 A + \cdots$$

or

$$I = A \sum_i n_i q_i v_i \qquad (21.3a)$$

For convenience we shall drop the summation sign; it will be used when needed in a particular situation. We write Equation 21.3a as

$$I = nqvA \qquad (21.3b)$$

Thus, I depends upon A—a geometrical factor. A quantity independent of A is called *current density J* and defined as the current per unit cross-sectional area; that is,
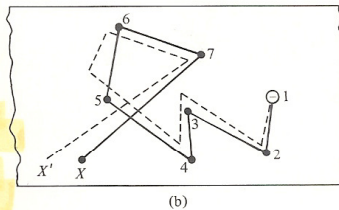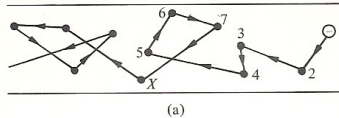

(a)

E

(b)

FIGURE 21.5. *Random motion of an electron in a conductor (a) when there is no electric field and (b) a portion of part (a) in the presence of an electric field results in the dashed-line path.*
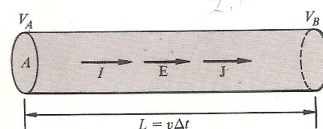

$L = v\Delta t$

FIGURE 21.6. *Current in a conductor of length L and cross-sectional area A.*

$$J = \frac{I}{A} = nqv \qquad (21.4)$$

Actually, current density is a vector quantity. The current density vector **J** is defined as

$$\mathbf{J} = nq\mathbf{v} \qquad (21.5)$$

For positive charge carriers the direction of **v** is the same as that of **E**. Thus, the direction of **J** is the same as that of **E** for positive charge carriers. Suppose that the charge carriers are negative, say electrons. The direction of **v** will be opposite to **E**. Since q is negative, the direction of **J** will be opposite to that of **v**; that is, **J** has the same direction as **E**. *Thus, whether the charge carriers are positive or negative, the current density vector **J** is always parallel to **E**.* Also, the direction of motion of positive charge carriers is parallel to **J**, while those of negative charge carriers is parallel to $-\mathbf{J}$.

EXAMPLE 21.1   Consider a copper wire of 3 mm² cross-sectional area in which there is a current of 6 A. Calculate (a) the number of free electrons per m³; (b) the total charge that crosses any cross-sectional area in 1 h; (c) the current density; and (d) the drift velocity.

(a) Copper has a density $\rho = 8.93 \times 10^3$ kg/m³, an atomic mass $A = 63.54$ u, while Avogadro's number is $N_A = 6.022 \times 10^{26}$/kmol. Thus, the number of electrons per unit volume (since there is one free electron associated with each atom) is

$$n = \rho \frac{N_A}{A} = 8.93 \times 10^3 \text{ kg/m}^3 \frac{6.022 \times 10^{26}/\text{kmol}}{63.54 \text{ kg/kmol}} = 0.85 \times 10^{29}/\text{m}^3$$

(b) $Q = It = 6 \text{ A} \times 1 \text{ h} = 6 \text{ C/s} \times 3600 \text{ s} = 2.16 \times 10^4 \text{ C}$

(c) From Equation 21.4,

$$J = \frac{I}{A} = \frac{6 \text{ A}}{3 \text{ mm}^2} = \frac{6 \text{ A}}{3 \times 10^{-6} \text{ m}^2} = 2 \times 10^6 \text{ A/m}^2$$

(d) From Equation 21.4, $J = nqv$. Since $q = e$.

$$v = \frac{J}{nq} = \frac{2 \times 10^6 \text{ A/m}^2}{(0.85 \times 10^{29}/\text{m}^3)(1.6 \times 10^{-19} \text{ C})} = 1.47 \times 10^{-4} \text{ m/s}$$

Note how small this drift velocity is.

EXERCISE 21.1   Repeat Example 21.1 for the case of an aluminum wire of the same cross-sectional area. Which of the two wires, Cu or Al, would you prefer for electrical conduction? The atomic mass of Al is 26.98 kg/mol and its density is 2700 kg/m³. [*Ans.:* (a) $n = 6.026 \times 10^{28}$ electrons/m³; (b) $Q = 2.16 \times 10^4$ C; (c) $J = 2 \times 10^6$ A/m²; (d) $v = 2.08 \times 10^{-4}$ m/s. Cu is preferred because of the greater density of free electrons.]

21.3. Resistivity and Resistance

When an object falls vertically through a viscous medium, in spite of the fact that there is a gravitational force that accelerates the object, because of the resistance offered by the medium the object reaches a terminal velocity and subsequently moves without any acceleration. Similarly, in conductors when an electric field $\mathbf{E}$ is applied, there is a net force $q\mathbf{E}$ on the charge, hence an acceleration. But over a very short distance, because of the resistance of the medium to the flow of the charges (as a result of the collisions) the charges achieve a terminal velocity, that is, the drift velocity we have referred to in the previous section. In this section we investigate how the flow of charge is related to the characteristics of the conductor and the applied field $\mathbf{E}$.

The current density $\mathbf{J}$ depends upon the value of $\mathbf{E}$ and the type of conductor. We define the *resistivity* ρ of a conductor as the ratio of the electric intensity to the current density. Thus,

$$\rho = \frac{E}{J} \tag{21.6}$$

In vector form we may write

$$\boxed{\mathbf{J} = \frac{\mathbf{E}}{\rho} = \sigma \mathbf{E}} \tag{21.7}$$

[This is Ohm's law, in general form, if σ is constant (see Section 21.4).] That is,

$$\sigma = \frac{1}{\rho} \tag{21.8}$$

where σ is called the *conductivity* of the material and is equal to the reciprocal of the resistivity ρ. Thus, from Equation 21.7, the greater the resistivity of a material, the larger the value of the electric intensity $\mathbf{E}$ needed to produce a given current density, and vice versa. Thus, a perfect insulator will have infinite resistivity (or zero conductivity). The resistivities of different material at 20°C are listed in Table 21.1. Note the change in ρ from insulators to metals by a factor of the order of 10^{22}. Also, the resistivities of the semiconductors are between those of the conductors and insulators.

To calculate ρ from Equation 21.6, we must know the values of E and J. But the values of E and J are not directly measurable. Thus, we must express ρ in some other form. Let us consider a cylindrical conducting wire of length L and cross-sectional area A. When a potential difference of $V = V_A - V_B$ is established between the two ends, it results in a field $\mathbf{E}$ inside the wire and establishes a current I as shown in Figure 21.6. From the definition of E and J,

$$E = \frac{V}{L} \quad \text{and} \quad J = \frac{I}{A}$$

Substituting these in Equation 21.6 and rearranging,

$$\rho = \frac{E}{J} = \frac{V/L}{I/A}$$

TABLE 21.1
Resistivity of Different Materials at 20°C and their
Mean Temperature Coefficient

Material	ρ (Ω-m)	$\bar{\alpha}$ (per C°)
Conductors		
Aluminum	2.8×10^{-8}	0.0039
Brass	7×10^{-8}	0.0020
Constantan	49×10^{-8}	0.00002
Copper	1.79×10^{-8}	0.0039
Gold	2.27×10^{-8}	0.0034
Iron	11×10^{-8}	0.0052
Manganin	44×10^{-8}	0.0000
Mercury	96×10^{-8}	0.00089
Nichrome	100×10^{-8}	0.0004
Platinum	11×10^{-8}	0.0039
Silver	1.6×10^{-8}	0.0038
Tungsten	5.6×10^{-8}	0.0045
Semiconductors		
Carbon	3.5×10^{-5}	-0.0005
Germanium	0.60	-0.048
Silicon	2300	-0.075
Insulators		
Amber	5×10^{14}	
Glass	$10^{10}-10^{14}$	
Lucite	$>10^{13}$	
Mica	$10^{11}-10^{15}$	
Quartz	7.5×10^{16}	
Wood	$>10^{13}$	

or

$$\frac{V}{I} = \rho \frac{L}{A} \tag{21.9}$$

In this particular situation ρ, A, and L are all fixed; hence, the ratio V/I will be constant. The ratio of the potential difference V to the current I is called the *resistance* R. That is,

$$\boxed{R = \frac{V}{I} = \rho \frac{L}{A}} \tag{21.10}$$

The quantities V, I, L, and A are directly measurable; hence, R, as well as ρ, may be easily calculated.

From Equation 21.10, if V is measured in volts and current I in amperes, the resistance is given in ohms (Ω). That is,

$$1 \text{ ohm} = \frac{1 \text{ volt}}{1 \text{ ampere}} \tag{21.11}$$

EXAMPLE 21.2 Calculate the resistance of a wire 20 m long that has a radius of 2 mm and resistivity of 5.5×10^{-8} Ω-m. If the two ends of the wire are connected to a source of 24 V, what are the values of I, J, and E?

The cross-sectional area of the wire is $A = \pi r^2 = \pi(2 \times 10^{-3}\text{ m})^2 = 4\pi \times 10^{-6}$ m². Substituting the values of ρ, L, and A in Equations 21.10, we get

$$R = \rho \frac{L}{A} = 5.5 \times 10^{-8}\text{ Ω-m} \frac{20\text{ m}}{4\pi \times 10^{-6}\text{ m}^2} = 8.75 \times 10^{-2}\text{ Ω}$$

From Equation 21.10, $R = V/I$ or

$$I = \frac{V}{R} = \frac{24\text{ V}}{8.75 \times 10^{-2}\text{ Ω}} = 274\text{ A}$$

Therefore,

$$J = \frac{I}{A} = \frac{274\text{ A}}{4\pi \times 10^{-6}\text{ m}^2} = 21.8 \times 10^6\text{ A/m}^2$$

while from Equation 21.6,

$$E = \rho J = 5.5 \times 10^{-8}\text{ Ω-m} \times 21.8 \times 10^6\text{ A/m}^2 = 1.2\text{ V/m}$$

EXERCISE 21.2 To be safe we do not want the current in the wire used in Example 21.2 to exceed 100 A, while keeping the length the same but changing the cross-sectional area. For this case calculate R, A, J, and E. [*Ans.:* $R = 0.24$ Ω, $A = 4.58 \times 10^{-6}$ m², $J = 2.18 \times 10^7$ A/m², $E = 1.2$ V/m.]

21.5. Energy and Power in Electric Circuits

Energy and Power

Consider a source of emf $\mathcal{E}$ connected to a load AB through conducting wires of negligible resistance, as shown in Figure 21.13. There is a potential difference $V_{AB} = V_B - V_A$ across the load and a steady current I in the circuit directed from A to B. The load in the shaded box may be a resistor, a storage battery, a motor, and so on.

Suppose that in a time interval Δt a quantity of positive charge $\Delta q = I\,\Delta t$ enters the terminal A while the same charge leaves the terminal B. Thus, the charge Δq passing through the load experiences a change of *potential energy* ΔU given by

$$\Delta U = V_B\,\Delta q - V_A\,\Delta q = (V_B - V_A)\,\Delta q$$

or

$$\Delta U = V_{AB}\,\Delta q = V_{AB}I\,\Delta t \tag{21.16}$$

If the potential V_A at A is greater than V_B at B, the charge Δq will lose potential energy in going through AB. Thus, there is an *energy input* by the charge to the portion AB of the circuit. On the other hand, if V_A is less than V_B, the charge passing through AB will gain potential energy. This implies that there will be an *energy output* from this portion AB of the circuit.

The rate of energy transfer between the circulating charge and the circuit is called the power, P. Thus, from Equation 21.16,

$$\boxed{P = \frac{\Delta U}{\Delta t} = V_{AB}I} \tag{21.17}$$

which is a general expression for the electrical power input or output to or from the load between A and B. In Equation 21.17, the unit of V is the volt and the unit of I is the ampere, which yield the unit of power to be the joule/s or watt, as shown below.

$$(\text{volt})(\text{ampere}) = \left(\frac{\text{joule}}{\text{coulomb}}\right)\left(\frac{\text{coulomb}}{\text{second}}\right) = \left(\frac{\text{joule}}{\text{second}}\right) = \text{watt}$$

If the portion of the circuit AB contains a motor, the potential energy appears in the form of mechanical energy, while if it is a storage battery that is being charged (for this purpose the positive terminal of the storage battery will be connected to the positive terminal of the source), the potential energy will be used as stored chemical energy in the battery. On the other hand, if the circuit AB is a resistor, the potential energy appears as heat energy in the resistor, as discussed below.
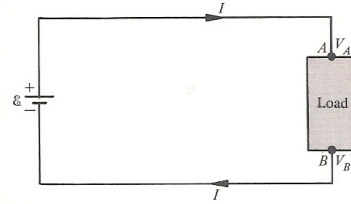

FIGURE 21.13. *Load connected to a source of emf.*

9

Joule's Law of Heating

Let us assume that the load AB in Figure 21.11 is purely resistive. In this case the circulating charge loses potential energy in going from A to B; hence, there will be an energy input to the resistor. The potential energy lost by the circulating charges appear as thermal energy in the resistor. The temperature of the resistor rises until the heat flowing out of the resistor equals the energy input to the circuit. Thus, we say that the potential energy is being dissipated in the resistor, and the process is called *Joule heating*.

We can explain Joule heating from the microscopic point of view by considering the motion of electrons in a metallic resistor. After the initial acceleration the electrons colliding with the atoms settle down to a steady drift velocity. Any further loss in potential energy does not appear as the increased kinetic energy of the electrons. In collisions between electrons and atoms, this energy is transferred to the atoms of the crystal, thereby increasing the amplitude of thermal vibrations of the crystal lattice. This leads to an increase in temperature.

If the circuit is an ohmic circuit element, $V_{AB} = IR$; hence, the power dissipated P_R in a resistor according to Equation 21.17 takes the form

$$P_R = I^2R \qquad (21.18)$$

which is known as *Joule's law*.

Unless it is our intention to convert electrical energy into heat energy, the power loss as given by Equation 21.18 is undesirable. This power loss is called the I^2R *loss* or the *Joule heating loss*. Since every electrical circuit contains resistance, there will always be some I^2R loss. This loss becomes quite significant when electricity is transferred from a generating station to homes. To keep this loss to a minimum, besides using a wire of very low resistance, it is necessary and desirable to use very low current I. But to keep the power supplied $P = VI$ constant, if I is low, V must be high. This means that from a generating station, the electric power is transmitted at low current and high voltage. Because high voltage is dangerous, for safety reasons when this electric power is received in homes and businesses it is changed into high current and low voltage. Change from high current and low voltage into low current and high voltage is accomplished by means of a step-up transformer; the reverse is accomplished by means of a step-down transformer. We shall discuss these transformers in Chapter 26.

Source Power Conversion

The purpose of a source of emf is to convert nonelectrical energy into electrical potential energy. The rate at which this energy is converted is the power supplied by the source, and may be calculated as shown below.

Suppose in time Δt the charge Δq is transported within the source from the low-potential terminal to the high-potential terminal. If $\mathcal{E}$ is the emf, the work done ΔW by the nonelectrical force will be

$$\Delta W = \mathcal{E} \, \Delta q$$

Then the rate at which the source does work, called power P_S, is given by

$$P_S = \frac{\Delta W}{\Delta t} = \mathcal{E}\frac{\Delta q}{\Delta t}$$

Since $\Delta q/\Delta t = I$,

$$\boxed{P_S = \mathcal{E}I} \qquad\qquad (21.19)$$

If the source is a storage battery or a dry cell, the chemical energy is converted into electrical potential energy according to Equation 21.19. On the other hand, if the source is a generator or a dynamo, it is the mechanical energy that is converted into electrical potential energy.

Normally, the conventional current direction within the source is from a low potential to a high potential. If the current in the source is made to go in the backward direction, that is, from the high potential terminal to the low potential terminal by connecting the source to another source of higher emf, the electrical potential energy will be converted into nonelectrical energy. Such is the case when the storage battery is being charged so that the electrical potential energy will be converted into chemical potential energy. Similarly, a dynamo working backward is an electric motor, because electrical potential energy is being converted into mechanical energy.

EXAMPLE 21.4   Consider the circuit shown in the accompanying figure, consisting of a source of emf $\mathcal{E}$ with an internal resistance $r = 0.1\,\Omega$ and an electric appliance L with resistance R that operates at 300 W. The voltage V is 110 V while the current through the circuit is 5 A. Calculate the following quantities: (a) the power output from the source; (b) the power used in the joule heating of R; (c) the magnitude of R; (d) the power loss in the source itself; and (e) the emf of the source.
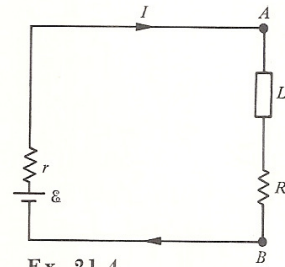
Ex. 21.4

(a) The power output from the source which is delivered to R and L is

$$P = V_{AB}I = 110\text{ V} \times 5\text{ A} = 550\text{ W}$$

(b) Of the 550 W, 300 W ($= P_A$) is used by L. Therefore, P_R power loss in Joule heating of R is

$$P_R = P - P_A = 550\text{ W} - 300\text{ W} = 250\text{ W}$$

(c) From the relation $P_R = I^2R$,

$$R = \frac{P_R}{I^2} = \frac{250\text{ W}}{(5\text{ A})^2} = 10\,\Omega$$

(d) Joule heating loss in the internal resistance r of the source is

$$P_r = I^2r = (5\text{ A})^2(0.1\,\Omega) = 2.5\text{ W}$$

(e) Thus, the total power provided by the source, or rate at which the source does work, is

$$P_S = P_A + P_R + P_r = 300\text{ W} + 250\text{ W} + 2.5\text{ W} = 552.5\text{ W}$$

From the relation $P_S = \mathcal{E}I$, the emf of the source is

$$\mathcal{E} = \frac{P_S}{I} = \frac{552.5 \text{ W}}{5 \text{ A}} = 110.5 \text{ V}$$

EXERCISE 21.4 Suppose in Example 21.4 that $V_{AB} = 100$ V, $R = 8\,\Omega$, $I = 6$ A, and $r = 0.2\,\Omega$. Calculate the power used by the appliance and its resistance, P_r, P_S, and $\mathcal{E}$. [*Ans.:* $P_A = 312$ W, $P_R = 288$ W, $P_r = 7.2$ W, $P_S = 607.2$ W, $\mathcal{E} = 101.2$ V.]

SUMMARY

The source emf $\mathcal{E}$ is equal to the work done in carrying 1 C of charge through the source; $\mathcal{E} = W/q$. Current is the net charge flowing across the area per unit time, $I = \Delta Q/\Delta t$. The direction of current is that in which the positive charge will drift, that is, in the direction of **E**. The unit of current is the *ampere*. The *current density* **J** is defined as the current per unit cross-sectional area, $J = I/A = nqv$.

The *resistivity* ρ of a conductor is the ratio of the electric intensity to the current density. Thus, $\rho = E/J$ and $J = E/\rho = \sigma E$, where $\sigma = 1/\rho$ is called the *conductivity* of the material. The ratio of the potential difference V to the current I is called the *resistance R*. That is, $R = V/I = \rho L/A$. The unit of resistance is the *ohm* (Ω), while the unit of resistivity ρ is in *ohm-m*.

The variation of ρ with temperature T is given by $\rho = \rho_0[1 + \bar{\alpha}(T - T_0)]$ or $R = R_0[1 + \bar{\alpha}(T - T_0)]$, where $\bar{\alpha}$ is the *mean temperature coefficient of resistivity*. In the neighborhood of 4 K, many metals exhibit abnormally low resistivity. This is the *superconducting state*, and the property is called *superconductivity*. The *critical temperature* T_c is the temperature below which the material is superconducting.

The resistance of a metallic conductor is $R = V/I$, which is the statement of *Ohm's law*. Any resistor that obeys Ohm's law is a *linear circuit element*. For a nonlinear circuit element, the plot of V versus I is not linear.

The rate of energy transfer between the circulating charge and the circuit is called the *power, P*; $P = \Delta U/\Delta t = V_{AB}I$. The unit of power, J/s, is called the *watt*. The process of dissipation of potential energy in the resistor is called *Joule heating*, and $P_R = I^2 R$ is known as *Joule's law*. The rate at which the source does work called power is $P_S = \mathcal{E}I$.

QUESTIONS

1. When charge carriers reach the terminals of the source of an emf, what happens to their velocity or kinetic energy?
2. When a potential difference is maintained between two ends of an insulator, what effect has it on the charges in regard to the velocity, force, and energy?
3. How do you measure drift velocity and distinguish it from the random motion of the charge carriers?
4. In a gas where there are both positive and negative charge carriers, what are the relative directions of **v**, **J**, and the force **F** on the charge carries if the direction of **E** is from left to right?
5. Are collisions between the charge carriers and the atoms in a conductor elastic or inelastic? What types of collisions contribute to Joule heating?

6. Should the conductivity of a conductor decrease or increase with temperature? Explain.

7. Can you use a resistance thermometer in the vicinity of absolute zero? Explain.

8. Suppose that there is a current I in (a) a conductor; and (b) a semiconductor. What effect will an increase or decrease in temperature have on the current?

9. Why are thermistors used as automatic temperature-control switching devices?

10. Give an example of each of the following systems in which electrical energy is converted into (a) mechanical energy; (b) heat energy; and (c) chemical energy. Also, give an example in which (d) chemical energy is converted into electrical energy; and (e) mechanical energy is converted into electrical energy.

PROBLEMS

1. Calculate the emf of a source if the work done by the nonelectrical forces on a charge of 1.5 C is 30 J.

2. Considering the electrons as charge carriers in a battery of 12 V, calculate the change in potential energy in terms of joules as the electrons migrate from one pole to another.

3. If there are 10^{18} electrons flowing across any cross section of a wire in 1 min, what is the current in the wire?

4. There is a current of 5 mA in a wire. How much charge passes any cross section of the wire in 10 s? If this current is due to the flow of electrons, how many electrons pass through the wire in 10 s?

5. An automobile battery is rated at 120 ampere-hours, that is, 1 A for 120 h, 10 A for 12 h, and so on. If a starter in the automobile draws a current of 400 A, how long could the starter be kept on before the battery runs down? How much is the total charge that goes through the starter in this time?

6. An electron going around a proton in Bohr-type orbit makes 6.6×10^{15} revolutions per second. What is the current in such an orbit caused by the electron motion?

7. A gas between two electrodes is ionized by applying a very high potential difference between two electrodes. The gas ionizes and every minute 5×10^{20} electrons, and 2×10^{20} positive ions move in opposite directions. Calculate the current and its direction.

8. A 0.5-m-wide belt of a Van de Graaff machine runs at 20 m/s. How much charge in C/s must be sprayed on the belt to give a current of 5 mA?

9. A metal wire 1 mm in diameter contains 3×10^{28} electrons/m^3, and a charge of 100 C/h crosses any cross-sectional area. Calculate the current and current density in the wire and the drift velocity of the electrons.

10. Consider a copper wire of 1 mm diameter in which there is a current of 5 A. Calculate (a) the number of free electrons per m^3; (b) the total charge that crosses any cross-sectional area in 1 h; (c) the current density; and (d) the drift velocity.

11. Calculate the resistance of a wire 10 m long that has a diameter of 2 mm and resistivity of 2.63×10^{-8} Ω-m.

12. A copper wire in household wiring has a cross section of 0.32×10^{-6} m^2

(called 22-gauge wire). The resistivity of such a wire is 1.72×10^{-8} Ω-m. What is the resistance of such a wire that is 30 m long?

13. A wire has a resistance of 200 Ω. What will be the resistance of a wire of the same material if it is 2 times as long and its cross section is 2 times as large?

14. Consider a copper wire of length 30 m and cross-sectional area 2×10^{-6} m^2. The IR drop across this wire is 3 V. Calculate I, J, and E.

15. Calculate the resistance of a copper plate of length 15 cm and rectangular cross section of 3 cm $\times$ 0.5 cm.

16. A 22-gauge copper wire has a diameter of 0.64 mm. Calculate the resistance of such a wire 5 km long at 0°C.

17. What is the resistance of a wire made of (a) copper; (b) tungsten; (c) iron; and (d) constantan if each is 1 m long and has a diameter of 1 mm?

18. A 10-m-long wire of 1 mm diameter has a resistance of 10 Ω. What should be the length of a wire of the same material having a 2-mm diameter so that it has a resistance of 5 Ω.

19. A copper wire of length L and cross-sectional area A is to be replaced by an aluminum wire of the same length and resistance but of different cross-sectional area A_1. Calculate A_1 in terms of A.

20. Show that the relation $R = \rho L / A$ may also be written as $R = \rho d L^2 / m = \rho m / d A^2$, where $\rho, L, A, m,$ and d are the resistivity, length, cross-sectional area, mass, and density, respectively, of a wire.

21. A total of 50 g of copper is stretched into a wire 100 m long at 0°C. What is the resistance of the wire? (*Hint:* Find A from $m = dV$, $d =$ density.)

22. A wire of 50 Ω has a certain length. If it is stretched to 4 times its length while keeping its volume the same, what is the new resistance?

23. If 1 g of gold, 1 g of copper, and 1 g of aluminum are drawn into uniform wires of length 10 m each, calculate the resistance of each. What is the ratio of the resistance to the diameter in each case?

24. An electrical system operating at 12 V draws a current of 0.05 A. What is the resistance of the system?

25. Calculate the resistance of the heating element of a toaster if a current of 5 A passes through it when connected across 120 V.

26. A voltmeter reads 24 V across a resistor and an ammeter reads a current of 2 A through it. Calculate R. What will be the current through the resistor if the voltage changes to 12 V?

27. A wire carries a current of 100 A and has a resistance of 120 $\mu\Omega$/m. What is the potential difference across 1 m?

28. A copper wire at 0°C has a resistance of 10 Ω. (a) What will be the resistance when this wire is immersed in steam? (b) At what temperature will its resistance become double, that is, 20 Ω?

29. A certain wire has a resistance of 10 Ω at 0°C and 10.2 Ω at 200°C. Calculate the temperature coefficient of resistance. What is the value of this resistance at 70°C?

30. Calculate the resistance of a carbon rod at 100°C which has a resistance of 0.032 Ω at 0°C.

31. Consider a copper wire and a tungsten wire each 1 m long and of 1 mm diameter. Compare their resistances at 0°C and 100°C.

32. Calculate the resistance of a tungsten wire at 20°C which has a resistance of 10 Ω at 1000°C. (First calculate the resistance at 0°C.)

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33. The resistance of constantan wire at room temperature (20°C) is 100 Ω. What will be its resistance at (a) 0°C; (b) −100°C; (c) +100°C; and (d) 1000°C?

34. A heater is rated at 1200 W when the current is 5 A. Calculate (a) the resistance of the heater; and (b) the voltage across the heater.

35. Suppose that a current of 5 A is drawn from a battery to obtain 1200 J of work over a time interval of 1 min. Calculate (a) the available power; and (b) the voltage across the battery.

36. Calculate the current supplied by the 12-V battery of an automobile to a starter that develops a power of 2 kW (1 kW = 1.34 hp).

37. Calculate the cost of lighting a 100-W bulb every night for 6 h for 1 year at the rate of 3 cents/kWh.

38. The maximum energy a certain 100,000-Ω resistor can dissipate is 2 W; that is, the power rating is 2 W. What is the maximum voltage that can be applied to this resistor? What is the maximum voltage if the power rating is 5 W?

39. How much current is drawn from a 12-V automobile battery to operate a 1.0-hp (=746-W) starter? How much energy is used up if it takes 5 s to start the automobile?

40. A motor is designed to operate at 120 V and 5 A. If its efficiency is 50 percent, calculate the energy lost in cal/s.

41. A $\frac{1}{2}$-hp (1 hp = 746 W) motor draws a current of 4.00 A from a line source of 110 V. What is its efficiency?

42. The headlights of an automobile are left on by mistake. This results in a 12-V battery going dead in 4 h. If the two lamps have a total power of 400 W, what was the total chemical energy in the battery that was available for conversion into electrical energy?

43. A 720-W heater operates from a 120-V line voltage. Calculate (a) the resistance of the heater; and (b) the current drawn by the heater. What fluctuations in power take place if the line voltage fluctuates between 110 and 130 V?

44. A 2500-W heater has to be made out of a constantan wire that can carry a maximum current of 5 A. What is the length of this wire if its diameter is 0.8 mm?

45. The current through a hot-water heater is 20 A when connected to a voltage of 240 V. If the voltage drops to 110 V, what will be the current? How much longer will it take to heat the same amount of water through the same temperature interval at the lower voltage?

46. An electric iron draws 10 A of current from a 120-V line voltage. Half of the energy is radiated while the other half heats the iron. If the mass of the iron is 1.5 kg and its specific heat is 0.1 kcal/kg-C°, how long will it take to heat the iron from 20 to 110°C?

47. A typical 100-W lamp operates at 120-V line voltage. Calculate (a) the current through the filament; (b) the resistance of the filament; and (c) the number of electrons through the filament/second.

48. A commercial 12-V battery has a resistance of 0.01 Ω. It has to be charged by connecting it to a 120-V line voltage. Show the circuit diagram for this purpose. If it is charged for 6 h, drawing a current of 10 A, what is the cost of the electrical energy used at the rate of 3 cents/kWh?